3. Structure functions and quark distributions

Elastic electron-parton scattering:

Parton i, charge z_i, mass m_i, parton density $f_i(x)$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\cdot z_i^2 \cdot f_i(x)\left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_i^2}\sin^2\frac{\theta}{2}\right)$$

Elastic scattering

$$1 = \frac{Q^2}{2m_i\nu} \quad \text{and } x = \frac{Q^2}{2M\nu} \Rightarrow m_i = xM$$

Inelastic ep→eX scattering in parton model:

$$\frac{d\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\cdot \sum_i z_i^2 \cdot f_i(x)\left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2x^2}\sin^2\frac{\theta}{2}\right)$$

Compared to the macroscopic expression:

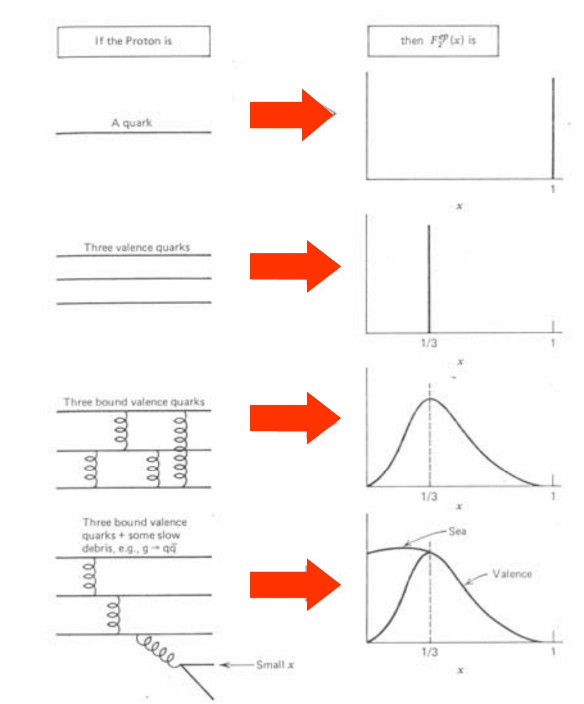
$$\frac{d\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\cdot \frac{F_2(x)}{x}\left(\cos^2\frac{\theta}{2} + \frac{2xF_1(x)}{F_2(x)}\frac{Q^2}{2M^2x^2}\sin^2\frac{\theta}{2}\right)$$

Structure functions

$$F_2(x) = x\cdot\sum_i z_i^2 f_i(x)$$

$$2xF_1(x) = F_2(x)$$

Proton model



$F_2(x)$

3.1 Parton Spin

Callan-Gross relation for spin ½ partons $\frac{2xF_1}{F_2} = 1$

⬇

Proton constituents are spin ½ partons

e.g.: for spin 0 partons, $\sin^2\theta/2$ term in cross section disappears: $F_1(x)=0$

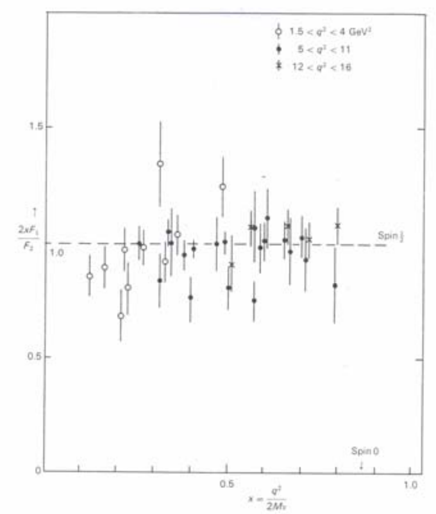
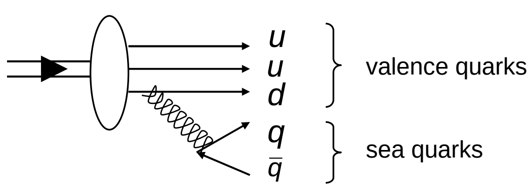


Fig. 7.19 The ratio $2xF_1/F_2$ measured in SLAC electron-nucleon scattering experiments. For spin-½ partons, with $g = 2$, a ratio of unity is expected in the limit of large q^2 – the Callan-Gross relation. (Data compiled from published SLAC data.)

