2.4 Lepton couplings to the Z boson

In the following ignore the difference between chirality and helicity:
good approximation as leptons are produced with energies >> mass.

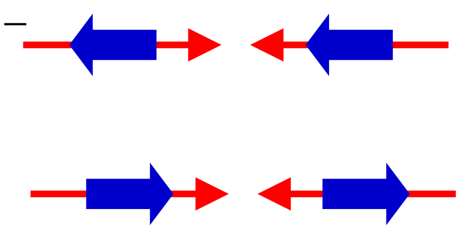
Z boson couples differently to LH and RH leptons:

$$\left| g_L = \frac{1}{2}(g_V + g_A) \right| \quad > \quad \left| g_R = \frac{1}{2}(g_V - g_A) \right|$$

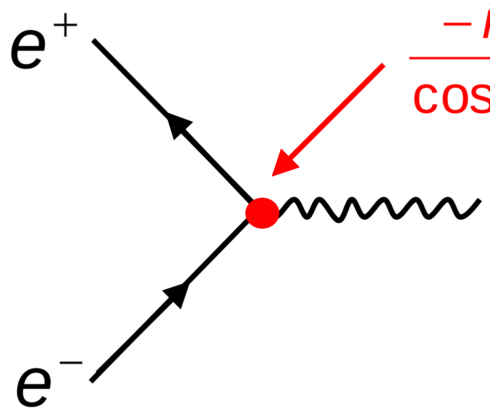
Coupling to LH leptons stronger

Z produced in e+e- collisions is polarized.

Experimental configuration:

