IV. Weak interaction

1. Phenomenology of weak decays
2. Parity violation and neutrino helicity
3. V-A theory
4. Neutral currents

The weak interaction was and is a topic with a lot of surprises:

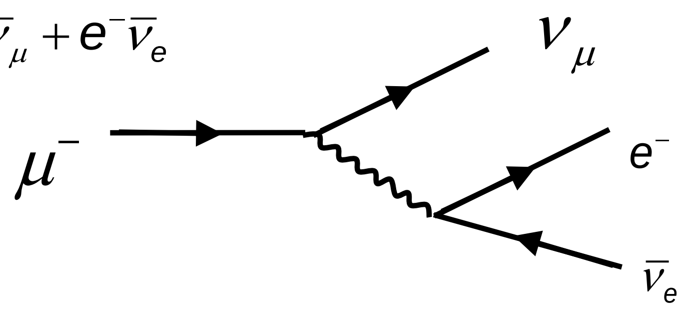
Past: Flavor violation, P and CP violation.
Today: Weak decays used as probes for new physics

1. Phenomenology of weak decays

All particles (except photons and gluons) participate in the weak interaction. At small q^2 weak interaction can be shadowed by strong and electro-magnetic effects.

- Observation of weak effects only possible if strong/electro-magnetic processes are forbidden by conservation laws:

$\mu^- \to \bar{\nu}_\mu + e^- \bar{\nu}_e$

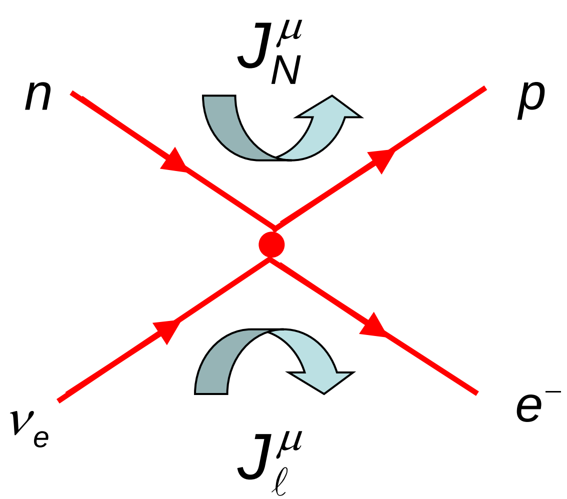


$\tau = 2.6 \cdot 10^{-6}\,\text{s}$

Electromagnetic decay $\mu^- \to e^- \gamma$ forbidden by lepton number conservation

1.1 Weak interaction and nuclear β-Decay: $n \to p\, e^- \,\bar{\nu}_e$

Fermi's explanation (1933/34) of the nuclear β-decay:



Two fermionic vector currents coupled by a **weak coupling const.** at single point (4-fermion interact.)

Apply "Feynman Rules"

$$M = \frac{G_F}{\sqrt{2}} \cdot J_{N,\mu} \cdot J_e^{\mu^+} = \frac{G_F}{\sqrt{2}} \cdot \left(\bar{u}_p \gamma_\mu u_n\right) \cdot \left(\bar{u}_e \gamma_\mu v_\nu\right)$$

↳ Fermi coupling constant, dimension = $(1/M)^2$

Coupling of the currents described by coupling constant G_F – a very small number ~10^{-5} GeV^{-2}. Explains the "weakness" of the force.

Fermi's ansatz was inspired by the structure of the electromagnetic interaction and the fact that there is essentially no energy dependence observed.

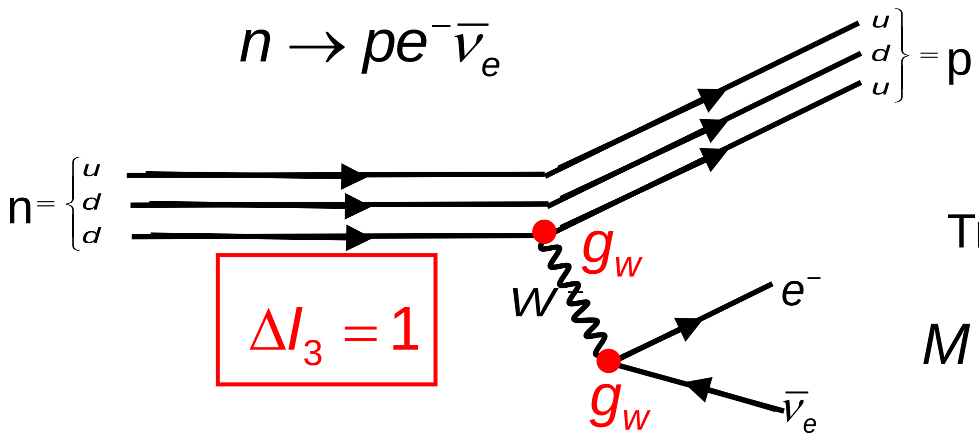
Problem: Ansatz cannot explain parity violation (was no a problem in 1933)

Today's picture of the β-decay

- Nucleons are composed of quarks, which are the fundamental fermions. The quarks couple to the fundamental forces.
- The weak interaction is mediated by a massive vector field (gauge boson, W). Coupling constant g_w.