3.2 Sea and valence quarks

Parton distribution function $f_i(x)$ = Probability to find parton with $x_i \in [x, x+dx]$

Partons = Quarks and Gluons



Quark composition of the proton

$$u_v + u_v + d_v + \underbrace{(u_s + \bar{u}_s) + (d_s + \bar{d}_s) + (s_s + \bar{s}_s)}$$

Sea: Heavy quark contribution strongly suppressed

$$\frac{F_2^{ep}(x)}{x} = \sum_i z_i^2 \cdot f_i(x)$$

$$= \frac{4}{9}(u^p(x) + \bar{u}^p(x)) + \frac{1}{9}(d^p(x) + \bar{d}^p(x)) + \frac{1}{9}(s^p(x) + \bar{s}^p(x))$$

u(x), d(x) (anti) quark densities of u and d

Quark composition of the neutron

$$\frac{F_2^{en}(x)}{x} = \sum_i z_i^2 \cdot f_i(x)$$

$$= \frac{4}{9}(u^n(x) + \bar{u}^n(x)) + \frac{1}{9}(d^n(x) + \bar{d}^n(x)) + \frac{1}{9}(s^n(x) + \bar{s}^n(x))$$

$$\frac{F_2^{en}(x)}{x} = \frac{4}{9}(d(x) + \bar{d}(x)) + \frac{1}{9}(u(x) + \bar{u}(x) + s(x) + \bar{s}(x))$$

Iso-spin symmetry

$$u^n(x) = d^p(x) = d(x)$$
$$d^n(x) = u^p(x) = u(x)$$
$$s^n(x) = s^p(x) = s(x)$$
$$\bar{q}^n(x) = \bar{q}^p(x) = \bar{q}(x)$$

In total 6 unknown quark distributions ⬅

Sum rules

$$q(x) = q_v(x) + q_s(x)$$
$$\bar{q}(x) = \qquad \bar{q}_s(x)$$

valence:
$$\int_0^1 u_v(x)dx = 2$$
$$\int_0^1 d_v(x)dx = 1$$

sea:
$$\int_0^1 (q_s(x) - \bar{q}_s(x))dx = 0$$

For F_2^p and F_2^n the following relation can be obtained (Nachtmann):

$$\frac{1}{4} < \frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{\frac{4}{9}(d + \bar{d}) + \frac{1}{9}(u + \bar{u} + s + \bar{s})}{\frac{4}{9}(u + \bar{u}) + \frac{1}{9}(d + \bar{d} + s + \bar{s})} < 4$$

(for all x)

The ratio F_2^n / F_2^p allows to probe the sea and valence quark contributions.

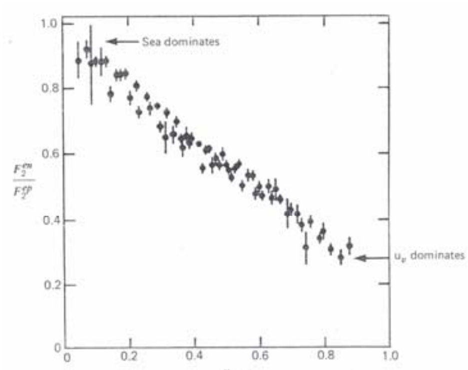


Fig. 9.6 The ratio F_2^{en}/F_2^{ep} as a function of x, measured in deep inelastic scattering. Data are from the Stanford Linear Accelerator.

For x→0: Sea quarks dominate

- Created by gluon splitting
- Gluon spectrum ~1/x

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \to 1$$

use $u_s \approx d_s \approx s_s \approx \bar{u}_s \approx \bar{d}_s \approx \bar{s}_s$

For x→0: Valence quarks dominate (no momentum left for sea)

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \to \frac{u_v + 4d_v}{4u_v + d_v} \geq \frac{1}{4}$$

Observation for protons:

$u_v >> d_v$ for x→ 1

Valence quarks

$$\frac{F_2^{ep}(x)}{x} = \frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}+s+\bar{s})$$
$$= \frac{1}{9}[4u_v + d_v] + \frac{4}{3}S$$
$$\frac{F_2^{en}(x)}{x} = \frac{4}{9}(d+\bar{d}) + \frac{1}{9}(u+\bar{u}+s+\bar{s})$$
$$= \frac{1}{9}[u_v + 4d_v] + \frac{4}{3}S$$

$$\frac{1}{x}\left[F_2^{ep}(x) - F_2^{en}(x)\right] = \frac{1}{3}[u_v(x) - d_v(x)]$$

As $u_v >> d_v$:

