g) Neutrino Masses

* In the SM neutrinos are exactly massless & occur only as ν_L (left-handed) within doublet $\begin{pmatrix}\nu_L \\ \ell_L\end{pmatrix}$.

* Neutrino oscillation experiments (mid 1990s) suggest: mass differences for neutrinos:
  In mass eigenstates
$$\Delta m^2_{12} \sim 10^{-5}\, eV^2 \,, \quad |\Delta m^2_{23}| \sim 10^{-3}\, eV^2$$

Note: only mass differences known, no absolute numbers

* Modelling neutrino masses requires introduction of a right-handed neutrino ν_R (at least in most straight-forward models)

* Simplest scenario: mimic mass generation for up-quarks

$$L_N = \begin{pmatrix}\nu'_L \\ \ell'_L\end{pmatrix}_N \,, \quad R_{N-} = (\ell'_R)_N \,, \quad R_{N0} = (\nu'_R)_N$$

$$\mathcal{L}_{lept, \Phi} = -\sqrt{2}\left(\bar{L}_N f^-_{NM} \phi R_{M-} + \bar{L}_N f^0_{NM} \phi^c R_{M0} + h.c.\right)$$

$$U(1)_Y : Y(\bar{L}_N) = +\tfrac{1}{2} \,, \quad Y(\phi^c) = -\tfrac{1}{2}$$

$$\Rightarrow Y(R_{M0}) = 0 \iff (\nu_R)_N \text{ completely neutral}$$

These masses are of the form:

$$\mathcal{L} = -(\bar{\nu}'_L)_N (M_D)_{NM} (\nu'_R)_M + h.c. \quad M_D: \text{ Dirac mass matrices}$$

* Since ν_R is neutral, there exists another possible mass bilinear:

As for quarks, Dirac masses introduces mixing in flavour space in charged current

$$J^\mu = 2\,\bar{L}_N \gamma^\mu \sigma_+ L_N$$

$$= 2\,(\bar{\nu}_\tau' \;\; \bar{\nu}_\mu' \;\; \bar{\nu}_e')_L \, \gamma^\mu \begin{pmatrix} \tau' \\ \mu' \\ e' \end{pmatrix}_L$$

in terms of weak eigenstates

Diagonalising Dirac mass terms for neutrinos & lepton masses

$\longrightarrow$ mass eigenstates $\left.\begin{matrix} (\nu_\tau, \nu_\mu, \nu_e) \\ (\tau, \mu, e) \end{matrix}\right\}$ as for quarks

$$J^\mu = (\bar{\nu}_\tau, \bar{\nu}_\mu, \bar{\nu}_e)\, \gamma^\mu (1-\gamma_5) \cdot V_{lept.} \begin{pmatrix} \tau \\ \mu \\ e \end{pmatrix}$$

* V_{lept} governs Family mixing of neutrinos & can be measured in neutrino oscillation experiments.

* Like the CKM matrix $V_{lept.}$ introduces CP violation.
If only Dirac masses (as above) then: V_{lept} has 3 angles & 1 phase — just as V_{CKM}.

2 Challenges:

a) Naturalness:
Why are the Dirac masses for neutrinos smaller by factor of $\sim 10^{-6}$ than those for down-type quarks?
(Note however: only mass differences known experimentally)

b) There exists another possibility for neutrino masses — Majorana masses. One would need a mechanism to suppress / prevent them if neutrino masses are (only) of Dirac type

* Since ν_R is neutral, there exists another possible mass bilinear — the Majorana mass term:

Recall: For Dirac spinor $\psi(x)$:

$$\psi^c(x) = C\,\overline{\psi}^T(x) = C\gamma^0\psi^*(x)$$

$$\overline{\psi}^c(x) = -\psi^T C^{-1} = \psi^T C^*$$

$$(\psi_R)^c = \frac{1}{2}(1-\gamma^5)\psi^c = (\psi^c)_L$$

$$(\overline{\psi_R})^c = \overline{\psi}^c \cdot \frac{1}{2}(1+\gamma^5)$$

Therefore: $(\overline{\psi_R})^c \cdot \psi_R \neq 0$

Note: If ψ_R carries charge, then $(\overline{\psi_R})^c\psi_R$ is not gauge invariant because

$$(\overline{\psi_R})^c \cdot \psi_R = \psi_R^T C^* \psi_R$$

Consider only one family of neutrinos:

$\Rightarrow$ Majorana mass: $\mathcal{L}_M = -M_M\left(\overline{\nu}_R^c\,\nu_R + \overline{\nu}_R\,\nu_R^c\right)$

Dirac mass: $\mathcal{L}_D = -M_D\left(\overline{\nu}_R\nu_L + \overline{\nu}_L\nu_R\right)$

Consider only one family of neutrinos.