Using the the "quark level" decay one can describe weak hadron decays (treating the quarks which are not weakly interacting as spectators)

$$n \to p e^- \bar{\nu}_e$$



Currents:
V-A structure

Transition matrix:

$$M = (-ig_w)^2 J^\mu_{quark} \left(\frac{g^{\mu\nu}}{q^2 - M_w^2} \right) J^\nu_{lepton}$$

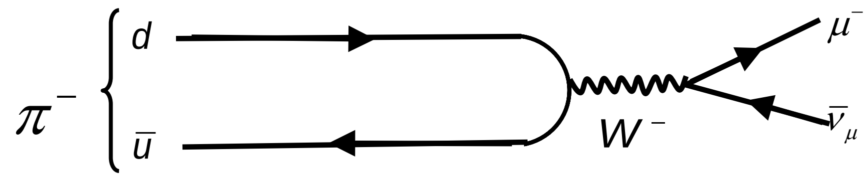
Strong isospin I_3 not conserved.

"weakness" result of $(1/M_w)^2$ suppression

1.2 Weak hadronic decays

a) Dominant decay modes

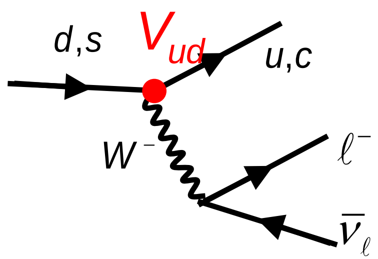
$$\pi^- \to \mu^- \bar{\nu}_\mu$$

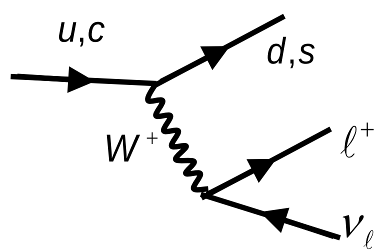


$$d \to u\, \ell^- \bar{\nu}_\ell \qquad u \to d\, \ell^+ \nu_\ell$$

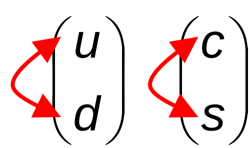
$$s \to c\, \ell^- \bar{\nu}_\ell \qquad c \to s\, \ell^+ \nu_\ell$$

$$\Delta I = 1 \quad \Delta I_3 = 1$$





If q^2 is large enough the W can also decay to (u, $\bar{d}$) or ($\bar{u}$ d) quark pairs