$u_v - d_v$ is a measure for valence quark distribution

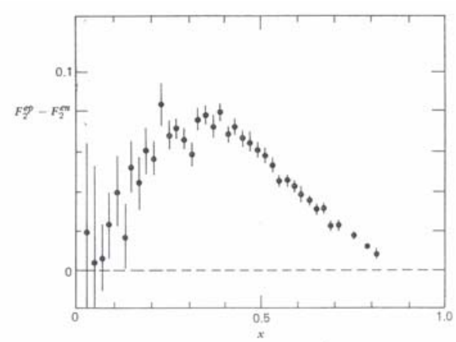


Fig. 9.8 The difference $F_2^{ep} - F_2^{en}$ as a function of x, as measured in deep inelastic scattering. Data are from the Stanford Linear Accelerator.

Valence quarks mostly at large x

Sum of quark momentum

Scattering at an iso-scalar target: #p = #n (e.g. C, Ca)

$$F_2^{eN} = \frac{1}{2}\left[F_2^{ep} + F_2^{en}\right] = \frac{5}{18} x \cdot [u+\bar{u}+d+\bar{d}] + \frac{1}{9} x \cdot [s+\bar{s}]$$
$$\approx \underbrace{\frac{5}{18} x \cdot [u+\bar{u}+d+\bar{d}]} = \frac{5}{18}[\text{Sum of all quark momenta}]$$

Small s quark distribution neglected

Naively one expects: $\frac{18}{5} \cdot \int_0^1 F_2^{eN}(x)dx \approx 1$

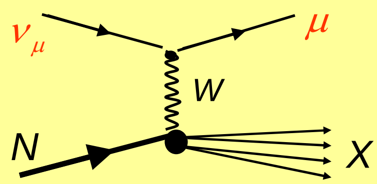
Experimental observation: $\frac{18}{5} \cdot \int_0^1 F_2^{eN}(x)dx \approx 0.5$

- probed quarks and anti-quarks carry only 50% of nucleon momentum
- Remaining momentum carried by gluons (see later)

3.3 Neutrino nucleon scattering

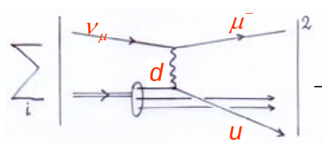
$$\nu_\mu N \rightarrow \mu^\pm X$$

- More information on quark distribution
- Separation between quarks / anti-quarks



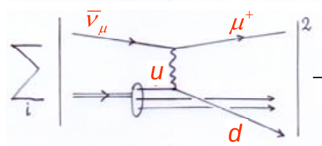
QPM: $x = \frac{Q^2}{2M\nu}$ $y = \frac{\nu}{E}$ $\nu = E - E'$

$$\frac{d\sigma(\nu_\mu N \rightarrow \mu^- X)}{dy} =$$



$\rightarrow F_i^{\nu p}(x)$ and $F_i^{\nu n}(x)$

$$\frac{d\sigma(\bar{\nu}_\mu N \rightarrow \mu^+ X)}{dy} =$$



$\rightarrow F_i^{\bar{\nu} p}(x)$ and $F_i^{\bar{\nu} n}(x)$

Structure functions for neutrino scattering

$$F_i^{\nu n} = F_i^{\bar{\nu} p}$$
$$F_i^{\nu p} = F_i^{\bar{\nu} n}$$

Equal because of Charge symmetry

$$F_i^{\nu N} = \frac{1}{2}(F_i^{\nu p} + F_i^{\nu n}) = \frac{1}{2}(F_i^{\bar{\nu} n} + F_i^{\bar{\nu} p}) = F_i^{\bar{\nu} N} \quad \text{for i = 1,2}$$

$$F_3^{\bar{\nu} N} = -F_3^{\nu N}$$

Additional structure function to account for parity violation

Double differential cross section: Scattering at iso-scalar target

$$\frac{d^2\sigma(\nu N, \bar{\nu} N)}{dxdy} = 2ME \frac{G_F^2}{2\pi} \left[(1-y)F_2^{\nu N}(x) + \frac{y^2}{2} 2xF_1^{\nu N}(x) \pm \underbrace{y(1-\frac{y}{2})xF_3^{\nu N}(x)}_{\text{to account for parity violation}} \right]$$

