III. QED for “pedestrians”

1. Dirac equation for spin ½ particles
2. Feynman rules
3. Fermion-fermion scattering
4. Higher orders

1. Dirac Equation for spin ½ particles

Idea: Avoid negative energy values using a linearized ansatz

“ E = p + m ”

Weyl Eq.: $$i\frac{\partial}{\partial t}\psi = -i\left(\alpha_1 \frac{\partial}{\partial x_1}\psi + \alpha_2 \frac{\partial}{\partial x_2}\psi + \alpha_3 \frac{\partial}{\partial x_3}\psi\right) + \beta m\psi$$

$$E\psi = (\vec{\alpha}\cdot\vec{p} + \beta\cdot m)\psi$$

Coefficients α_i and β are determined demanding that the free particle solution satisfies the relativistic E-p relation: $E^2\psi = (\vec{p}^2 + m^2)\psi$

Cannot be solved by scalar coefficients:

→ 4×4 martices: $\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ σ_i are Pauli matrices

Dirac Equation:

$$i\left(\beta \frac{\partial}{\partial t}\psi + \beta\vec{\alpha}\cdot\vec{\nabla}\psi\right) - m\cdot 1\cdot\psi = 0$$

$$i\left(\gamma^0 \frac{\partial}{\partial t}\psi + \vec{\gamma}\cdot\vec{\nabla}\psi\right) - m\cdot 1\cdot\psi = 0$$

where $\gamma^0 = \beta$ and $\gamma^i = \beta\alpha_i$, $i = 1,2,3$

$$\gamma^\mu \partial_\mu \psi - m\psi = 0$$

Solutions ψ are four-component spinors.

They describes the fundamental spin ½ particles: $\psi = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix}$

Extremely compressed description $\quad j = 1...4: \quad \sum_{k=1}^{4}\left(\sum_\mu i\cdot(\gamma^\mu)_{jk}\frac{\partial}{\partial x^\mu} - m\delta_{jk}\right)\psi_k$

1.1 γ Matrices

$\gamma^0 = \beta$

$\gamma^i = \beta\alpha_i$, $i = 1,2,3$

$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^2\gamma^3$

$\alpha_i = \begin{pmatrix}0 & \sigma_i\\ \sigma_i & 0\end{pmatrix}$ and $\beta = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

$\gamma^0 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

$\gamma^i = \begin{pmatrix}0 & \sigma_i\\ -\sigma_i & 0\end{pmatrix}$, $i = 1...3$

$\gamma^5 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

Rules

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}, \quad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 0 \text{ for } \mu \neq \nu$$

$$(\gamma^\mu)^+ = \gamma^0\gamma^\mu\gamma^0, \quad (\gamma^0)^+ = \gamma^0, \quad (\gamma^k)^+ = -\gamma^k$$

$$\gamma^0\gamma^0 = 1, \quad \gamma^k\gamma^k = -1, \quad k = 1..3$$

$$\gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0, \quad (\gamma^5)^+ = \gamma^5$$

1.2 Fermion current conservation

Construct currents similar to Klein-Godan eq.: here use hermitian conjugate equation (equation$^+$) instead of complex conjugate:

$$\text{Dirac eq.:} \quad i\left(\gamma^0 \frac{\partial}{\partial t}\psi + \vec{\gamma}\cdot\vec{\nabla}\psi\right) - m\cdot 1\cdot\psi = 0$$

$$\text{(Dirac eq)}^+ : \quad i\left(\frac{\partial}{\partial t}\psi^+\gamma^0 + \vec{\nabla}\cdot\psi^+(-\vec{\gamma})\right) + m\cdot 1\cdot\psi^+ = 0$$

Introducing the adjoint spinor $\overline{\psi} = \psi^+\gamma^0$ allows to write the (Dirac-Eq.)$^+$ in the covariant form:

$$\overline{\psi}(i\partial_\mu\gamma^\mu + m) = 0$$

Adjoint Eq.