Historically

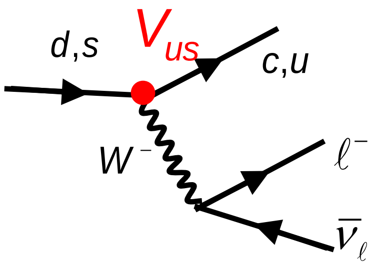
$$M^2 \sim \cos^2\theta_c \sim 0.95$$

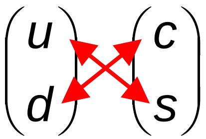
Cabibbo angle: $\theta_c \approx 0.22$

b) suppressed decay modes

$d \rightarrow c\ \ell^- \bar{\nu}_\ell \qquad u \rightarrow s\ \ell^+ \nu_\ell$

$s \rightarrow u\ \ell^- \bar{\nu}_\ell \qquad c \rightarrow d\ \ell^+ \nu_\ell$

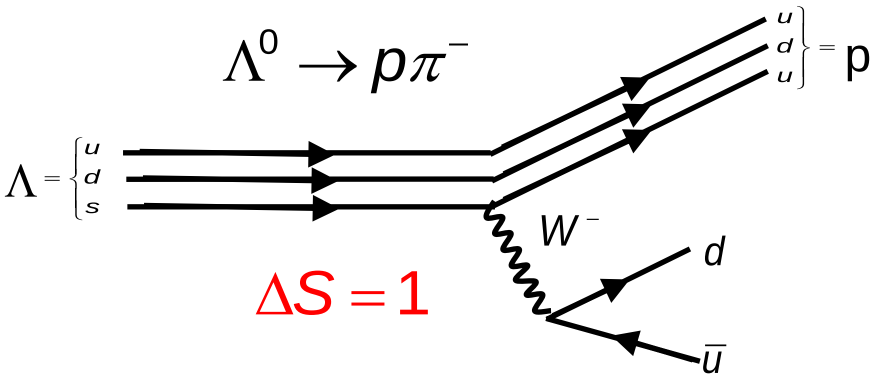




$M^2 \sim \sin^2\theta_c \sim 0.05$

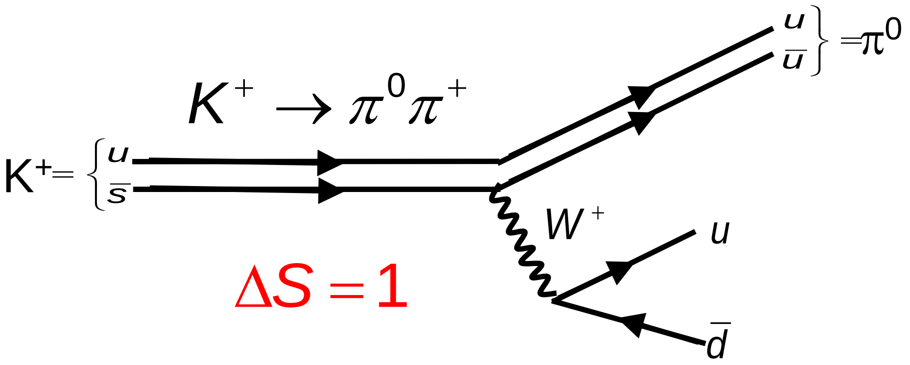
Quark flavor transition described by the CKM matrix.

$\Lambda^0 \rightarrow p\pi^-$



$\Delta S = 1$

$K^+ \rightarrow \pi^0\pi^+$

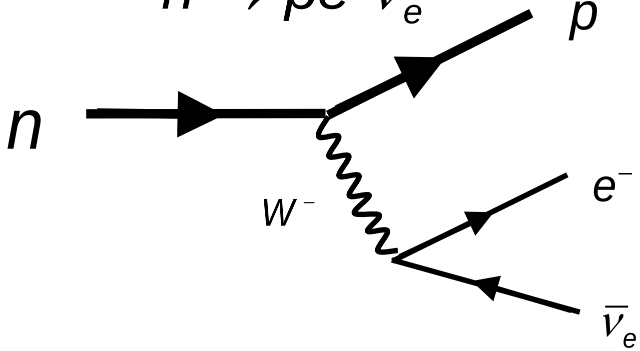


$\Delta S = 1$

Weak interaction does not conserve strong isospin, strangeness or other quark flavor numbers. Lepton number is conserved.

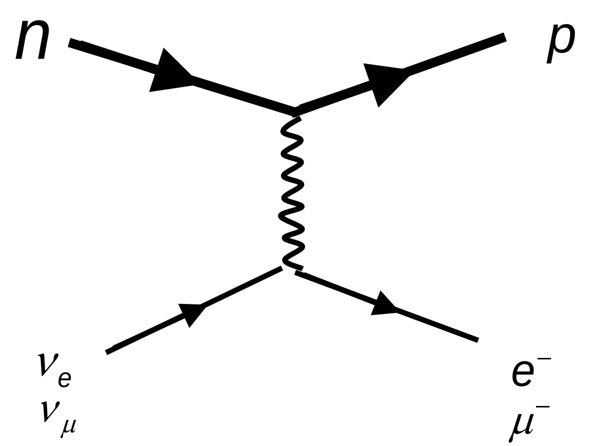
1.3 Neutrino interactions

$n \to p e^- \overline{\nu}_e$



β decay

crossing



Neutrino-nucleon scattering

Very small cross section for νN scattering: $\sigma(\nu N) \approx E_\nu[\text{GeV}] \times 10^{-38}\ \text{cm}^2$

$\approx E_\nu[\text{GeV}] \times 10\ \text{fb}$

- intense neutrino beams
- large instrumented targets

2. Parity violation

$$x'^{\mu} = \Lambda_P{}^{\mu}{}_{\nu}\, x^{\nu}$$

Parity transformations (P) = space inversion

$$\Lambda_P = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

P transformation properties: $P:$

$$\vec{r} \rightarrow -\vec{r}$$

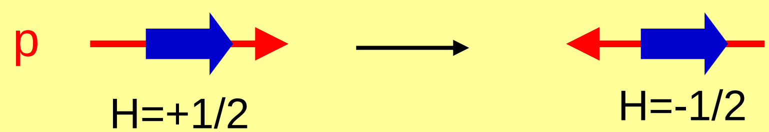
$$t \rightarrow t$$

$$\vec{p} \rightarrow -\vec{p}$$

$$\vec{\ell} = \vec{r} \times \vec{p} \rightarrow \vec{\ell}$$ Axial/pseudo vector

e.g.: Helicity operator

$$H = \frac{\vec{s} \cdot \vec{p}}{|\vec{p}|} \xrightarrow{P} -\frac{\vec{s} \cdot \vec{p}}{|\vec{p}|} \quad \text{(pseudo- scalar)}$$



Experimentally:

⇔ mirroring at plane + rotation around axis perpendicular to plane

⇒ To test parity it is sufficient to study the process in the "mirrored system": physics invariant under rotation

2.1 Historical θ/τ puzzle (1956)

Until 1956 parity conservation as well as T and C symmetry was a "dogma":
$\rightarrow$ very little experimental tests done

In 1956 Lee and Yang proposed parity violation in weak processes.

