Neutrino masses and global symmetries of SM

* SM Lagrangian is defined as the set of gauge invariant, renormalisable couplings between $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$, e_R; $\begin{pmatrix} u \\ d \end{pmatrix}_L$, u_R, d_R

$\Rightarrow$ In SM neutrino masses are absent.

* This Lagrangian exhibits ungauged continuous global symmetries = accidental symmetries (not built in by construction)

$\underbrace{L_e,\ L_\mu,\ L_\tau}_{\text{individual lepton numbers}}$, $\underbrace{B}_{\text{baryon number}}$

(see examples sheet 10)

But only $L_e - L_\mu$, $L_\mu - L_\tau$, $(B-L)$ are non-anomalous — remaining ones have chiral anomalies (see ex. 10)

* In addition to tree-level Lagrangian of renorm. couplings, one must consider higher-dimension, non-renormalisable interactions. These constitute the full effective Lagrangian.

Rule: $\mathcal{L}_{eff.}$ is invariant under all non-anomalous symmetries of original Lagrangian.
The anomalous symmetries are preserved in perturbation theory, but can be violated non-perturbatively.

* Observe: $\exists$ only 1 possible dim-5 operator that is gauge invariant under SM gauge group:

Weinberg operator

$$\mathcal{O}_W = \frac{\lambda}{M_*} (L\phi)^T C^{-1} (L\phi)$$

where $L\phi \equiv \varepsilon^{ij} L_i \phi_j$ i,j: $SU(2)$ indices

But: $\mathcal{O}_W$ violates $(B-L)$ & can therefore not be present within SM effective Lagrangian in above sense.

Note: $\mathcal{O}_W$ would generate neutrino Majorana masses of order $\frac{\lambda}{M_*} \cdot v^2$ after SSB (where $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix}$ after SSB)

* See-Saw mechanism = attempts to create this Weinberg operator. But this requires introduction of new degrees of freedom & their renormalisable couplings
$\Longrightarrow$ accidental symmetries change.

Type I See-Saw

$\rightsquigarrow$ new fermions ν_R (singlets)

$\longrightarrow$ $L\phi\,\nu_R$, $\nu_R^c\,\nu_R$ $\longrightarrow$ B-L not accidental symmetry

Type II See-Saw

$\rightsquigarrow$ new extra Higgs fields (bosons) whose couplings break Lepton number explicitly or spontaneously
(Ex. 11)

Remark: Some authors define SM as set of all gauge-inv. couplings — renorm. or not. Then neutrino masses via $\mathcal{O}_W$ are not "Beyond" SM by definition.

Quantum Chromodynamics (QCD)

* Based on unbroken gauge group $SU(3)$
  $\longrightarrow$ SM: $SU(3) \times SU(2) \times U(1)_Y$

* Leptons, Higgs, EW bosons uncharged under $SU(3)$

* $SU(3)$: 8 massless gauge bosons gluons $\{A_{\mu a}\}$
  $a = 1, \ldots, 8 = \dim(\text{adj.})$

* Quarks are in fundamental repr. of $SU(3)$ [in addition to $SU(2) \times U(1)_Y$]
  "coloured"
* coupling g_s strongly scale dependent

$$\alpha_s = \frac{g_s^2}{4\pi} \equiv \alpha_s(\mu)$$

$\mu \gg \Lambda_{QCD} \sim 1\,\text{GeV}$: $\alpha_s \ll 1$ $\longleftrightarrow$ perturbative definition

UV "quark-gluon" phase

$\mu \lesssim \Lambda_{QCD}$: non-perturbative regime
"hadronisation" $\longleftrightarrow$ mesons, baryons

1) Symmetries & Interactions in the UV

Consider the "fundamental" "quark-gluon" phase at $\mu > \Lambda_{QCD}$
$\longleftrightarrow$ UV theory
* $SU(3)$ generated by $T_a = \frac{1}{2}\lambda_a$ λ_a: Gell-Mann matrices
  3x3 matrices
  $T_a^\dagger = T_a$, $\mathrm{tr}\, T_a = 0$

$$F_{\mu\nu a} = \partial_\mu A_{\nu a} - \partial_\nu A_{\mu a} - g_s f_{abc} A_{\mu b} A_{\nu c}$$

* quark q_f : SU(3) - triplet : 3 - representation

$$q_f = \begin{pmatrix} q_f^1 \\ q_f^2 \\ q_f^3 \end{pmatrix}$$

f : flavour - index

$f = \{u, d, c, s, t, b\}$

$$q_f \xrightarrow{SU(3)} e^{-i\xi_a(x)\cdot\frac{\lambda_a}{2}} q_f = U \cdot q_f$$

$$A_{\mu a} \longrightarrow U A_{\mu a} U^{-1} - \frac{1}{g_s}(\partial_\mu U)U^{-1}$$

$$D_\mu q_f = \partial_\mu q_f + i g_s A_{\mu a} \cdot \frac{\lambda_a}{2} q_f$$

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_{\mu\nu a} F^{\mu\nu}_a + \sum_{f=1}^{n_f} \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f$$