From the Dirac Eq. and its adjoint form you can derive a continuity equation for a 4-vector current:

Continuity Equation

$$\overline{\psi}\gamma^\mu\partial_\mu\psi + (\partial_\mu\overline{\psi})\gamma^\mu\psi = 0$$

$$\Rightarrow \quad \partial_\mu(\overline{\psi}\gamma^\mu\psi) = 0$$

Fermion current

$$j^\mu = (\overline{\psi}\gamma^\mu\psi)$$

here: $\rho = j^0 > 0$

$$j^0 = \overline{\psi}\gamma^0\psi = \psi^+\gamma^0\gamma^0\psi = \psi^+\psi > 0$$

Instead of probability current charge current is often used:

Electron current: $j_e^\mu = (-e)\cdot(\overline{\psi}\gamma^\mu\psi)$

Boson current: $j^\mu = (-e)\cdot 2p^\mu$

(for comparison)

1.3 Free particle solutions for Dirac Eq. $\gamma^\mu \partial_\mu \psi - m\psi = 0$

Ansatz: $\psi(x) = u(p)\cdot \exp(\mp ipx)$ for $E = \pm\sqrt{p^2 + m^2}$

4-comp. spinor: $u(p) = \begin{pmatrix} \varphi(p) \\ \chi(p) \end{pmatrix}$ and φ, χ 2-comp. spinors

Solutions for positive energy: $E = +\sqrt{p^2 + m^2}$

For the spinors φ and χ one finds from Dirac eq.: $\chi = \frac{\vec\sigma \cdot \vec p}{\mathrm{E+m}}\varphi$ $\oplus$ $\varphi_1 = \mathrm{N}\cdot\begin{pmatrix}1\\0\end{pmatrix}$ and $\varphi_2 = \mathrm{N}\cdot\begin{pmatrix}0\\1\end{pmatrix}$

solution spin ↑

$$u_1(p) = N\cdot\begin{pmatrix} \varphi_1 \\ \frac{\vec\sigma\cdot\vec p}{E+m}\varphi_1 \end{pmatrix} = N\cdot\begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x + ip_y}{E+m} \end{pmatrix}$$

solution spin ↓

$$u_2(p) = N\cdot\begin{pmatrix} \varphi_2 \\ \frac{\vec\sigma\cdot\vec p}{E+m}\varphi_2 \end{pmatrix} = N\cdot\begin{pmatrix} 0 \\ 1 \\ \frac{p_x - ip_y}{E+m} \\ \frac{p_z}{E+m} \end{pmatrix}$$

$N = \sqrt{E+m}$ (norm.)

1.4 Spin and helicity

- u_1 and u_2 are both solutions to the same energy value
- operator to distinguish the 2 solutions:

Helicity $\frac{\vec\Sigma\cdot\vec p}{|\vec p|} = \begin{pmatrix} \frac{\vec\sigma\cdot\vec p}{|\vec p|} & 0 \\ 0 & \frac{\vec\sigma\cdot\vec p}{|\vec p|} \end{pmatrix}$

$$\left[\frac{\vec\Sigma\cdot\vec p}{|\vec p|}, H\right] = 0$$

$$H = \begin{pmatrix} m & \vec\sigma\cdot\vec p \\ \vec\sigma\cdot\vec p & m \end{pmatrix}$$

- u_1 and u_2 are eigenstates of $\frac{\vec\Sigma\cdot\vec p}{|\vec p|}$ with eigenvalues $\pm$**1**
- u_1 and u_2 in particles rest frame

$$u_1 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \text{ and } u_2 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

1.5 Solutions for negative energies $E=-\sqrt{p^2+m^2}$

$\Rightarrow \phi = \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi$ and using $\chi_1 = N\cdot\begin{pmatrix}1\\0\end{pmatrix}$ and $\chi_2 = N\cdot\begin{pmatrix}0\\1\end{pmatrix}$

solution spin $\uparrow$

$$u_3(p)=N\cdot\begin{pmatrix}\dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_1\\ \chi_1\end{pmatrix}=N\cdot\begin{pmatrix}\dfrac{p_z}{E+m}\\ \dfrac{p_x+ip_y}{E+m}\\ 1\\ 0\end{pmatrix}$$

solution spin $\downarrow$

$$u_4(p)=N\cdot\begin{pmatrix}\dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_2\\ \chi_2\end{pmatrix}=N\cdot\begin{pmatrix}\dfrac{p_x-ip_y}{E+m}\\ \dfrac{-p_z}{E+m}\\ 0\\ 1\end{pmatrix}$$

$N=\sqrt{E+m}$ (norm.)