Historical names

Starting point: Observation of two particles **θ^+** and **τ^+** with exactly equal mass, charge and strangeness **but** with different parity:

$$\theta^+ \rightarrow \pi^+\pi^0 \quad w/ \quad P(\theta^+) = P(\pi)^2(-1)^\ell \rightarrow J^P(\theta^+) = 0^+ \qquad P(\pi)=-1$$

$$\tau^+ \rightarrow \pi^+\pi^+\pi^- \qquad P(\tau^+) = P(\pi)^3(-1)^{2\ell} \rightarrow J^P(\tau^+) = 0^-, 2^-$$

Lee + Yang: θ^+ and τ^+ same particle, but decay violates parity

$\Rightarrow$ particle is K^+:

$K^+(0^-) \rightarrow \pi^+\pi^0$ P is violated

$K^+(0^-) \rightarrow \pi^+\pi^+\pi^-$ P is conserved

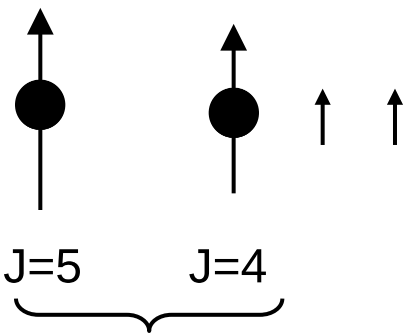
To search for possible P violation, a number of experimental tests of parity conservation in weak decays has been proposed:

1957 Observation of P violation in nuclear β decays by Chien-Shiung Wu et al.

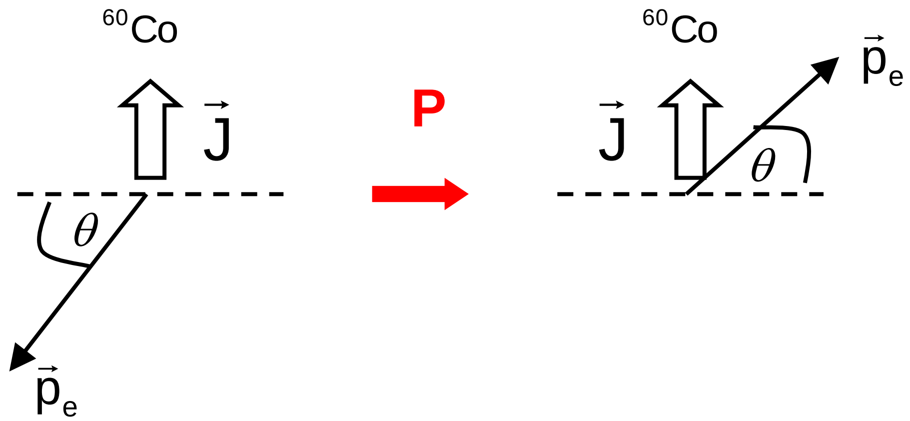
2.2 Observation of parity violation, C.S. Wu et al. 1957

Idea: Measurement of the angular distribution of the emitted e^- in the decay of polarized ^{60}Co nuclei

$$^{60}\text{Co} \rightarrow {}^{60}\text{Ni}^* + e^- + \bar{\nu}_e$$



Gamov-Teller Transition $\Delta J=1$



Observable: $\vec{J} \cdot \vec{p}_e \rightarrow -\vec{J} \cdot \vec{p}_e$

If P is conserved, the angular distribution must be symmetric in θ (symmetric to dashed line): transition rates for $\vec{J} \cdot \vec{p}_e$ and $-\vec{J} \cdot \vec{p}_e$ are identical.

Experiment: Invert Co polarization and compare the rates at the same position θ.