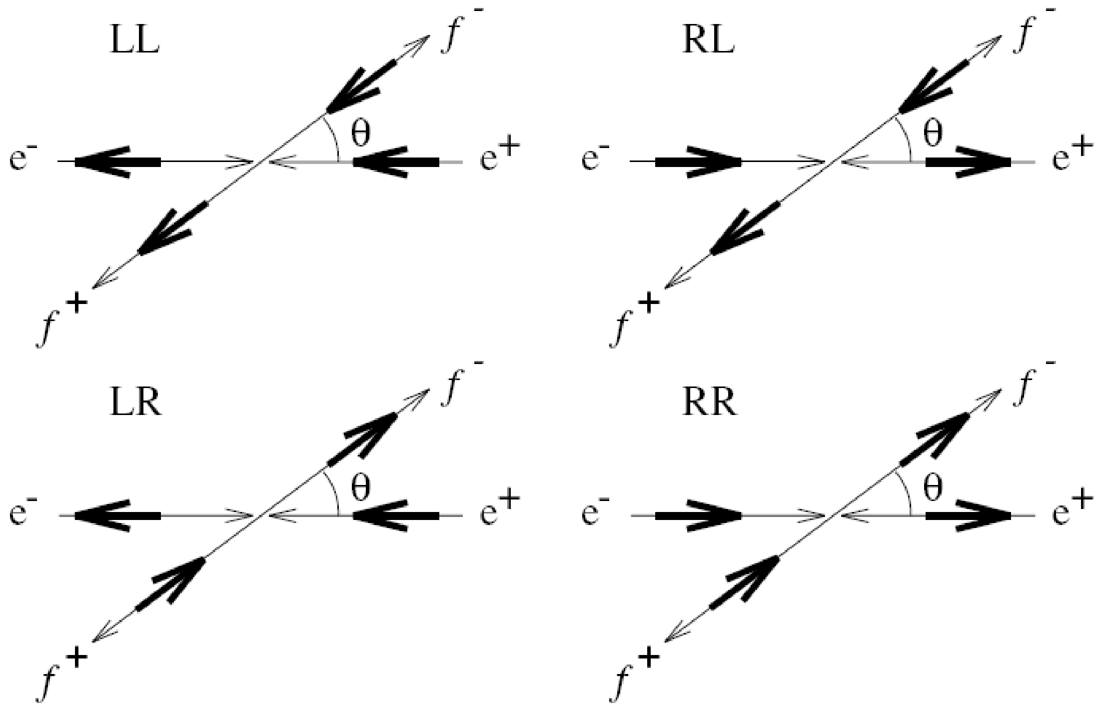
e^-  e^+ $\Rightarrow$ g_L

$\Rightarrow$ g_R

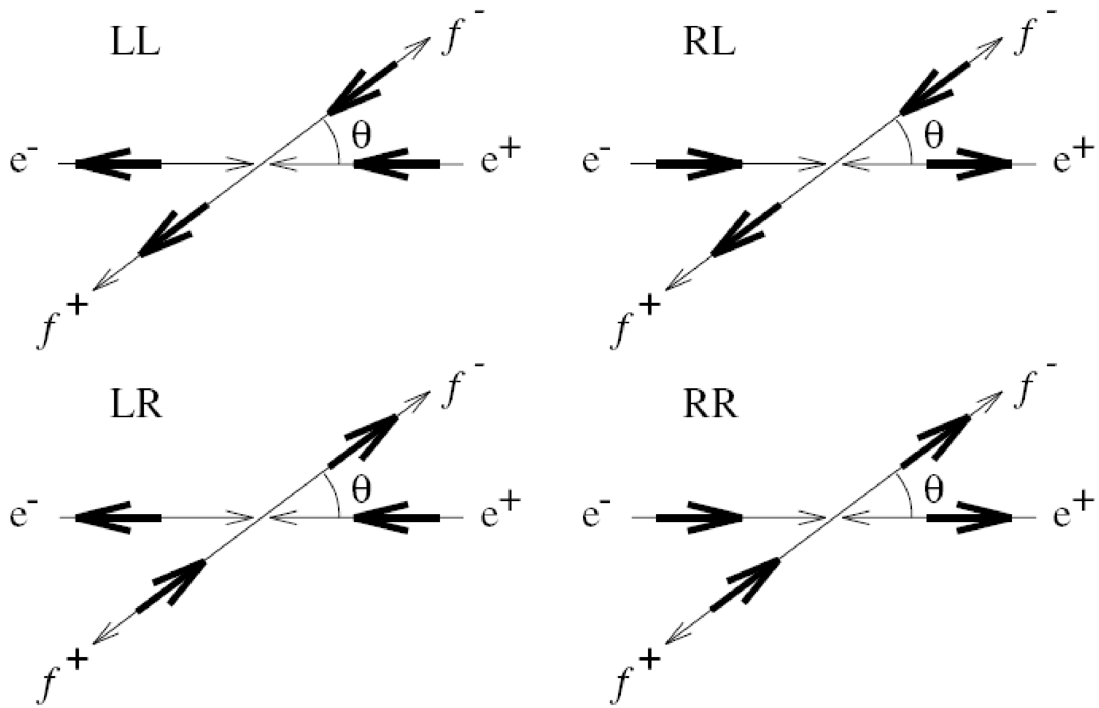


$$\frac{-ig}{\cos\theta_W}\gamma^\mu \frac{1}{2}\left(g_R^e + g_L^e\right)$$

Instead of measuring the spin averaged transition amplitudes try to decompose the different "chirality" components to the cross section:



Chirality e	f	amplitude	
L	L	$\mathcal{M}_{\mathrm{LL}} \propto g_L^f g_L^e d_{11}^1(\theta)$	$\propto g_L^f g_L^e (1 + \cos\theta)$
R	L	$\mathcal{M}_{\mathrm{RL}} \propto g_L^f g_R^e d_{11}^1(\theta+\pi)$	$\propto g_L^f g_R^e (1 - \cos\theta)$
L	R	$\mathcal{M}_{\mathrm{LR}} \propto g_R^f g_L^e d_{11}^1(\theta+\pi)$	$\propto g_R^f g_L^e (1 - \cos\theta)$
R	R	$\mathcal{M}_{\mathrm{RR}} \propto g_R^f g_R^e d_{11}^1(\theta)$	$\propto g_R^f g_R^e (1 + \cos\theta)$



Observables:

$\sigma_F = \sigma_{LL} + \sigma_{RR}$ $\sigma_B = \sigma_{RL} + \sigma_{LR}$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

Forward-backward asym. (final)

$\sigma_L = \sigma_{LL} + \sigma_{LR}$ $\sigma_R = \sigma_{RL} + \sigma_{RR}$

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

Left right asym. (initial)