Particles: $E<0, \vec{p}$

$u_3^{\uparrow}, u_4^{\downarrow}$

Anti-particles: $E>0, -\vec{p}$

$v_2^{\downarrow}, v_1^{\uparrow}$

$$v_2(p)=N\cdot\begin{pmatrix}\dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_1\\ \chi_1\end{pmatrix}=N\cdot\begin{pmatrix}\dfrac{p_z}{E+m}\\ \dfrac{p_x+ip_y}{E+m}\\ 1\\ 0\end{pmatrix}$$

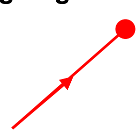
solution spin $\downarrow$

$$v_1(p)=N\cdot\begin{pmatrix}\dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_2\\ \chi_2\end{pmatrix}=N\cdot\begin{pmatrix}\dfrac{p_x-ip_y}{E+m}\\ \dfrac{-p_z}{E+m}\\ 0\\ 1\end{pmatrix}$$

solution spin $\uparrow$

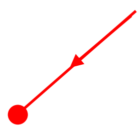
In- and out-going (anti)-particles

In-going electron:



$$\psi_{e-}(x)=\psi_{e-}(x)=u_{1,2}(p)\cdot\exp(+i\vec{p}\cdot\vec{x})\exp(-iEt)$$

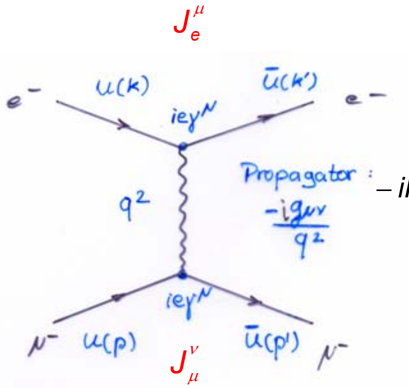
Out-going positron:



$$\psi_{e+}(x)=\psi_{e+}(x)=v_{1,2}(p)\cdot\exp(-i\vec{p}\cdot\vec{x})\exp(+iEt)$$

To describe **out-going electrons** or **in-going positrons** the adjoint spinors $\overline{\psi}_{e-}=\psi_{e-}\gamma^0$ or $\overline{\psi}_{e+}=\psi_{e+}\gamma^0$ and $\overline{u}_{1,2}$ or $\overline{v}_{1,2}$ are used.