$$\frac{4\pi\alpha^2}{Q^4} \mapsto \frac{G_F^2}{2\pi}$$

to account for parity violation

Structure functions in QPM

$$F_1^{\nu N} = \frac{1}{2x} F_2^{\nu N}$$

$$F_2^{\nu p} = 2x[d + \overline{u}] \qquad xF_3^{\nu p} = 2x[d - \overline{u}]$$

$$F_2^{\nu n} = 2x[d^n + \overline{u}^n] \qquad xF_3^{\nu n} = 2x[d^n - \overline{u}^n]$$

$$= 2x[u + \overline{d}] \qquad = 2x[u - \overline{d}]$$

Iso-scalar target

$$F_2^{\nu N} = x[u + \overline{u} + d + \overline{d}] \qquad xF_3^{\nu N} = x[(u + d) - (\overline{u} + \overline{d})]$$

$$F_2^{\nu N} = x[Q(x) + \overline{Q}(x)] \qquad xF_3^{\nu N} = x[Q(x) - \overline{Q}(x)]$$

Measures sum of quarks and anti-quarks

Measures valence quarks

Measurement: $F_2^{\nu N} + xF_3^{\nu N} = 2xQ(x)$ ➡ Sea and valence quarks

$F_2^{\nu N} - xF_3^{\nu N} = 2x\overline{Q}(x)$ ➡ Sea quarks

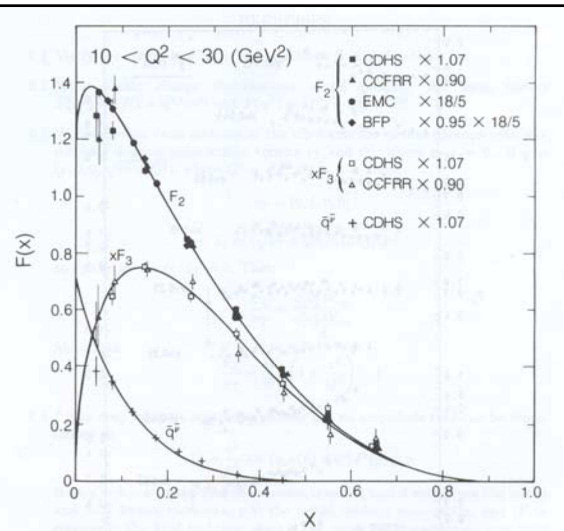


Figure 8.2: A compilation of data from neutrino and muon scattering experiments. The structure function F_2 is essentially proportional to the sum of the quark and antiquark distributions: $F_2(x) = x[q(x) + \overline{q}(x)]$. The structure function xF_3 is similarly related to the difference of the quark and anti-quark distributions: $xF_3(x) = x[q(x) - \overline{q}(x)]$. The third combination shown is $\overline{q}^{\overline{\nu}}(x) = x[\overline{u}(x) + \overline{d}(x) + 2\overline{s}(x)]$. The data shown are from the CDHS, CCFRR, EMC (European Muon Collaboration), and BFP (Berkeley, Fermilab, Princeton) groups [Compilation taken from *Review of Particle Properties*, *Phys. Lett.*, **170B**, 79 (1986)]. The normalizations of the data sets have been modified as indicated to bring them into better agreement. A factor 18/5, the inverse of the average charge squared of a light quark, is applied to the muon data to compare them with the neutrino data.

Parton distribution in eN and νN scattering

Question:

Do the parton distribution seen in electro-magnetic (F_2^{eN}) and in weak interaction (F_2^{eN}) agree ?

$$\frac{F_2^{\nu N}(x)}{F_2^{eN}(x)} = \frac{x[Q(x)+\overline{Q}(x)]}{\frac{5}{18}\cdot x[Q(x)+\overline{Q}(x)]} = \frac{18}{5}$$

↑ Factor from fractional charge

Answer:

- e.m. and weak quark structure is the same
- Factor 18/5 → fractional quark charge