NaJ detector to measure e rate

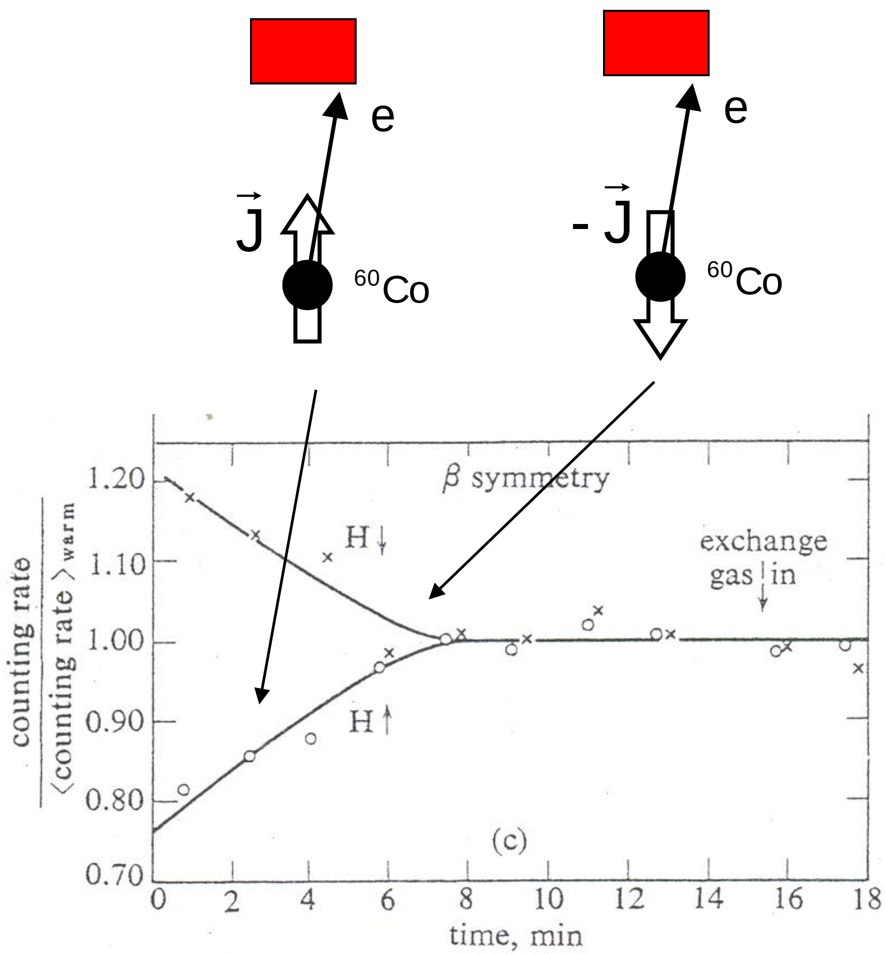


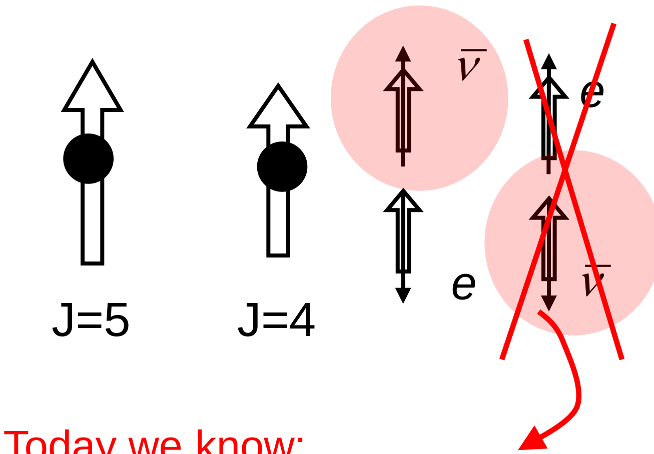
Figure 9-12 Gamma anisotropy (as determined from the two NaI counters) and beta asymmetry for the polarizing field pointing up and down as a function of time. The times for disappearance of the beta and gamma asymmetry coincide; this is the warm-up time. The warm-up time for the sample is approximately 6 min and the counting rates for the warm unpolarized sample are independent of the field direction. [From C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *Phys. Rev.*, **105**, 1413 (1957).]

<u>Result:</u>

Electron rate opposite to Co polarization is higher than along the ^{60}Co polarization:

parity violation

<u>Qualitative explanation:</u>



Today we know:

Consequence of existence of only left-handed (LH) neutrinos (RH anti-neutrinos)

Electron polarization in β decays

$$H_{e^-} = -\frac{1}{2}\frac{v}{c}$$

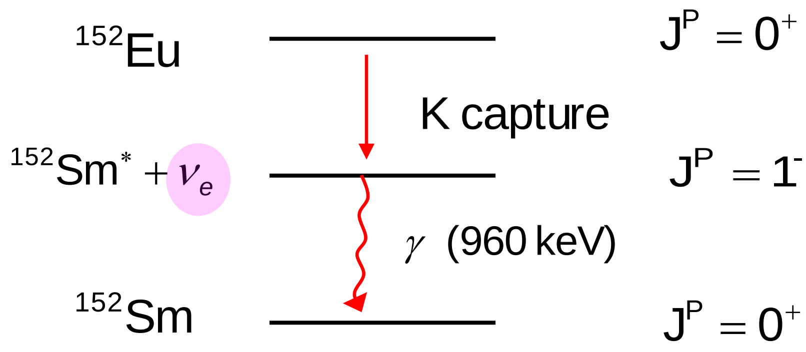
2.3 Determination of the neutrino helicity

Goldhaber et al., 1958

Indirect measurement of the neutrino helicity in a K capture reaction:

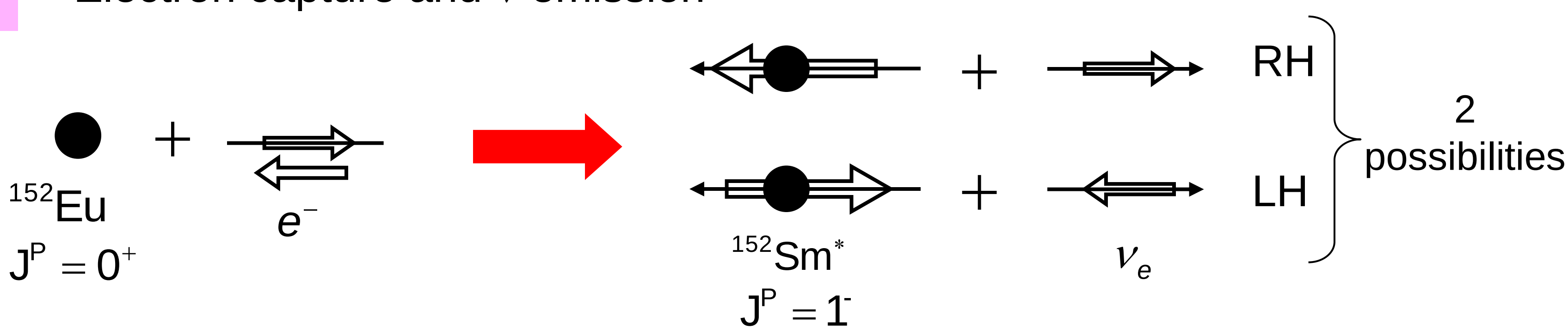
$$^{152}Eu + e^{-} \rightarrow \; ^{152}Sm^{*} + \nu_e$$