Define: $\chi = \nu_L + (\nu_L)^c \Rightarrow \chi^c = \chi \Leftrightarrow$ Majorana spinor

$\omega = \nu_R + (\nu_R)^c \qquad \omega^c = \omega$

then: $\mathcal{L}_M = -M_m\,\overline{\omega}\,\omega$ and

$$\mathcal{L}_{mass} = -M_D\left(\overline{\nu}_R\nu_L + \overline{\nu}_L\nu_R\right) - M_m\left(\overline{\nu}_R^c\nu_c + \overline{\nu}_R\nu_R^c\right)$$

$$= -(\overline{\chi}, \overline{\omega})\begin{pmatrix} 0 & \frac{1}{2}M_D \\ \frac{1}{2}M_D & M_m \end{pmatrix}\begin{pmatrix} \chi \\ \omega \end{pmatrix}$$

Note: $\frac{1}{2}(1-\gamma_5)\chi = \frac{1}{2}(1-\gamma_5)\nu_L \equiv \nu_L$

$\frac{1}{2}(1+\gamma_5)\omega \equiv \nu_R$

$\frac{1}{2}(1-\gamma_5)\omega \equiv \nu_R^c$

* Diagonalise:

eigenvalues: $m_{1,2} = \frac{1}{2}\left(M_m \pm \sqrt{M_m^2 + M_D^2}\right)$

eigenvectors: $\eta_1 = \cos\Theta\,\chi - \sin\Theta\,\omega \longleftrightarrow$ lower sign

$\eta_2 = \sin\Theta\,\chi + \cos\Theta\,\omega \longleftrightarrow$ upper sign

$\tan(2\Theta) = -M_D/M_m$

$\rightarrow$ For general M_D, M_m get admixture of χ/ω in mass eigenstates.

* See-saw mechanism:

To explain smallness of observed mass eigenvalues we take:

$M_D \sim$ order electro-weak masses $\lesssim 10^3$ GeV

$M_M \sim \gtrsim 10^{15}$ GeV $\Big\}\ M_D \ll M_M$

$\rightarrow |m_1| \cong \frac{1}{4}\cdot\frac{M_D^2}{M_m} \sim 1\,\text{eV} \longleftrightarrow \eta_1 \cong \chi = \nu_L + (\nu_L)^c$

$m_2 \cong M_m \longleftrightarrow \eta_2 \cong \omega = \nu_R + (\nu_R)^c$

i.e.: $\nu_L = \frac{1}{2}(1-\gamma_5)\chi \cong (\eta_1)_L$ where η_1 of small mass $\sim M_D^2/M_M$

$\nu_R = \frac{1}{2}(1+\gamma_5)\omega \cong (\eta_2)_R \longleftrightarrow$ heavy eigenstate

Note:

$M_M = 10^{15}$ GeV is upper limit in order to

eV - scale (light) mass eigenvalue

$\frac{M_D^2}{M_M}$ for $M_D \leqq \mathcal{O}\left(\begin{smallmatrix}\text{quark}\\ \text{mass}\end{smallmatrix}\right)$

Recall: $m_{top} \sim 170$ GeV $\Rightarrow M_D \leqq 10^3$ GeV

$\Rightarrow \frac{M_D^2}{M_M} \sim \frac{10^6}{10^{15}}$ GeV $\sim 10^{-9}$ GeV ✓

More natural is $M_D \leqq 10 - 10^2$ GeV

$\Rightarrow M_M \sim 10^{11} \sim 10^{15}$ GeV

Compare: $M_{Planck} = 10^{19}$ GeV

$M_{GUT} = 10^{16}$ GeV

$\Rightarrow$ Majorana mass scale is "New Physics"
& begs for theoretical explanation

In particular: Why is $M_M \neq M_{GUT}$?

$\rightsquigarrow$ Beyond Standard Model Physics

Generalisations to more families

* Assume 3 families of ν_L' as in $\begin{pmatrix} \nu_L' \\ l_L' \end{pmatrix}_i$ $\quad i = 1,2,3$
  $\hookrightarrow$ EW eigenstates
* Add m copies of $(\nu_R')_\alpha$ $\quad \alpha = 1, \ldots, m$
  $(\nu_R')_\alpha$: SM singlets (as above)

  $\rightsquigarrow$ most natural choice: $m = 3$

  $\rightsquigarrow$ $m = 2$ possible if only 2 neutrino families massive
  (Recall: only Δm_{12}^2, $|\Delta m_{13}^2|$ known)

  $\rightsquigarrow$ $n \geq 4$: consistent $\rightsquigarrow$ extended models w/ extra sterile (singlet) neutrinos

* Introduce: $\quad \vec{\chi}' \equiv (\nu_L' + (\nu_L')^c)_i \qquad i = 1,2,3$

$$\vec{\omega}' \equiv (\nu_R' + (\nu_R')^c)_\alpha \qquad \alpha = 1, \ldots, m$$

$$\mathcal{L}_{mass} = - (\vec{\chi}'^T, \vec{\omega}'^T) \underbrace{\begin{pmatrix} 0 & \frac{1}{2} M_D \\ \frac{1}{2} M_D & M_M \end{pmatrix}}_{\mathcal{M}} \begin{pmatrix} \vec{\chi}' \\ \vec{\omega}' \end{pmatrix}$$