2. Feynman rules

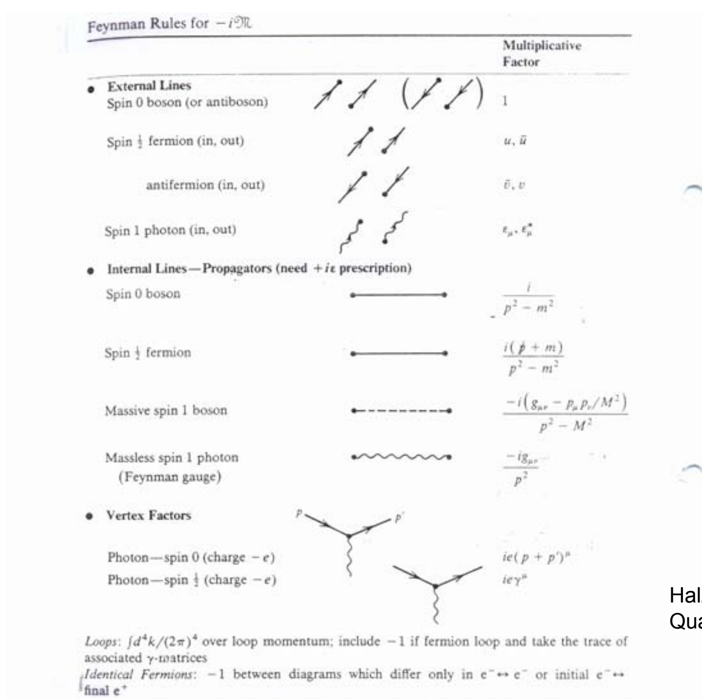


$$e^-\mu^- \rightarrow e^-\mu^-$$

$$-iM = \underbrace{\bar{u}(k')(ie\gamma^\mu)u(k)}_{-ie\cdot J_e^\mu} \cdot \frac{-ig_{\mu\nu}}{q^2} \cdot \underbrace{\bar{u}(p')(ie\gamma^\nu)u(p)}_{-ie\cdot J_\mu^\nu}$$

$$M = -\frac{e^2}{q^2}\bar{u}(k')\gamma_\mu u(k)\cdot\bar{u}(p')\gamma^\nu u(p)$$

There are similar rules for other Feynman diagrams



Feynman Rules for $-i\mathcal{M}$

		Multiplicative Factor
• **External Lines**		
Spin 0 boson (or antiboson)		1
Spin $\frac{1}{2}$ fermion (in, out)		$u, \bar{u}$
antifermion (in, out)		$\bar{v}, v$
Spin 1 photon (in, out)		$\epsilon_\mu, \epsilon_\mu^*$
• **Internal Lines—Propagators (need $+i\varepsilon$ prescription)**		
Spin 0 boson		$\frac{i}{p^2 - m^2}$
Spin $\frac{1}{2}$ fermion		$\frac{i(\not{p} + m)}{p^2 - m^2}$
Massive spin 1 boson		$\frac{-i(g_{\mu\nu} - p_\mu p_\nu/M^2)}{p^2 - M^2}$
Massless spin 1 photon (Feynman gauge)		$\frac{-ig_{\mu\nu}}{p^2}$
• **Vertex Factors**		
Photon—spin 0 (charge $-e$)		$ie(p+p')^\mu$
Photon—spin $\frac{1}{2}$ (charge $-e$)		$ie\gamma^\mu$

Loops: $\int d^4k/(2\pi)^4$ over loop momentum; include -1 if fermion loop and take the trace of associated γ-matrices
Identical Fermions: -1 between diagrams which differ only in $e^- \leftrightarrow e^-$ or initial $e^- \leftrightarrow$ final e^+

Halzen, Martin: Quark&Leptons

3. Fermion-fermion scattering

3.1 Process $e^- \mu^- \to e^- \mu^-$

Sect. II.5 →

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|} \cdot |M_{fi}|^2$$

Sect. III.2 →

$$M = -\frac{e^2}{q^2} \bar{u}(k')\gamma_\mu u(k) \cdot \bar{u}(p')\gamma^\nu u(p)$$

Spinors decribe a specific spin state of the fermions

For non-polarized ingoing particles and for non-observation of final state spin one observes unpolarized cross sections $\Rightarrow$ need to **average over possible initial spin states** and **sum over all final spin states**.

$$\overline{|M|^2} = \frac{1}{(2s_e+1)(2s_\mu+1)} \cdot \sum_{s_e, s_\mu} \sum_{s'_e, s'_\mu} |M|^2$$

$$\overline{|M|^2} = \frac{1}{(2s_e+1)(2s_\mu+1)} \cdot \sum_{s_e, s_\mu} \sum_{s'_e, s'_\mu} |M|^2$$

$$= \frac{1}{4} \cdot \frac{e^4}{q^4} \sum_{s_e, s_\mu} \sum_{s'_e, s'_\mu} [\bar{u}(k')\gamma^\mu u(k)][\bar{u}(k')\gamma^\nu u(k)]^* \cdot$$

$$[\bar{u}(p')\gamma_\mu u(p)][\bar{u}(p')\gamma_\nu u(p)]^*$$

$$= \frac{e^4}{q^4} L_e^{\mu\nu} \cdot L_{muon,\mu\nu}$$

Electron tensor $\quad L_e^{\mu\nu} = \frac{1}{2} \sum_{s_e, s'_e} [\bar{u}(k')\gamma^\mu u(k)][\bar{u}(k')\gamma^\nu u(k)]^*$