$$E_\nu \approx 950 KeV$$



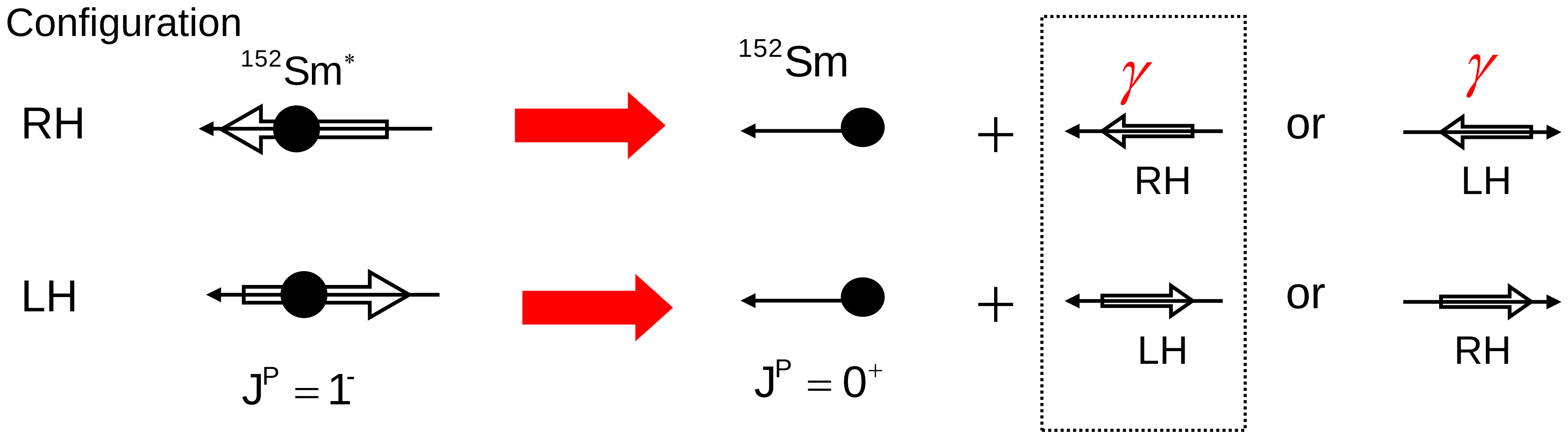
<u>Idea of the experiment:</u>

1. Electron capture and ν emission



Sm undergoes a small **recoil (p_{recoil} =950 KeV)**. Because of angular momentum conservation Spin J=1 of Sm^* is opposite to neutrino spin. Important: **neutrino helicity is transferred to the Sm nucleous.**

2. γ emission: ${}^{152}Sm^*(J^P = 1^-) \rightarrow {}^{152}Sm(J^P = 0^+) + \gamma$



Photons along the Sm recoil direction carry the polarization of the Sm* nucleus

- How to select photons along the recoil direction ? $\Rightarrow$ 3
- How to determine the polarization of these photons ? $\Rightarrow$ 4

3. Resonant photon scattering: $\gamma + {}^{152}\text{Sm} \rightarrow {}^{152}\text{Sm}^* \rightarrow {}^{152}\text{Sm} + \gamma$

<u>Resonant scattering:</u>

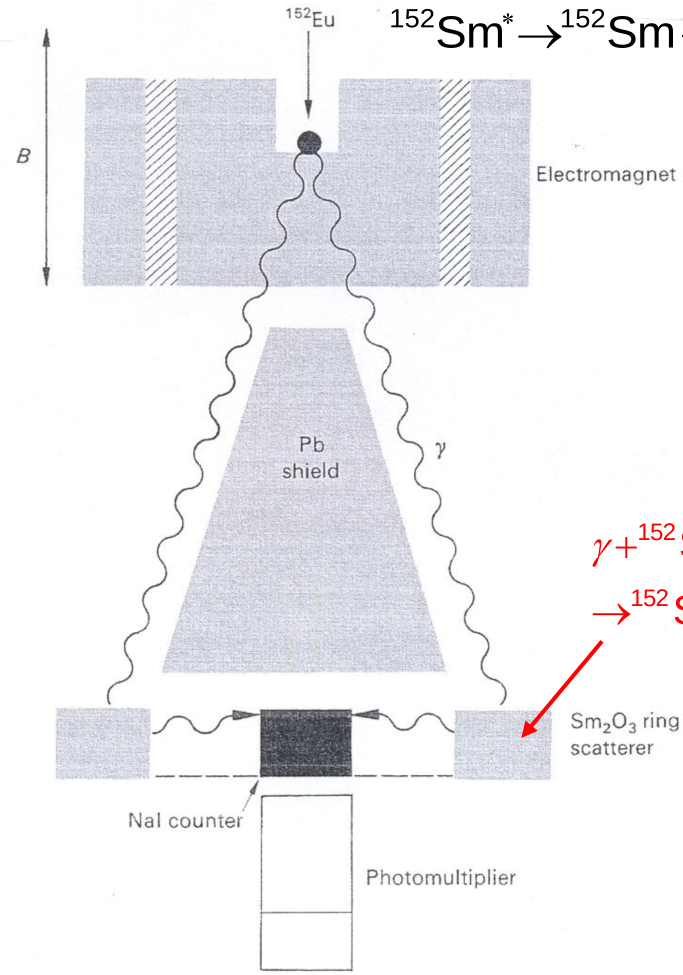
To compensate the nuclear recoil, the photon energy must be slightly larger than 960 keV.