$\mathcal{M}$: symmetric, but complex

Diagonalise: $\quad \mathcal{M} = \mathcal{V}^T \tilde{\mathcal{M}}_D \mathcal{V} \qquad \mathcal{V}$: unitary

$$\tilde{\mathcal{M}}_D = \begin{pmatrix} \varepsilon^2 \mathcal{M}_{\nu_L} & 0 \\ 0 & \mathcal{M}_{\nu_R} \end{pmatrix} \qquad \varepsilon : \text{parameter} \ll 1 \; \left(\mathcal{O}\left(M_D / M_M \right) \right)$$

$$\mathcal{V} = \begin{pmatrix} U_{11} & \varepsilon U_{12} \\ \varepsilon U_{21} & U_{22} \end{pmatrix} \qquad \mathcal{V}^+ = \mathcal{V}^{-1}$$

Charged current:

$$\vec{\nu}_L' = \frac{1}{2}(1 - \gamma_5)\, \vec{\chi}' = \frac{1}{2}(1 - \gamma^5) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathcal{V}^{-1} \underbrace{\begin{pmatrix} \vec{\chi} \\ \vec{\omega} \end{pmatrix}}_{\text{mass eigenstates}}$$

$$\begin{pmatrix}1&0\\0&0\end{pmatrix} \mathcal{V}^{-1} \cdot \begin{pmatrix}\vec{\nu}_L\\ \vec{\nu}_R^c\end{pmatrix} = U_{11}^\dagger \vec{\nu}_L + \varepsilon U_{21}^\dagger \vec{\nu}_R^c$$

$$\vec{\nu}_L' = U_{11}^\dagger \vec{\nu}_L + \underbrace{\varepsilon U_{21}^\dagger \vec{\nu}_R^c}_{\substack{\text{small admixture from heavy} \\ \text{right-handed neutrino mass eigenstates}}}$$

$\vec{\nu}_R^c$: heavy neutrino eigenstates

$\vec{\nu}_L$: light neutrino eigenstates

$\Rightarrow$ charged current:

$$J^\mu = \overline{\vec{\nu}}_L \gamma^\mu (1-\gamma^5) V_{PMNS} \vec{\ell}_L \quad \Big\} \text{ primary mixing}$$

$$+ \overline{\vec{\nu}}_R^c \gamma^\mu (1-\gamma^5) \varepsilon \cdot \tilde{V} \vec{\ell}_L \quad \Big\} \text{ suppressed}$$

$\Rightarrow$ good approximation to truncate mixing to (3×3) Pontecorvo - Maki - Nakagawa - Sakata (PMNS) matrix

complete mixing depends on

$3(3+2n-1)/2$ mixing angles

$3(3+2n-1)/2$ phases

(Details: Schechter, Valle Phys Rev D 22, 2227 (1980))

V_{PMNS} is analogue of V_{CKM} in Majorana models.

For Majorana masses phase shifts of fermions do <u>not</u> leave the mass matrix intact — unlike for Dirac masses!

→ cannot eliminate the (6−1) phases as in V_{CKM}
<u>but only 3 phases</u> from rotation of $\vec{e}_L$

→ V_{PMNS} =

$$= V_{23} \cdot V_{31} \cdot V_{12} \cdot K$$

$$V_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_3 \\ 0 & -s_3 & c_2 \end{pmatrix}, \quad V_{31} = \begin{pmatrix} c_3 & 0 & s_3 e^{-i\delta} \\ 0 & 1 & 0 \\ -s_3 e^{i\delta} & 0 & c_3 \end{pmatrix}, \quad V_{12} = \dots$$

$$K = \mathrm{diag}\left(1, e^{i\phi_1}, e^{i\phi_2}\right) \Longleftarrow$$ not present in CKM & not present if masses are of Dirac type!

<u>Exp.:</u> $\sin^2(2\Theta)_{23} > 0.87$
$0.7 \leq \sin^2(2\Theta_{12}) \leq 0.94$ } 2 large mixings, 1 small mixing
$\sin^2\Theta_{13} < 0.051$

$|\Delta m^2_{13}| \sim 10^{-3}\, eV^2$
$\Delta m^2_{12} \sim 10^{-5}\, eV^2$ } 2 possibilities
a) usual hierarchy $m_1 < m_2 < m_3$
b) inverted hierarchy $m_3 < m_1 < m_2$

only δ relevant for neutrino oscillations
ϕ_1, ϕ_2 relevant for neutrinoless double-beta decay

* Models w/ Majorana masses also lead to $\vec{\nu}_L / \vec{\nu}_R^c$ mixing in weak current interactions

* Majorana masses <u>explicitly</u> break lepton number
  → cosmological implications on "Leptogenesis" & Baryogenesis via (B−L) breaking through sphalerons