Muon tensor $\quad L_{muon,\mu\nu} = \frac{1}{2} \sum_{s_\mu, s'_\mu} [\bar{u}(p')\gamma_\mu u(p)][\bar{u}(p')\gamma_\nu u(p)]^*$

After a lengthy calculation

Berechnung von $L_e^{\mu\nu}$:

$$\left(\bar{u}(k)\gamma^\mu u(k)\right)^* = \text{1x1 Matrix, deswegen } (\,)^* = (\,)^\dagger$$
$$= (\bar{u}\gamma^\mu u)^\dagger = \left(u^\dagger(k')\gamma^0\gamma^\mu u(k)\right)^\dagger$$
$$= u^\dagger(k)\gamma^{\mu\dagger}\gamma^0 u(k') \leftarrow \gamma^{0\dagger} = \gamma^0$$
$$= u^\dagger(k)\gamma^0\gamma^\mu u(k')$$
$$= \bar{u}(k)\gamma^\mu u(k')$$

$$L_e^{\mu\nu} = \frac{1}{2}\sum_{s,s'} \bar{u}(k')\gamma^\mu u(k) \cdot \bar{u}(k)\gamma^\nu u(k')$$

mit $(A \cdot B \cdot C)_{\alpha\beta} = A_{\alpha\gamma} B_{\gamma\delta} C_{\delta\beta}$

$$= \frac{1}{2}\sum_{s,s'} \bar{u}_\alpha(k')\gamma^\mu_{\alpha\beta} u_\beta(k)\bar{u}^{(k)}_\chi \gamma^\nu_{\chi\lambda} u_\lambda(k')$$
$$= \frac{1}{2}\sum_{s'} u_\lambda(k')\bar{u}_\alpha(k')\gamma^\mu_{\alpha\beta} \cdot \sum_s u_\beta(k)\bar{u}_\chi(k)\gamma^\nu_{\chi\lambda}$$

mit Vollständigk.-Relation $\sum_s u\bar{u} = \not{k} + m$

$$= \frac{1}{2}(\not{k}' + m)_{\lambda\alpha}\gamma^\mu_{\alpha\beta}(\not{k} + m)_{\beta\chi}\gamma^\nu_{\chi\lambda}$$
$$= \frac{1}{2}\left[(\not{k}' + m)\gamma^\mu(\not{k} + m)\gamma^\nu\right]_{\lambda\lambda} =$$
$$= \frac{1}{2}\,\mathrm{Sp}\left[(\not{k}' + m)\gamma^\mu(\not{k} + m)\gamma^\nu\right]$$

Bem.: Über Indizes μ und ν bisher nicht summiert

$$= \frac{1}{2}\,\mathrm{Sp}\left[\not{k}'\gamma^\mu\not{k}\gamma^\nu + \not{k}'\gamma^\mu m\gamma^\nu + m\gamma^\mu\not{k}\gamma^\nu + m\gamma^\mu m\gamma^\nu\right]$$

Spur ungerader Zahl von γ's $= 0$: $\not{a} = a_\mu\gamma^\mu$!!

$$= \frac{1}{2}\,\mathrm{Sp}\left[\not{k}'\gamma^\mu\not{k}\gamma^\nu + m^2\gamma^\mu\gamma^\nu\right]$$
$$= \frac{1}{2}\,\mathrm{Sp}\left[k'_\alpha\gamma^\alpha\gamma^\mu k_\beta\gamma^\beta\gamma^\nu + m^2\gamma^\mu\gamma^\nu\right]$$
$$= \frac{1}{2}k'_\alpha k_\beta \cdot \mathrm{Sp}\left[\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\right] + \frac{1}{2}m^2\,\mathrm{Sp}\left[\gamma^\mu\gamma^\nu\right]$$
$$= \frac{1}{2}k'_\alpha k_\beta \cdot 4 \cdot \left[g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\mu\beta} - g^{\alpha\beta}g^{\mu\nu}\right] + \frac{1}{2}m^2 \cdot 4g^{\mu\nu}$$
$$= 2\left[k'^\mu k^\nu + k'^\nu k^\mu - [k' \cdot k - m^2]g^{\mu\nu}\right]$$

also

$$L_e^{\mu\nu} = 2\left[k'^\mu k^\nu + k'^\nu k^\mu - [k' \cdot k - m^2]g^{\mu\nu}\right]$$
$$L^{Myon}_{\mu\nu} = 2\left[p'_\mu p_\nu + p'_\nu p_\mu - [p' \cdot p - M^2]g_{\mu\nu}\right]$$

