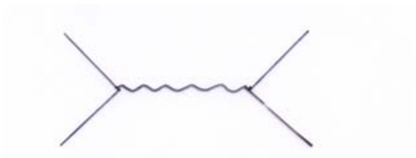
4. Higher orders

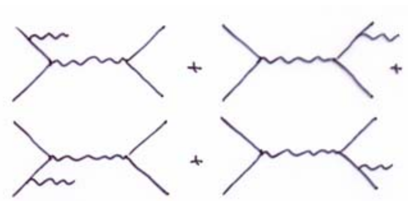
- Prediction of measurable higher order corrections
- Important to compare QED predictions to measurements

4.1 Radiative corrections to “Born” / ”tree” level predictions



Born diagram

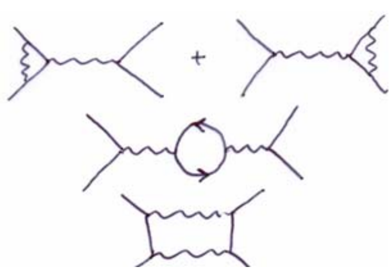
$$\frac{d\sigma^0_{f\bar{f}}}{d\Omega}$$



Bremsstrahlung corrections

$$\frac{d\sigma^{Brems}_{f\bar{f}\gamma}}{d\Omega}$$

Soft and **hard** photons



Virtual corrections:

$$\frac{d\sigma^{Virtual}_{f\bar{f}}}{d\Omega}$$

- Vertex corrections
- Propagator corrections
- Box corrections

Experimental cross section has to be compared to Born+corrections:

$$\boxed{\left(\frac{d\sigma_{ff(\gamma)}}{d\Omega}\right)_{exp} \Leftrightarrow \frac{d\sigma^0_{ff}}{d\Omega}\left(1+\delta_{Brems}+\delta_{Virtual}\right)}$$

Remark: at higher energies also electro-weak corrections are important

4.2 Bremsstrahlung

a.) soft photon radiation: $E_\gamma < \Delta E << \sqrt{s}$

No influence on ff prod./kinematics

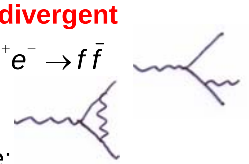
$$\frac{d\sigma_{f\bar{f}\gamma}}{d\Omega} = \frac{d\sigma^0_{f\bar{f}}}{d\Omega} \cdot \underbrace{R(p_+,p_-,q_+,q_-,k)\cdot k^2 dk\, d\Omega_\gamma}$$

Radiative corrections factorize:

$$\sim \frac{2\alpha}{\pi}\log\left(\frac{s}{m_f^2}\right)\frac{dk}{k}$$

Problems:

- Corrections are divergent for $k \to 0$ ($E_\gamma \to 0$): **infra-red divergent**
- If $E_\gamma < \Delta E$=detection threshold: $e^+e^- \to f\bar{f}\gamma \Leftrightarrow e^+e^- \to f\bar{f}$
- Vertex corrections to $e^+e^- \to f\bar{f}$ are also divergent



$\Rightarrow$Treat vertex + bremsstr. corrections at the same time:

$$\left.\frac{d\sigma_{f\bar{f}}}{d\Omega}\right|^{1st\ order} = \frac{d\sigma^0_{f\bar{f}}}{d\Omega}\cdot\left[\beta(s,m_e,m_f)\cdot\log\frac{\Delta E}{\sqrt{s}}+...\right]$$

Divergences cancel

b.) hard photon radiation: $E_\gamma > \Delta E$

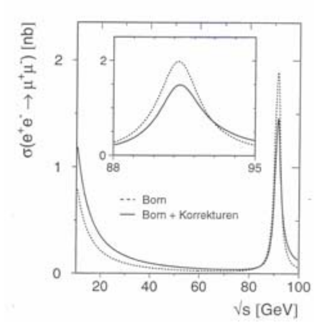
Final state with a detectable photon: $f\bar{f}\gamma$

$\Rightarrow$ Photon changes the kinematics and also production cross sections:

Initial state radiation (ISR): $\Rightarrow$ reduced effective CMS energy s'=zs

$$\boxed{\sigma_{ff(\gamma)} = \int_{4m_f^2/s}^{1} G(z)\sigma^0_{ff}(zs)\,dz} \qquad z = 1-\frac{2E_\gamma}{\sqrt{s}}$$

Radiator function G(z) describes photon radiation $\Rightarrow$ large effects if σ^0_{ff} has large s dependence.