This is the case for photons which have been emitted in the direction of the Eu→Sm recoil (Doppler-effect).

↓

Resonant scattering only possible for “forward” emitted photons, which carry the polarization of the Sm* and thus the polarization of the neutrinos.

$^{152}\text{Sm}^* \rightarrow {}^{152}\text{Sm} + \gamma$

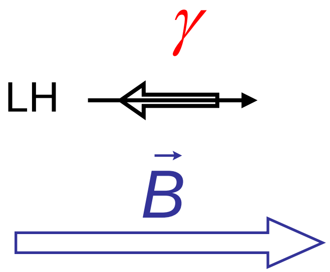


$\gamma + {}^{152}\text{Sm} \rightarrow {}^{152}\text{Sm}^*$
$\rightarrow {}^{152}\text{Sm} + \gamma$

Fig. 7.8. Schematic diagram of the apparatus used by Goldhaber *et al.*, in which γ-rays from the decay of $^{152}\text{Sm}^*$, produced following K-capture in ^{152}Eu, undergo resonance scattering in Sm_2O_3 and are recorded by a sodium iodide scintillator and photomultiplier. The transmission of photons through the iron surrounding the source depends on their helicity and the direction of the magnetic field **B**.

4. Determination of the photon polarization

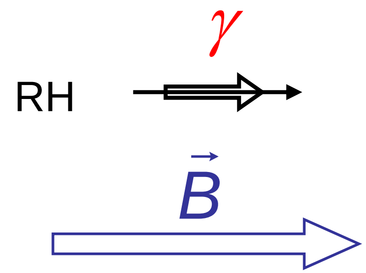
Exploit that the transmission index through magnetized iron is polarization dependent: <u>Compton scattering in magnetized iron</u>



Polarization of
electrons in iron
(to minimize pot. energy)

LH photons cannot be absorbed:

Good transmission



Polarization of
electrons in iron

Absorption leads to spin flip

RH photons undergo Compton scattering: Bad transmission

Photons w/ polarization anti-parallel to magnetization undergo less absorption

Experiment

Sm^* emitted photons pass through the magnetized iron. Resonant scattering allows the photon detection by a NaJ scintillation counter. The counting rate difference for the two possible magnetizations measure the polarization of the photons and thus the helicity of the neutrinos.

<u>Results:</u>

$$P_\gamma = -0.66 \pm 0.14$$

→ photons from Sm^* are left-handed. The measured photon polarization is compatible with a neutrino helicity of H=–1/2.

From a calculation with 100% photon polarization one expects a measurable value P_γ~0.75. Reason is the finite angular acceptance.
→ Also not exactly forward-going γ's can lead to resonant scattering.

Summary: Lepton polarization in β decays

	e^-	e^+	ν	$\bar{\nu}$
$H=\frac{1}{2}\cdot$	-v/c	+v/c	-1	+1

3. "V-A Theory" for charged current weak interactions

3.1 Lorentz structure of the weak currents

Fermi:

$$M = C_V \, J_{N,\mu} \cdot J_e^{\mu^+} = C_V \left(\bar{u}_p \gamma_\mu u_n \right) \cdot \left(\bar{u}_e \gamma_\mu v_\nu \right)$$

Cannot explain the parity violation in beta decays.

More general ansatz
(Gamov & Teller)

$$M = \sum_i C_i \left(\bar{u}_p \Gamma_i u_n \right) \cdot \left(\bar{u}_e \Gamma_i v_\nu \right)$$

$$i = \mathrm{S, P, V, A, T}$$

$$\bar{u}_p \; \Gamma_i \; u_n$$

bilinear Lorentz covariants:

$$\bar{\psi} \; (4 \times 4) \psi$$

$$\bar{u}_p \, \Gamma_i \, u_n = \begin{cases} S: & \bar{u}_p u_n & \text{scalar} \\ P: & \bar{u}_p \gamma^5 u_n & \text{pseudo-scalar} \\ V: & \bar{u}_p \gamma^\mu u_n & \text{vector} \\ A: & \bar{u}_p \gamma^5 \gamma^\mu u_n & \text{pseudo-vector} \\ T: & \bar{u}_p \sigma^{\mu\nu} u_n & \text{tensor} \end{cases}$$

$$\sigma^{\mu\nu} = \frac{i}{2}\left(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu\right)$$

$$\gamma^5 = i\,\gamma^0 \gamma^1 \gamma^2 \gamma^3$$

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \Rightarrow \quad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\left(\gamma^5\right)^2 = 1 \qquad \gamma^5 \gamma^\mu + \gamma^\mu \gamma^5 = 0$$

Remark: Pure P or A couplings do not lead to observable parity violation!