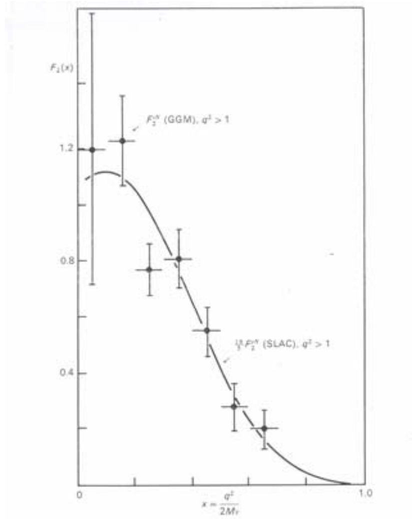


Fig. 7.21 Comparison of $F_2^{\nu N}$ measured in neutrino-nucleon scattering in the Gargamelle heavy-liquid bubble chamber in a PS neutrino beam at CERN, with SLAC data on F_2^{eN} from electron-nucleon scattering, in the same region of q^2. The data points are the neutrino results, and the curve is a fit through the electron data, multiplied by the factor $\frac{18}{5}$, which is the reciprocal of the mean squared charge of u- and d-quarks in the nucleon. This is a confirmation of the fractional charge assignments for the quarks. Note that the total area under the curve, measuring the total momentum fraction in the nucleon carried by quarks, is about 0.5. The remaining mass is ascribed to gluon constit[illegible], which are the postulated carriers of the interquark color field

Summary: eN and νN scattering

eN scattering

$$\frac{d^2\sigma^{eN}}{dxdy} = \frac{2\pi\alpha^2}{Q^4} x\left[1+(1-y)^2\right]\cdot\frac{5}{18}\left[Q(x)+\overline{Q}(x)\right]$$

$$F_2^{eN}(x) = \frac{5}{18} x\left[Q(x)+\overline{Q}(x)\right]$$

$\nu N + \overline{\nu} N$ scattering

$$\frac{d^2\sigma^{\nu N}}{dxdy} = \frac{G_F^2}{2\pi} xs\left[Q(x)+(1-y)^2\cdot\overline{Q}(x)\right]$$

$$\frac{d^2\sigma^{\overline{\nu} N}}{dxdy} = \frac{G_F^2}{2\pi} xs\left[Q(x)+(1-y)^2\cdot\overline{Q}(x)\right]$$

$$F_2^{\nu N}(x) = x\left[Q(x)+\overline{Q}(x)\right] \quad F_3^{\nu N}(x) = x\left[Q(x)-\overline{Q}(x)\right]$$

$$F_2^{\overline{\nu} N}(x) = x\left[Q(x)+\overline{Q}(x)\right] \quad F_3^{\overline{\nu} N}(x) = x\left[\overline{Q}(x)-Q(x)\right]$$

Summary: nucleon structure

- Nucleons contain point-like constituents: evidenced by approximate scale invariance of the structure function $F_2(x)$
- Constituents have spin 1/2: $2xF_1 \approx F_2$
- Same quark distribution for e.m. and weak interaction

But

- Quarks amount only to ~50% of nucleon momentum $\rightarrow$ remainder carried by gluons
- Logarithmic Q^2 dependence of F_2 for very small/large x and large Q^2 observed

⬇

Understood within perturbative QCD

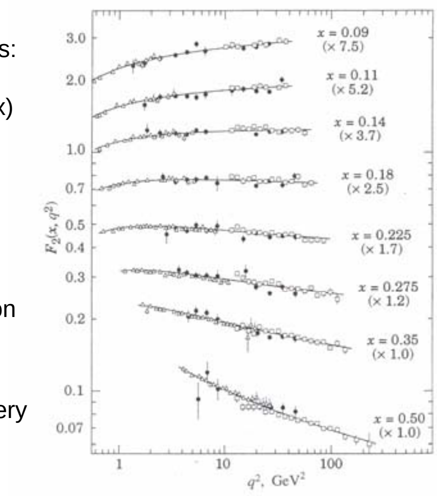
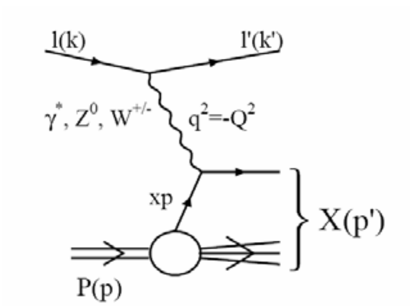


Fig. 6.14. The nucleon structure function $F_2(x, q^2)$ measured in deep inelastic muon and electron scattering off a deuterium target. The curves show the dependence expected from QCD, with Λ = 0.2 GeV. For clarity, the different curves have been multiplied by the factors shown in brackets. ●, NMC; △, SLAC; □, BCDMS. (After Montanet *et al.* 1994.)

4. Scaling-violation in deep-inelastic lepton-nucleon scattering



Scaling violation: $F_{2,3} = F_{2,3}(x, Q^2)$

?