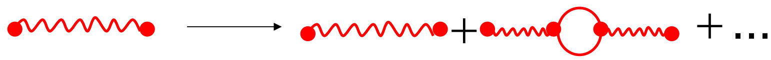
Final state radiation (FSR):

$$\sigma_{ff(\gamma)} = \sigma^0_{ff}\left(1+\frac{3\alpha}{4\pi}+...\right) \approx 1.0017\cdot\sigma^0_{ff}$$

4.3 Propagator corrections and running couplings

→ Leads to a change of the effective coupling constant

Propagator



$$\frac{e^2}{q^2} \longrightarrow \frac{e^2}{q^2}\left[1-\Pi^\gamma(q^2)\right]$$

divergent

$$e \longrightarrow e\left[1-\frac{1}{2}\Pi^\gamma(q^2)\right]$$

In the Thomson limit $q^2 \to 0$ effective charge should be equal to "e" but vacuum polarization leads to divergent correction $-\frac{1}{2}e\,\Pi^\gamma(0)$

⇒ Redefine the charge used in Feynman rules as "bare" charge $\mathbf{e_0}$ which is not measurable. $\mathbf{e_0}$ related to the physical charge $\mathbf{e}$:

$e_0 = e + \delta e$ with renormalization condition $\delta e = \frac{1}{2}e\,\Pi^\gamma(0)$

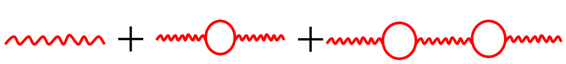
$$\frac{e_0^2}{q^2} \longrightarrow \frac{e^2}{q^2}\left[1+\Pi^\gamma(0)-\Pi^\gamma(q^2)\right] \quad \text{(to 1st order in } \Pi(q^2))$$

$$= \frac{e^2}{q^2}\left[1-\hat{\Pi}^\gamma(q^2)\right] \quad \text{with} \quad \hat{\Pi}^\gamma(q^2) = \Pi^\gamma(q^2) - \Pi^\gamma(0)$$

finite divergent

For $q^2 >> m^2$ (fermion masses)

$$\hat{\Pi}^\gamma(q^2) = -\frac{\alpha}{3\pi}\sum_f Q_f^2 \log\frac{q^2}{m_f^2}$$

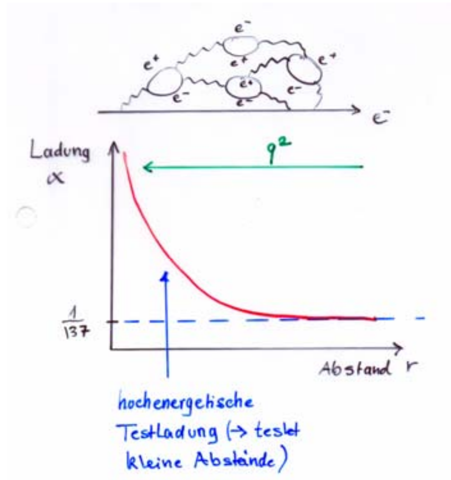


$$\left[1-\hat{\Pi}^\gamma(q^2)+[\hat{\Pi}^\gamma(q^2)]^2-\ldots\right]=\left[\frac{1}{1+\hat{\Pi}^\gamma(q^2)}\right] \Rightarrow \boxed{e^2(q^2)=\frac{e^2}{\left[1+\hat{\Pi}^\gamma(q^2)\right]}}$$

(geometric progression) (including higher orders)

Running coupling α

$$\boxed{\begin{aligned}\alpha(q^2) &= \frac{\alpha}{1+\hat{\Pi}^{\gamma}(q^2)} \\ &= \frac{\alpha}{1-\frac{\alpha}{3\pi}\cdot\sum_f Q_f^2\cdot\log\frac{q^2}{m_f^2}}\end{aligned}}$$