Spin averaged matrix element for $e^-\mu^- \rightarrow e^-\mu^-$

$$\overline{|M|^2} = \frac{1}{(2s_e + 1)(2s_\mu + 1)} \cdot \sum_{s_e, s_\mu}\sum_{s'_e, s'_\mu} |M|^2 = \frac{e^4}{q^4} L_e^{\mu\nu} \cdot L_{muon,\mu\nu}$$
$$= 8\frac{e^4}{q^4}\left[(k' \cdot p')(k \cdot p) + (k' \cdot p)(k \cdot p') - m^2 p' \cdot p - M^2 k' \cdot k + 2m^2M^2\right]$$

m electron mass
M muon mass

→ exact 1st order result for $e^-\mu^- \rightarrow e^-\mu^-$

Relativistic limit → neglect masses m and M

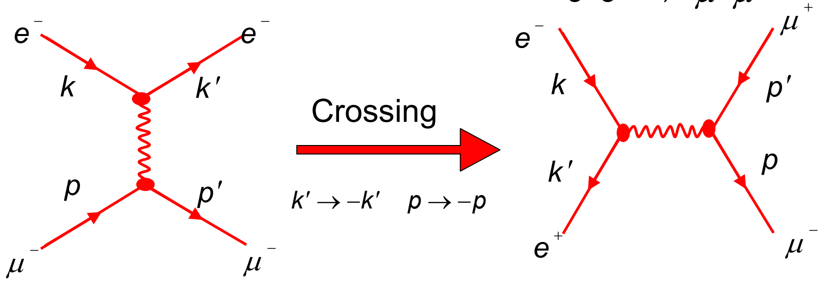
$$\overline{|M|^2} = 8\frac{e^4}{(k - k')^4}\left[(k' \cdot p')(k \cdot p) + (k' \cdot p)(k \cdot p')\right] = 2e^4\frac{s^2 + u^2}{t^2}$$

By using the Mandelstam variables in the relativistic limit

$$s = (k + p)^2 = m^2 + M^2 + 2kp \approx 2kp \approx 2k'p'$$
$$t = (k - k')^2 = m^2 + M^2 - 2kk' \approx -2kk' \approx -2pp'$$
$$u = (k - p')^2 = m^2 + M^2 - 2kp' \approx -2kp' \approx -2k'p$$

3.2 Process $e^+e^- \to \mu^+\mu^-$

$e^+e^- \to \mu^+\mu^-$



$$t = (k-k')^2 \quad \to \quad s' = (k-k')^2$$
$$s = (k+p)^2 \quad \to \quad t' = (k-p)^2$$
$$u = (k-p')^2 \quad \to \quad u' = (k-p')^2$$

$$\overline{|M|^2}_{e^-\mu^- \to e^-\mu^-}(s,t,u) = \overline{|M|^2}_{e^+e^- \to \mu^+\mu^-}(s',t',u')$$

$$\overline{|M|^2}_{e^-\mu^- \to e^-\mu^-}(s,t,u) = 2e^4 \frac{s^2+u^2}{t^2} \quad \Rightarrow \quad \overline{|M|^2}_{e^+e^- \to \mu^+\mu^-}(s',t',u') = 2e^4 \frac{t'^2+u'^2}{s'^2}$$

Differential cross section for $e^+e^- \to \mu^+\mu^-$ (CMS)

Reminder:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|} \cdot |M_{fi}|^2$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2} \cdot \frac{1}{s} \cdot \frac{t^2+u^2}{s^2}$$
$$= \frac{e^4}{64\pi^2} \cdot \frac{1}{s} \cdot (1+\cos^2\theta)$$