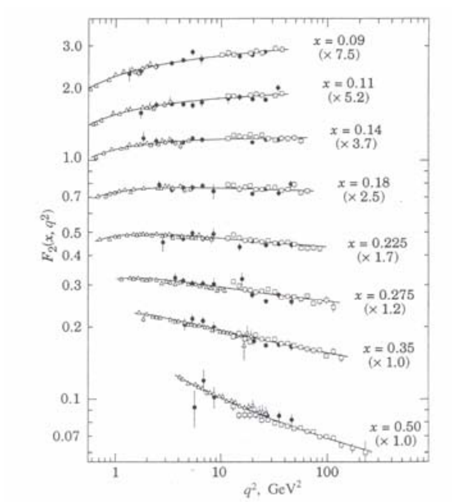
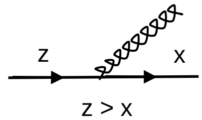


Fig. 6.14. The nucleon structure function $F_2(x, q^2)$ measured in deep inelastic muon and electron scattering off a deuterium target. The curves show the dependence expected from QCD, with Λ = 0.2 GeV. For clarity, the different curves have been multiplied by the factors shown in brackets. ●, NMC; △, SLAC; □, BCDMS. (After Montanet *et al.* 1994.)

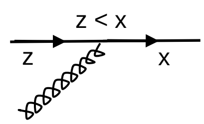
Scaling violation in a qualitative QCD picture

Quarks q(x) probed by the photon have history:

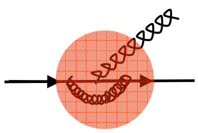
Gluon-emission



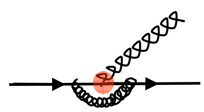
Gluon-absorption



The smaller the wavelength of the probe (Q^2 of the photon) the more quantum fluctuations can be resolved :



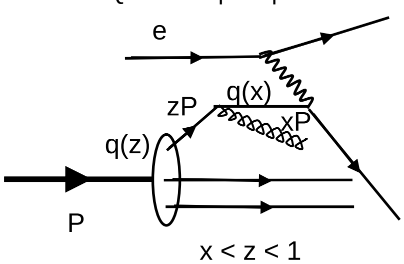
Small Q^2



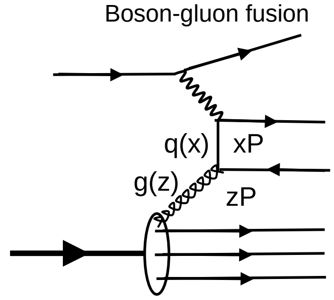
Large Q^2

2 effects:

QCD Compton process

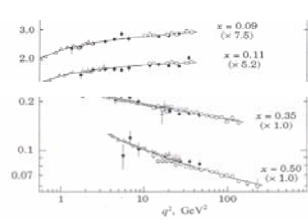


Boson-gluon fusion



Increase of sea quarks will soften the valence quark distribution as Q^2 increases

F_2 will rise with Q^2 at small x where the sea quarks dominate and will fall with Q^2 at large x where valence quarks dominate.



Quantitative description of scaling violation:

DGLAP evolution (**D**okshitzer, **G**ribov, **L**ipatov, **A**ltarelli, **P**arisi, 1972 – 1977)
of quark and gluon density functions q(x) and g(x) for large Q^2:

evolution of quark density with $\ln Q^2$

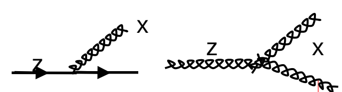
$$\frac{\partial q_i(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}z}{z}\left[\sum_j q_j(z,Q^2)P_{ij}\left(\frac{x}{z}\right) + g(z,Q^2)P_{ig}\left(\frac{x}{z}\right)\right]$$

Quark density — Gluon density

Splitting functions P_{ij} and P_{ig}

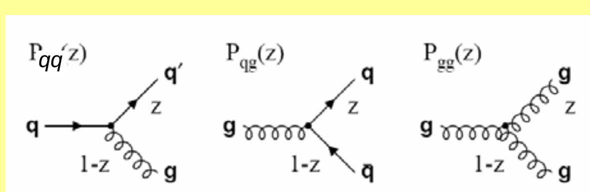
Probability that a parton j (quark or gluon) emits a parton **i** with momentum fraction $\varepsilon = x/z$ of the parent parton.