$\sigma_- = \sigma_{LL} + \sigma_{RL}$ $\sigma_+ = \sigma_{RR} + \sigma_{LR}$

$$\mathcal{P}_f = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

Polarization (final)

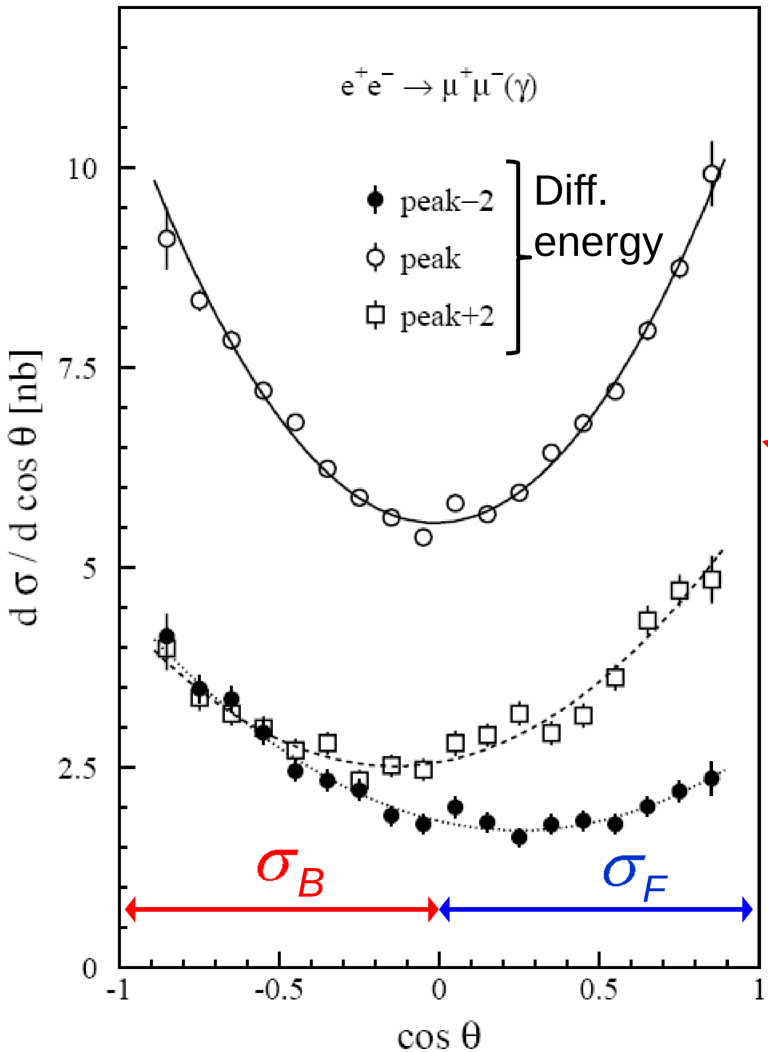
2.5 Forward-backward asymmetry and fermion couplings to Z