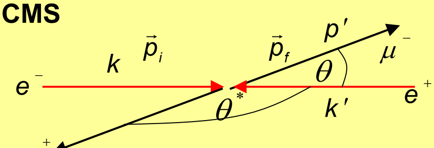
$e^2 = 4\pi\alpha$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CMS} = \frac{\alpha^2}{4s} \cdot (1+\cos^2\theta)$$

⟵1/s dependence from flux factor

Kinematics for high-relativistic particles

CMS



$\vec{p}_i = \vec{p}_f$

$$s = (k+k')^2 \approx 4E_i^2$$
$$t = (k-p)^2 \approx -2kp \approx -2E_i^2(1-\cos\theta^*)$$
$$\approx -\frac{s}{2}(1+\cos\theta)$$
$$u = (k-p')^2 \approx -2kp' \approx -2E_i^2(1-\cos\theta)$$
$$\approx -\frac{s}{2}(1-\cos\theta)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CMS} = \frac{\alpha^2}{4s}\cdot(1+\cos^2\theta)$$

$$\sigma_{tot} = \frac{4\pi\alpha^2}{3s} = \frac{86.86\,\text{nb}\,\text{GeV}^2}{s}$$

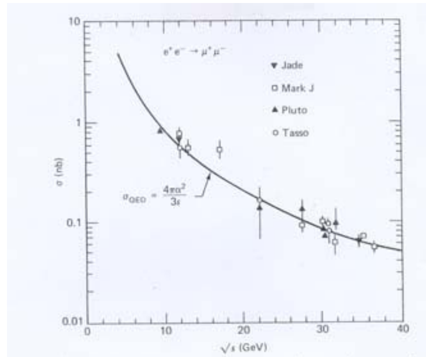


Fig. 6.6 The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

3.3 Particle spin and angular distribution

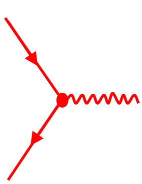
Chirality operator:

Projection of left- and right-handed components spin components of spinor u

$$u_L = \frac{1}{2}(1-\gamma^5)u$$
$$u_R = \frac{1}{2}(1+\gamma^5)u$$

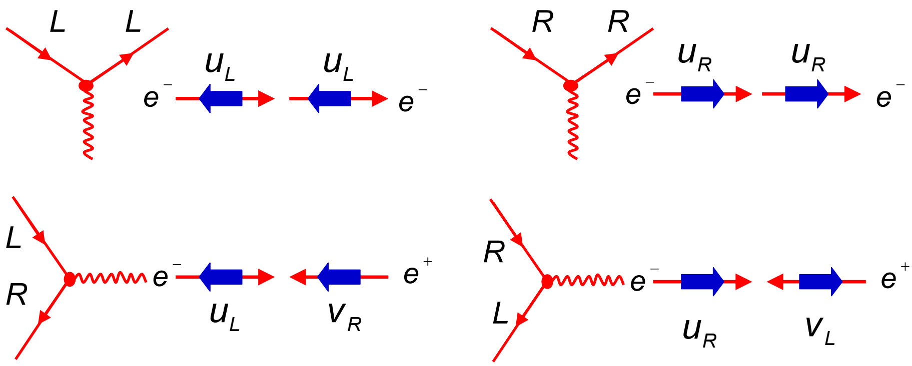
In the relativistic limit the eigenstates of the helicity operator $\frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}$ correspond to the chirality states.

Decomposition of the fermion current:

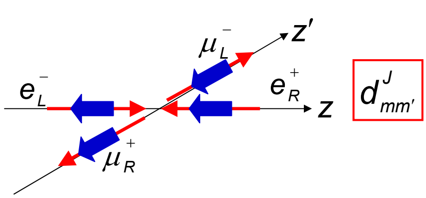


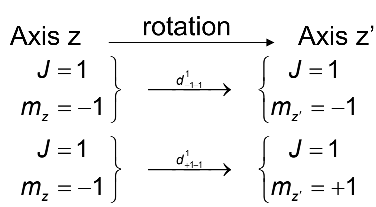
$$\bar{u}\gamma^\mu u = (\bar{u}_R + \bar{u}_L)\gamma^\mu(u_R + u_L)$$
$$= \bar{u}_R\gamma^\mu u_R + \bar{u}_L\gamma^\mu u_L$$