Evolution of gluon density



$$\frac{\partial g(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{\mathrm{d}z}{z}\left[\sum_j q_j(z,Q^2)P_{gj}\left(\frac{x}{z}\right) + g(z,Q^2)P_{gg}\left(\frac{x}{z}\right)\right]$$

Splitting functions are calculated as power series in α_s up to a given order:



$$P_{ij}(z,\alpha_s) = P^0_{ij}(z) + \frac{\alpha_s}{2\pi}P^1_{ij}(z) + \ldots$$

In leading order: $P_{ij}(z,\alpha_s) \equiv P_{ij}^0(z)$

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z} \qquad P_{gq}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}$$

$$P_{qg}(z) = \frac{z^2+(1-z)^2}{2} \qquad P_{gg}(z) = 6\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)$$

Deep-inelastic muon-nucleon scattering: NMC Experiment

μ beam

E_μ=90, 120, 200, 280 GeV

$F_2^p(x,Q^2) \quad F_2^n(x,Q^2)$

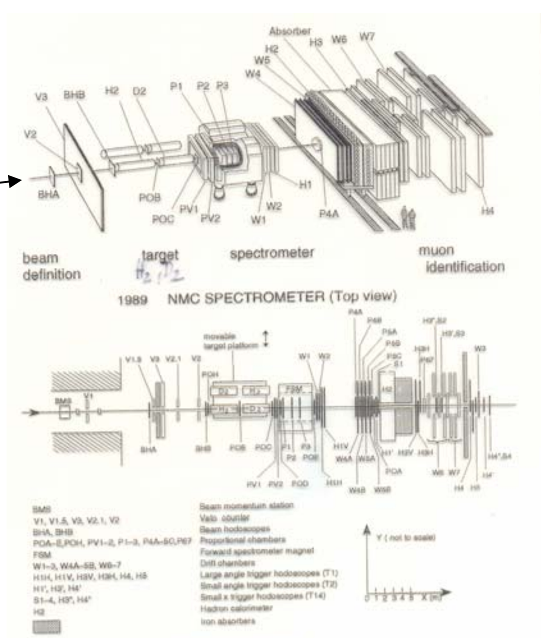
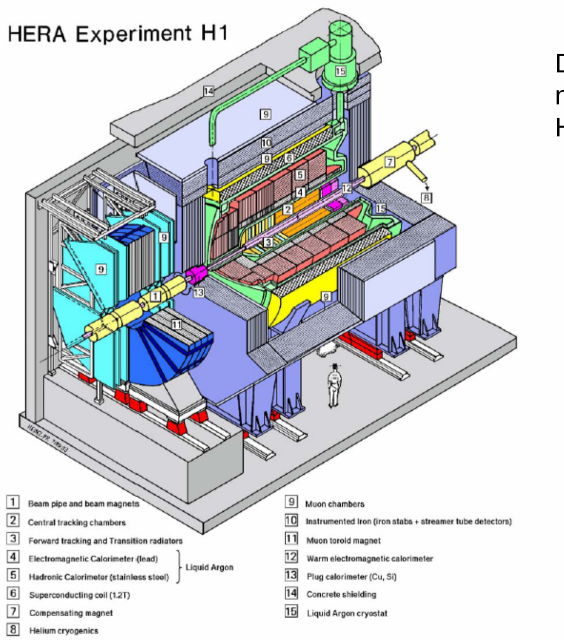
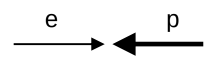


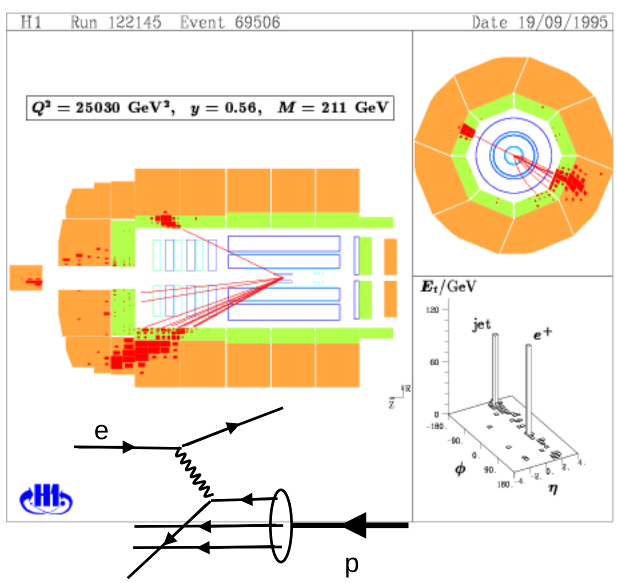
Fig. 15. The NMC detector.