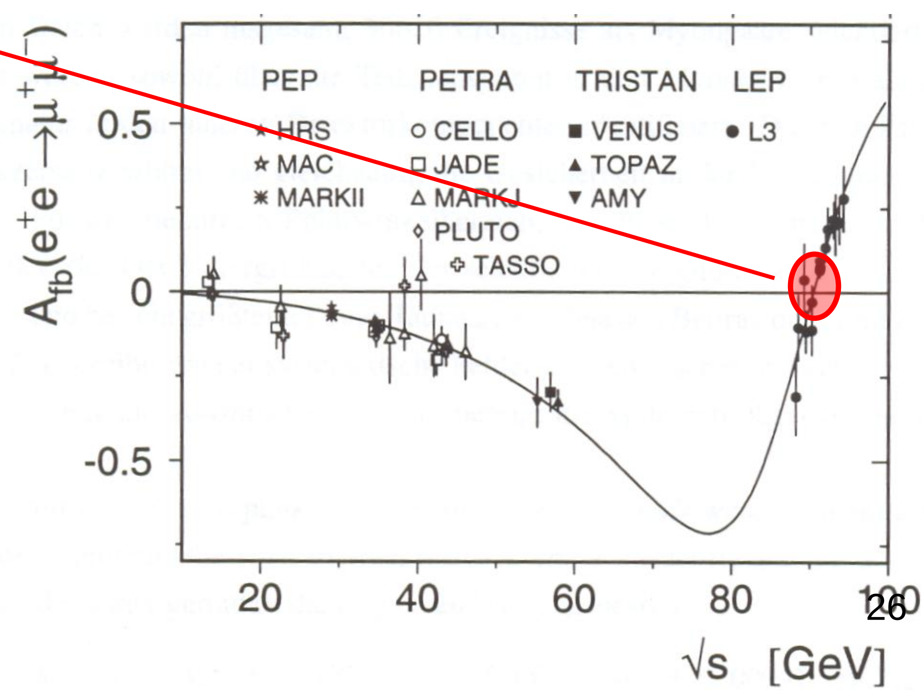
$$e^+ e^- \to Z \to \mu^+ \mu^-$$

$$\frac{d\sigma}{d\cos\theta} \sim (1+\cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta$$

with $$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_{F(B)} = \int_{0\,(-1)}^{1\,(0)} \frac{d\sigma}{d\cos\theta} d\cos\theta$$





Angular distribution:

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4\sin^2\theta_W \cos^2\theta_W}\left[2 g_V^e g_V^\mu (1+\cos^2\theta) + 4 g_A^e g_A^\mu \cos\theta\right]$$

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W \cos^4\theta_W}\left[(g_V^{e^2} + g_A^{e^2})(g_V^{\mu^2} + g_A^{\mu^2})(1+\cos^2\theta) + 8 g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta\right]$$

Forward-backward asymmetry A_{FB}

- Away from the resonance large → interference term dominates

$$A_{FB} \sim g_A^e g_A^f \cdot \frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \qquad \rightarrow \text{large}$$

- At the Z pole: Interference = 0

$$A_{FB} = 3 \cdot \frac{g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2} \cdot \frac{g_V^\mu g_A^\mu}{(g_V^\mu)^2 + (g_A^\mu)^2}$$

→ very small because g_V^l small in SM

Asymmetrie at the Z pole

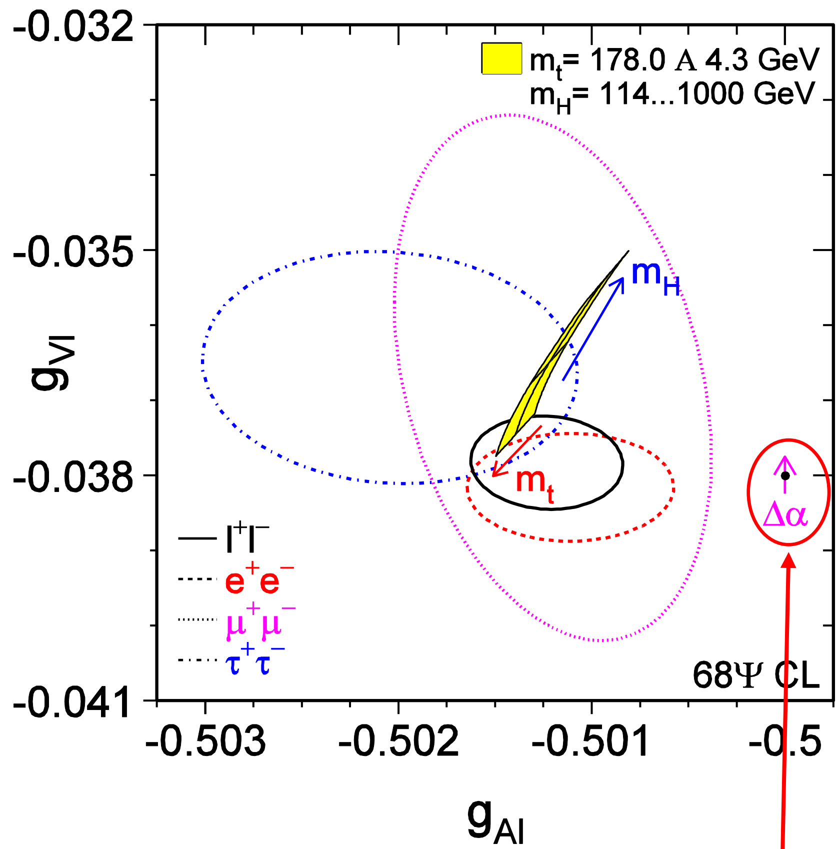
$$A_{FB} \sim g_A^e g_V^e g_A^f g_V^f$$

Cross section at the Z pole

$$\sigma_Z \sim \left[(g_V^e)^2 + (g_A^e)^2\right]\left[(g_V^\mu)^2 + (g_A^\mu)^2\right]$$