<u>Remarks</u> :

1) Quark masses m_f $\longleftrightarrow$ for QCD these are "input" parameters from EW sector

$m_u \approx 1-5$ MeV, $m_d \approx 4-9$ GeV, $m_s \sim 75-170$ MeV, $m_c \sim 1.3$ GeV, $m_b \sim 4$ GeV, $m_t \sim 170$ GeV

Note : The EW masses m_f (usually massless) make only up to $1/3$ of the mass of hadrons (mesons, baryons) (for GeV baryons)

Rest : non-pert. effects

$m_u, m_d, m_s \ll 1$ GeV $\Longrightarrow$ usually set to zero in above lagrangian

Common practice: Include only quarks of $m_f \lesssim Q$ if working at scale $\mu = Q$

e.g.: at scale $\mu = M_Z$ ignore t ($m_t \sim 170$ GeV)

$\Rightarrow$ $n_f = 5$ "active"/effective quark flavours

Global Symmetries

Depending on precise assumptions on m_f, $\mathcal{L}_{QCD}$ possesses extra global, continuous symmetries

Organise: $q(x) = \begin{pmatrix} q_1 \\ \vdots \\ q_f \end{pmatrix}$

* Even for $m_f \neq 0$: $q \rightarrow e^{i\xi} q$: $U(1)_V$ global symmetry

$\longleftrightarrow$ Baryon number : conserved ✓

* If all m_f equal (e.g. if one restricts to u, d, s and 'sets' $m_u \simeq m_d \simeq m_s$)

$$q \longrightarrow e^{i\xi_b \Theta_b} q \longleftrightarrow SU(n_f)_V \text{ flavour symmetry}$$

(rotation in flavour space)

Θ_b, $b = 1, \dots, (n_f^2 - 1)$ generate $SU(n_f)_V$

e.g. $n_f = 2 \longleftrightarrow$ Isospin

$n \longleftrightarrow p$

* If all $m_f = 0$ there are chiral symmetries

$$q_L = \begin{pmatrix} q_1 \\ \vdots \\ q_f \end{pmatrix}_L \qquad q_R = \begin{pmatrix} q_1 \\ \vdots \\ q_f \end{pmatrix}_R$$

$\rightsquigarrow U(1)_A$ $\quad q_L \rightarrow e^{i\xi} q_L \qquad q_R \rightarrow e^{-i\xi} q_R$

anomalous in quantum theory

$\rightsquigarrow SU(n_f)_A$ $\quad q_L \rightarrow e^{+i\xi_b\Theta_b} q_L$

$q_R \rightarrow e^{-i\xi_b\Theta_b} q_R$

Strong CP problem & Theta-Angle

$\exists$ another gauge invariant, renormalisable term:

$$\mathcal{L}_\Theta = \frac{\Theta}{16\pi^2} F_{\mu\nu a} \tilde{F}^{\mu\nu a} \equiv \frac{\Theta}{16\pi^2} \, tr\left(F_{\mu\nu} \tilde{F}^{\mu\nu}\right)$$

$$\tilde{F}_{\mu\nu a} = \frac{1}{2} \varepsilon_{\mu\nu\rho\lambda} F^{\rho\lambda} \longrightarrow \text{dual field strength}$$

(In form language: $\int -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} d^4x = \int -\frac{1}{2} F \wedge * F$

$$\int F_{\mu\nu} \tilde{F}^{\mu\nu} d^4x = \int F \wedge F \quad)$$

$\mathcal{L}_\Theta$ is a <u>total</u> derivative:

$$\int d^4x \; tr\left(F_{\mu\nu} \tilde{F}^{\mu\nu}\right) = \frac{1}{2} \int d^4x \; \partial_\mu K^\mu$$

$$K_\mu = 4 \, \varepsilon^{\mu\nu\lambda\rho} \; tr\left[A_\nu \partial_\lambda A_\rho + \frac{2}{3} A_\nu A_\lambda A_\rho\right]$$

(in form language: $K_A = A \wedge dA + \frac{2}{3} A \wedge A \wedge A$)

Chern-Simons-Form

$\Rightarrow \mathcal{L}_\Theta$ contains no propagating d.o.f. (topological)

K^μ is <u>not</u> gauge invariant $\Longrightarrow$

$\mathcal{L}_\Theta$ is relevant if at $|\infty|$ $F \to 0$ s.t.

$$A_\mu \to U^{-1} \partial_\mu U \quad \text{pure gauge but } A_\mu \neq 0$$

($\longleftrightarrow$ instanton solutions see e.g. Cheng/Li Chapter 16)

$\boxed{\mathcal{L}_\Theta \text{ leads to } P \;\&\; CP \text{ violation}}$

Experimental absence of major CP violation in QCD sector constrains

$\boxed{\Theta < 10^{-9} \longleftrightarrow \text{strong CP problem} \quad \text{Why is } \Theta \text{ so small?}}$

Important:

For SU(2) Θ-angle can be removed by chiral trafo of fermions w/ anomalous (B+L) symmetry. QCD: masses break $U(1)_A$ [illegible] CP viol. phases!!