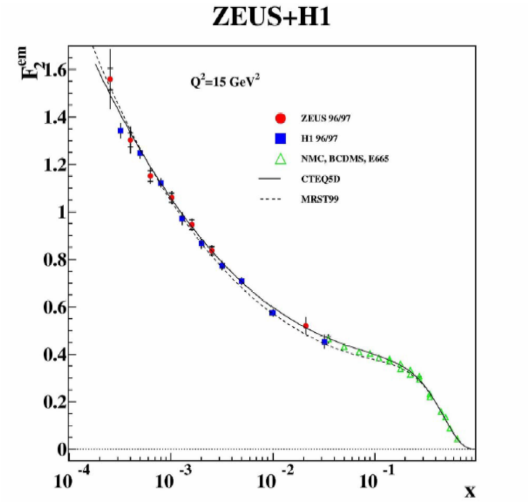


Deep-inelastic electron nucleon scattering: H1 and ZEUS at HERA



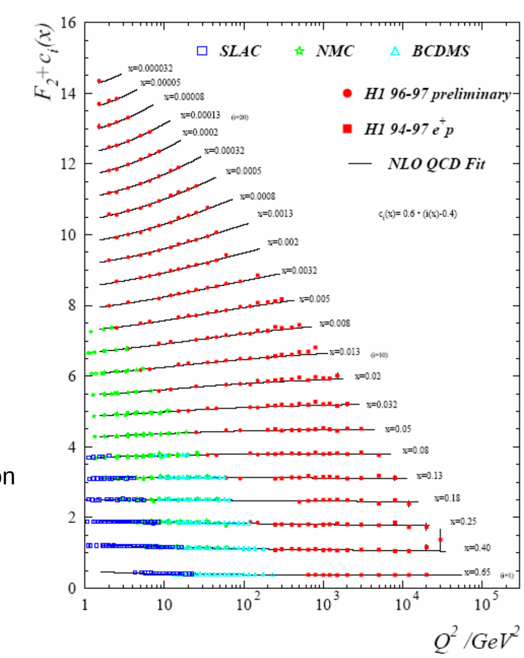
$$\sqrt{s} \approx 318\,\text{GeV}$$





$F_2(x)$

Large increase of $F_2(x)$ for very small x: unexpected



$F_2(x,Q^2)$

Described by QCD evolution

Experimental determination of the gluon density

Using the DGLAP evolution eq. one finds for $F_2(x,Q^2)$:

$$\frac{dF_2(x,Q^2)}{d\ln Q^2} = \sum_i e_i^2 \frac{\alpha_s(Q^2)}{2\pi} \cdot \int_x^1 \frac{dz}{z}\left[P_{qq}(\frac{x}{z}) q_i(z,Q^2) + P_{qg}(\frac{x}{z}) g(z,Q^2) \right]$$

For small x ($x<10^{-2}$):

quark pair production through gluon splitting dominant (1/x gluon spectrum):

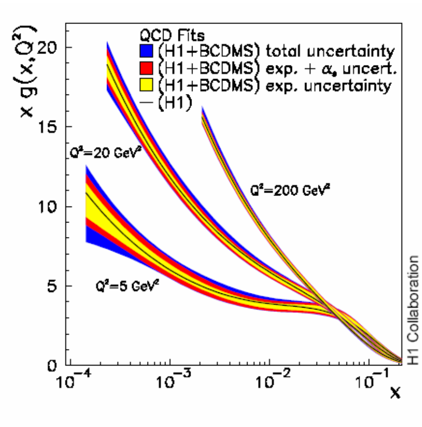
$$\rightarrow \quad P_{qg}(\frac{x}{z}) g(z,Q^2) \quad \text{dominant}$$

As an approximation one finds:

$$x \cdot g(x,Q^2) \approx \frac{27\pi}{10\alpha_s(Q^2)} \cdot \frac{dF_2(x,Q^2)}{d\ln Q^2}$$

i.e. scaling violation of F_2 at small x measures the gluon density.

Gluon density $g(x,Q^2)$:



In practice one makes a global DGLAP fit to the measured $F_2(x,Q^2)$

$$\alpha_s(M_Z^2) = 0.115$$

For larger Q^2 one sees more and more gluons at small x