Asymmetries together with cross sections allow the determination of the lepton couplings g_A and g_V.

Good agreement between the 3 lepton species confirms “lepton universality”



Lowest order SM prediction:

$$g_V = T_3 - 2q\sin^2\theta_W \quad g_A = T_3$$

Deviation from lowest order SM prediction is an effect of rad. correct.

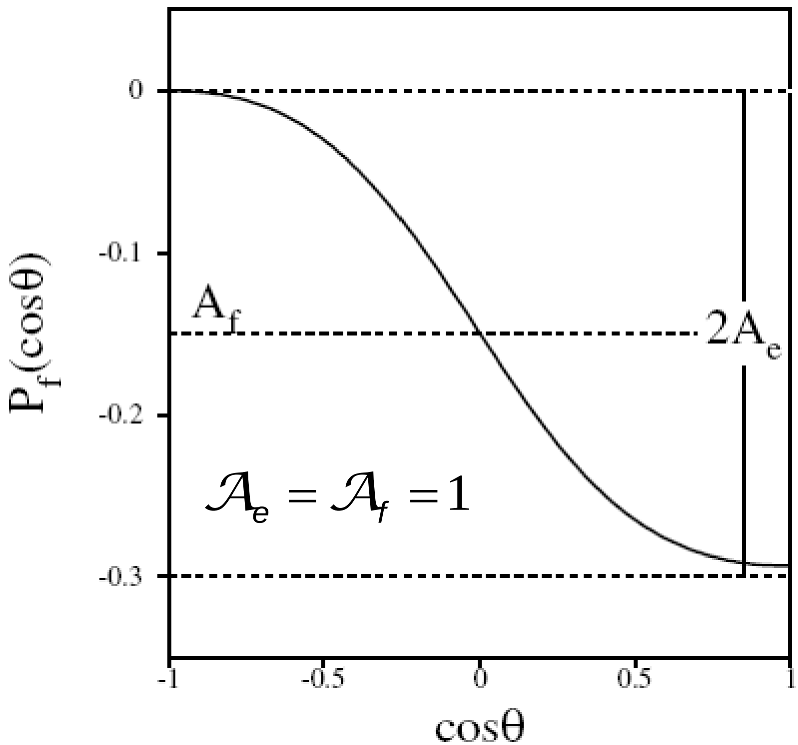
2.6 Polarization of final state leptons: tau pol.

$$\mathcal{P}_f(\cos\theta) = \frac{\frac{d\sigma_+}{d\cos\theta} - \frac{\sigma_-}{d\cos\theta}}{\frac{d\sigma_+}{d\cos\theta} + \frac{\sigma_-}{d\cos\theta}}$$

$$\mathcal{P}_f(\cos\theta) = \frac{\mathcal{A}_f(1+\cos^2\theta) + 2\mathcal{A}_e\cos\theta}{(1+\cos^2\theta) + 8/3\, A_{FB}\cos\theta}$$

with $$\mathcal{A}_i = \frac{2g_V^i g_A^i}{(g_V^i)^2 + (g_A^i)^2}$$

$$\mathcal{P}_\ell \approx -2\frac{2g_V^\ell}{g_A^\ell} = -2(1 - 4\sin^2\theta_W)$$