Photon (vector $ie\gamma^{\mu}$) coupling:



Angular distribution $e^+e^- \to \mu^+\mu^-$

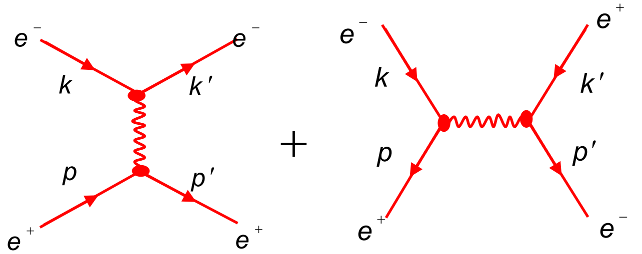


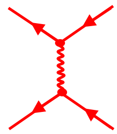
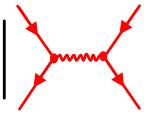


$$\frac{d\sigma}{d\Omega} \sim \left(d^1_{-1-1}\right)^2 + \left(d^1_{+1-1}\right)^2 \sim \frac{1}{4}(1+\cos\theta)^2 + \frac{1}{4}(1-\cos\theta)^2 \sim 1+\cos^2\theta$$

3.3 Bhabha scattering

$M =$

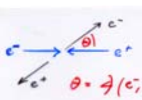


$\overline{|M|^2} = \Big|$  $\Big|^2 +$ interference $+ \Big|$  $\Big|^2$

($e^-\mu^- \to e^-\mu^-$) ($e^+e^- \to \mu^+\mu^-$)

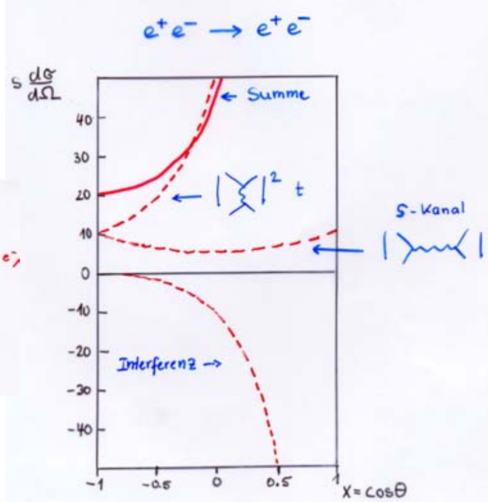
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{s^2+u^2}{t^2} + \frac{2u^2}{ts} + \frac{t^2+u^2}{s^2}\right)$$

<u>Im CMS</u>: mit $x = \cos\theta$ und



$\theta = \sphericalangle(e^-, e^-$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{4+(1+x)^2}{(1-x)^2} - \frac{(1+x)^2}{1-x} + \frac{1+x^2}{2}\right)$$



3.4 Summary

	Feynman Diagrams		$\overline{|\mathcal{M}|^2}/2e^4$		
	Forward peak	Backward peak	Forward	Interference	Backward
Møller scattering $e^-e^- \to e^-e^-$			$\frac{s^2+u^2}{t^2}$	$+\ \frac{2s^2}{tu}$	$+\ \frac{s^2+t^2}{u^2}$
				($u \leftrightarrow t$ symmetric)	
(Crossing $s \leftrightarrow u$)	Forward	"Time-like"	Forward	Interference	Time-like
Bhabha scattering $e^-e^+ \to e^-e^+$			$\frac{s^2+u^2}{t^2}$	$+\ \frac{2u^2}{ts}$	$+\ \frac{u^2+t^2}{s^2}$
$e^-\mu^- \to e^-\mu^-$			$\frac{s^2+u^2}{t^2}$	"Rutherford"	
(Crossing $s \leftrightarrow t$) $e^-e^+ \to \mu^-\mu^+$					$\frac{u^2+t^2}{s^2}$