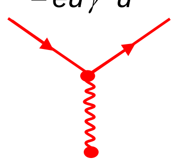
$\alpha(q^2=0) \quad = \frac{1}{137}$ (Thomson limit)

$\alpha(q^2=45^2 GeV^2) \quad = \frac{1}{129}$ (PETRA)

$\alpha(q^2=91^2 GeV^2) \quad = \frac{1}{128}$ (LEP)

4.4 Vertex corrections and anomalous magnetic moment

$-e\bar{u}\gamma^{\mu}u$



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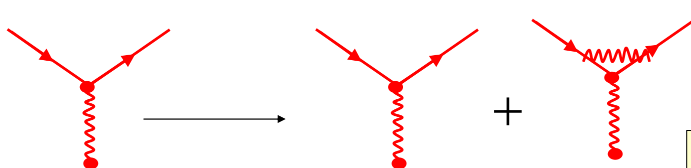
Gordon decomposition

Interaction due to spinless charges + Interaction due to spin

Magnetic moment in the non-relativistic limit:

$\vec{\mu} = -g\cdot\mu_B\cdot\vec{s} \qquad g=2$

Vertex corrections:



$\vec{\mu} = -2\cdot\mu_B\cdot\vec{s}$ $\qquad\longrightarrow\qquad$ $\vec{\mu} = -(2+\frac{\alpha}{\pi})\cdot\mu_B\cdot\vec{s}$

$$g = 2+\frac{\alpha}{\pi}$$

$$a = \frac{g-2}{2} = \frac{\alpha}{2\pi}$$

Higher order corrections to g-2

Radiative corrections g-2 are calculated to the 4-loop level:

Feynman Graphs	
O(α)	1
O(α²)	7
O(α³)	72
O(α⁴)	891
til O(α⁴)	971

Fig. 8.2 The Feynman graphs which have to be evaluated in computing the α^3 corrections the lepton magnetic moments (after Lautrup et al. 1972).

Most precise QED prediction.

Electron g-2 measurement

$$a_e = \frac{\alpha}{2\pi} - 0.328...\left(\frac{\alpha}{\pi}\right)^2 + 1.182....\left(\frac{\alpha}{\pi}\right)^3 - 1.505....\left(\frac{\alpha}{\pi}\right)^4$$

Theory

$$a_e = 0.001159\,652\,133\,(290)$$

$$a_{e^-} = 0.001159\,652\,188\,4\,(43)$$
$$a_{e^+} = 0.001159\,652\,187\,9\,(43)$$

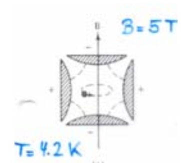
R.S. van Dyck et al. 1987

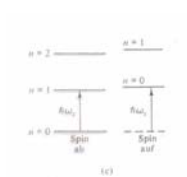
⇒ most precise value of α:

$$\alpha^{-1}(a_e) = 137.035\,999\,58\,(52)$$

For comparison α from Quanten Hall

$$\alpha^{-1}(qH) = 137.036\,003\,00\,(270)$$





Experimental method:

Storage of single electrons in a Penning trap (electrical quadrupole + axial B field)

⇒ complicated electron movement (cyclotron and magnetron precessions).

Idea: bound electron (**geonium**)

$$E_n = \left((2n+1) + m_s g \mu_B B\right)$$

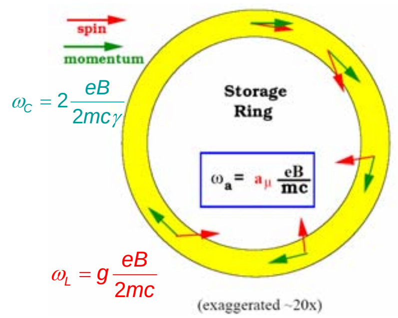
Trigger RF induced transitions between different n states or spin flips (=ω_s):

$$\omega_a = \omega_s - \omega_c = (g-2)\mu_B B$$
$$a = \frac{g-2}{2} = \frac{\omega_a}{\omega_c}$$

Muon g-2 experiment

Principle:

- store polarized muons in a storage ring; revolution with cyclotron frequency ω_c
- measure spin precession around the magnetic dipole field relative to the direction of cyclotron motion



Precession:

$$\vec{\omega}_a = -\frac{e}{m_\mu c}\left[\underbrace{a_\mu \vec{B}}_{} - \underbrace{(a_\mu - \frac{1}{\gamma^2 - 1})}_{}\vec{\beta}\times\vec{E}\right]$$

Difference between Lamor and cyclotron frequency

Effect of electrical focussing fields (relativistic effect).