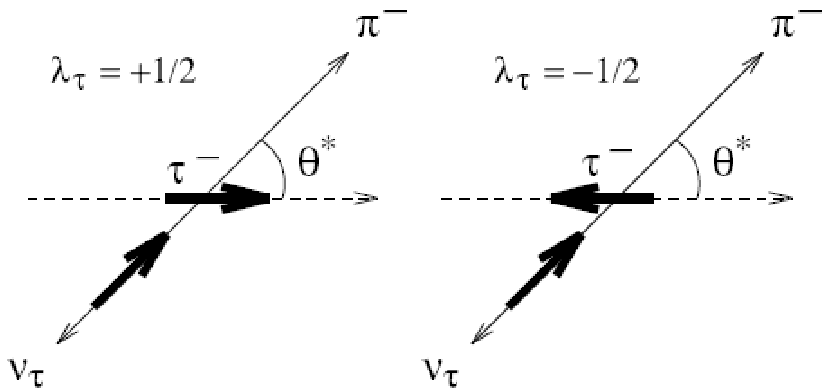


(cosθ is the fermion scattering angle)

Lepton polarization measures directly $\sin^2\theta_W$.
The only lepton for which polarization can be measured at LEP is the tau!

Experimental Method to measure tau polarization:

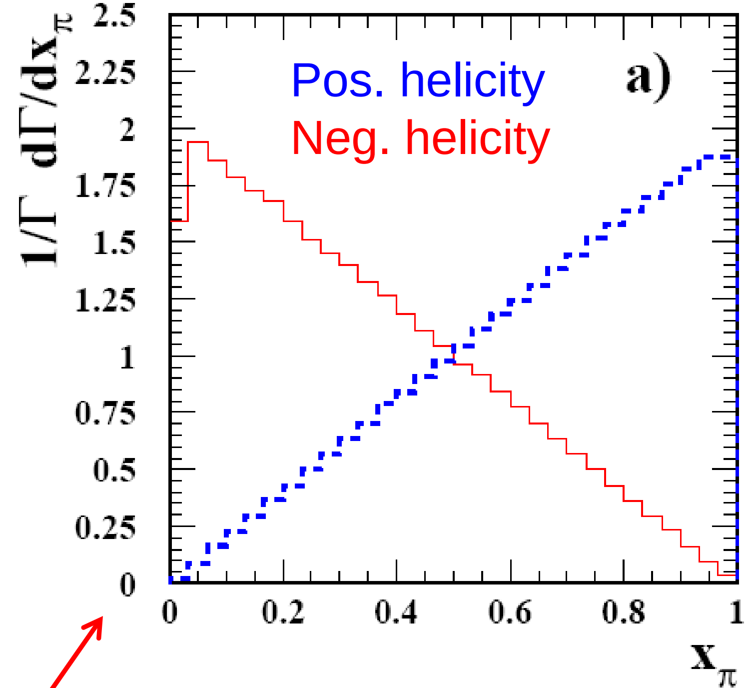
$$\tau^- \rightarrow \pi^- \nu_\tau$$

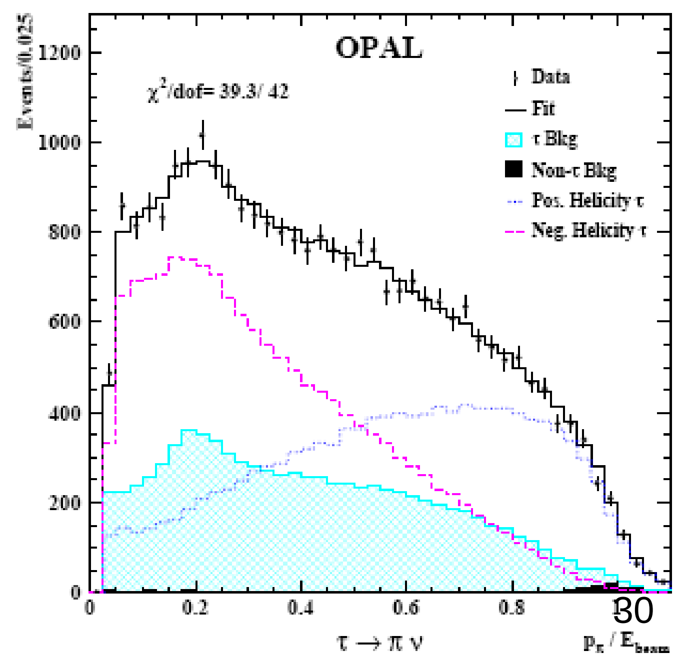


$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta^*} = \frac{1}{2}(1+\mathcal{P}_\tau \cos\theta^*)$$