Mixtures like $(1 \pm \gamma^5)$ or $\gamma^\mu (1 \pm \gamma^5)$ do violate parity.

3.2 Chirality

Operators

$$P_L \equiv \frac{1}{2}(1-\gamma^5)$$

$$P_R \equiv \frac{1}{2}(1+\gamma^5)$$

are projection operators:

$$(P_i)^2 = P_i\ , \quad P_L + P_R = 1, \quad P_L P_R = 0$$

(properties of γ^5)

working on the fermion spinors they result in the left / right handed chirality components:

$$u_L = \frac{1}{2}(1-\gamma^5)u$$

$$u_R = \frac{1}{2}(1+\gamma^5)u$$

Not observable!

In contrary to helicity, which is an observable: $\frac{1}{2}\frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}$

Reminder: Dirac spinors ➡ Eigenvectors of helicity operator

solution spin↑ i.e. helicity $\lambda = +\frac{1}{2}$

$$u_1(p) = \sqrt{E+m}\cdot\begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x + ip_y}{E+m} \end{pmatrix}$$

$\vec{p}$ along z

$$u_1(p) = \sqrt{E+m}\cdot\begin{pmatrix} 1 \\ 0 \\ p/(E+m) \\ 0 \end{pmatrix}$$

solution spin↓ i.e. helicity $\lambda = -\frac{1}{2}$

$$u_2(p) = \sqrt{E+m}\cdot\begin{pmatrix} 0 \\ 1 \\ \frac{p_x - ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}$$

$\vec{p}$ along z

$$u_2(p) = \sqrt{E+m}\cdot\begin{pmatrix} 0 \\ 1 \\ 0 \\ -p/(E+m) \end{pmatrix}$$

$$\frac{1}{2}\frac{\Sigma^k p^k}{|\vec{p}|}u_1 = \frac{1}{2}\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}u_1 = \frac{1}{2}u_1$$

$$\frac{1}{2}\frac{\Sigma^k p^k}{|\vec{p}|}u_1 = \frac{1}{2}\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}u_1 = -\frac{1}{2}u_2$$

Dirac spinors and chirality projection operators:

Positive helicity:

$$\frac{1-\gamma^5}{2}u_1 = \underbrace{\frac{1}{2}\sqrt{E+m}\cdot\left(1-\frac{p}{E+m}\right)}_{\approx 0 \text{ for } E \gg m}\cdot\begin{pmatrix}1\\0\\-1\\0\end{pmatrix} \quad \to 0 \quad \text{for } E \gg m$$

$$\gamma^5 = \begin{pmatrix}0 & I\\ I & 0\end{pmatrix}$$

Negative helicity:

$$\frac{1-\gamma^5}{2}u_2 = \underbrace{\frac{1}{2}\sqrt{E+m}\cdot\left(1+\frac{p}{E+m}\right)}_{\approx \sqrt{E} \quad \text{for } E \gg m}\cdot\begin{pmatrix}0\\1\\0\\-1\end{pmatrix} \quad \to u_2 \quad \text{for } E \gg m$$

In the **relativistic limit** helicity states are also eigenstates of the chirality operators.

Polarization for particles with finite mass

Left handed spinor component of unpolarized electron:

$$u_1, u_2 \rightarrow u_L, u_R$$

$$u_L = \frac{1-\gamma^5}{2} \underbrace{(u_1 + u_2)}_{\text{unpolarized}}$$

Not normalized

Helicity polarization of left handed chirality state u_L :

$$Pol = \frac{P(\lambda = +1/2) - P(\lambda = -1/2)}{P(\lambda = +1/2) + P(\lambda = -1/2)} = \frac{\left|\langle u_1 | u_L \rangle\right|^2 - \left|\langle u_2 | u_L \rangle\right|^2}{\left|\langle u_1 | u_L \rangle\right|^2 + \left|\langle u_2 | u_L \rangle\right|^2}$$

$$= \frac{\left(1 - p/(E+m)\right)^2 - \left(1 + p/(E+m)\right)^2}{\left(1 - p/(E+m)\right)^2 + \left(1 + p/(E+m)\right)^2} = -\frac{p}{E} = -\frac{v}{c}$$

i.e. the LH spinor component for a particle with finite mass is not fully in the helicity state “spin down” (λ=-½) .

For massive particles of a given chirality there is a finite probability to observe the “wrong” helicity state: $\mathcal{P}$ = 1- (p/E)