$= 0$ for $\gamma = 29.3$

$\Leftrightarrow p_\mu = 3.094$ GeV/c

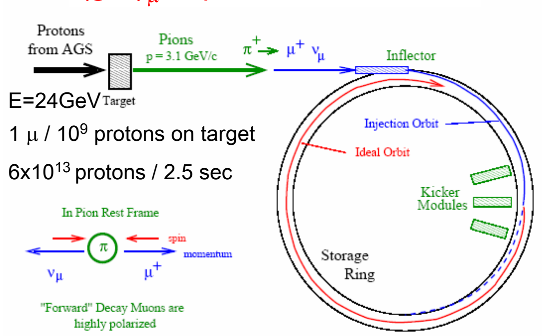
First measurements:

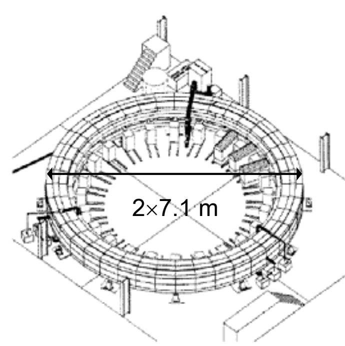
CERN 70s

$a_{\mu^-} = 0.001165937(12)$

$a_{\mu^+} = 0.001165911(11)$

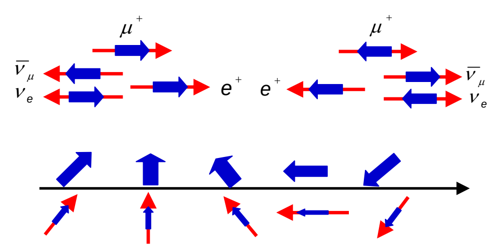
$(g\text{-}2)_\mu$ Experiment at BNL



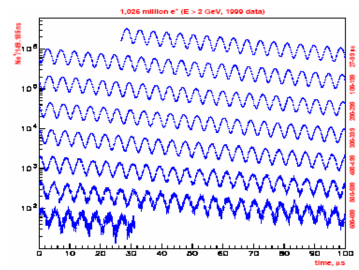


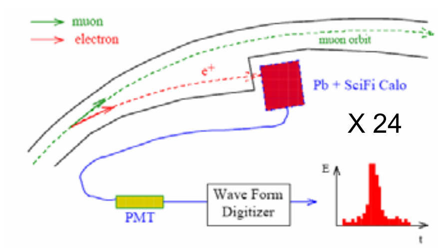
V-A structure of weak decay:

Use high-energy e^+ from muon decay to measure the muon polarization



Measure electron rate:





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$$N(t) = N_0\, e^{-\lambda t}\left[1 + A\cos(\omega_a t + \varphi)\right]$$

$$\frac{\omega_a}{2\pi} = 229023.59(16)\,\mathrm{Hz}$$

$$a_{\mu^+} = 11\,659\,203(8)\times 10^{-10}(0.7\,ppm)$$
$$a_{\mu^-} = 11\,659\,214(8)\times 10^{-10}(0.7\,ppm)$$
$$a_{\mu}^{SM} = 11\,659\,181(8)\times 10^{-10}$$

Remarks: Theoretical prediction of a_μ

Beside pure QED corrections there are weak corrections (W, Z) exchange and „hadronic corrections“

$$a_\mu = a_\mu^{QED} + a_\mu^{Had} + a_\mu^{EW}$$

(For the electron with much lower mass the hadronic and weak corrections are suppressed, and can be neglected.)

→ Determination of hadronic corrections is difficult and is in addition based on data.

→ For some evaluations of a_μ^{Had} the discrepancy between the prediction and the experiment becomes much smaller. (hot discussion amongst theoreticians)

Hadronic corrections