Boost into lab frame

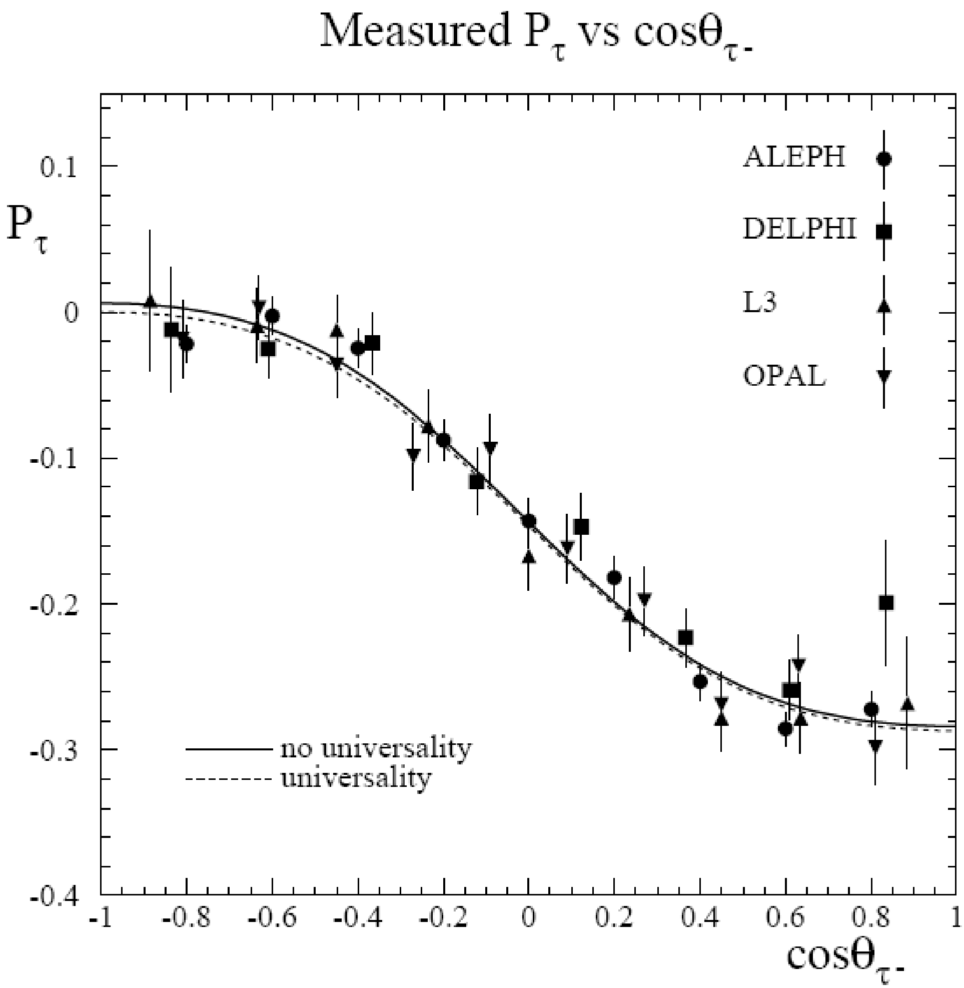
$$\frac{1}{\Gamma}\frac{d\Gamma}{dx_\pi} = 1+\mathcal{P}_\tau(2x_\pi - 1) \qquad x_\pi = E_\pi / E_\tau$$





Fit of the two theoretical distribution to data yields the polarization: ~ 0.15

Measured Tau Polarization



$$\mathcal{A}_\tau = 0.1439 \pm 0.0043$$

$$\mathcal{A}_e = 0.1498 \pm 0.0049$$

$$\mathcal{A}_\ell = 0.1465 \pm 0.0033$$

$$\sin^2\theta_w^{eff} = 0.23159 \pm 0.00041$$

hep-ex/0509008

2.7 Left-Right Asymmetry at SLC

Measure cross section σ_L (σ_R) for LH (RH) initial state electrons:

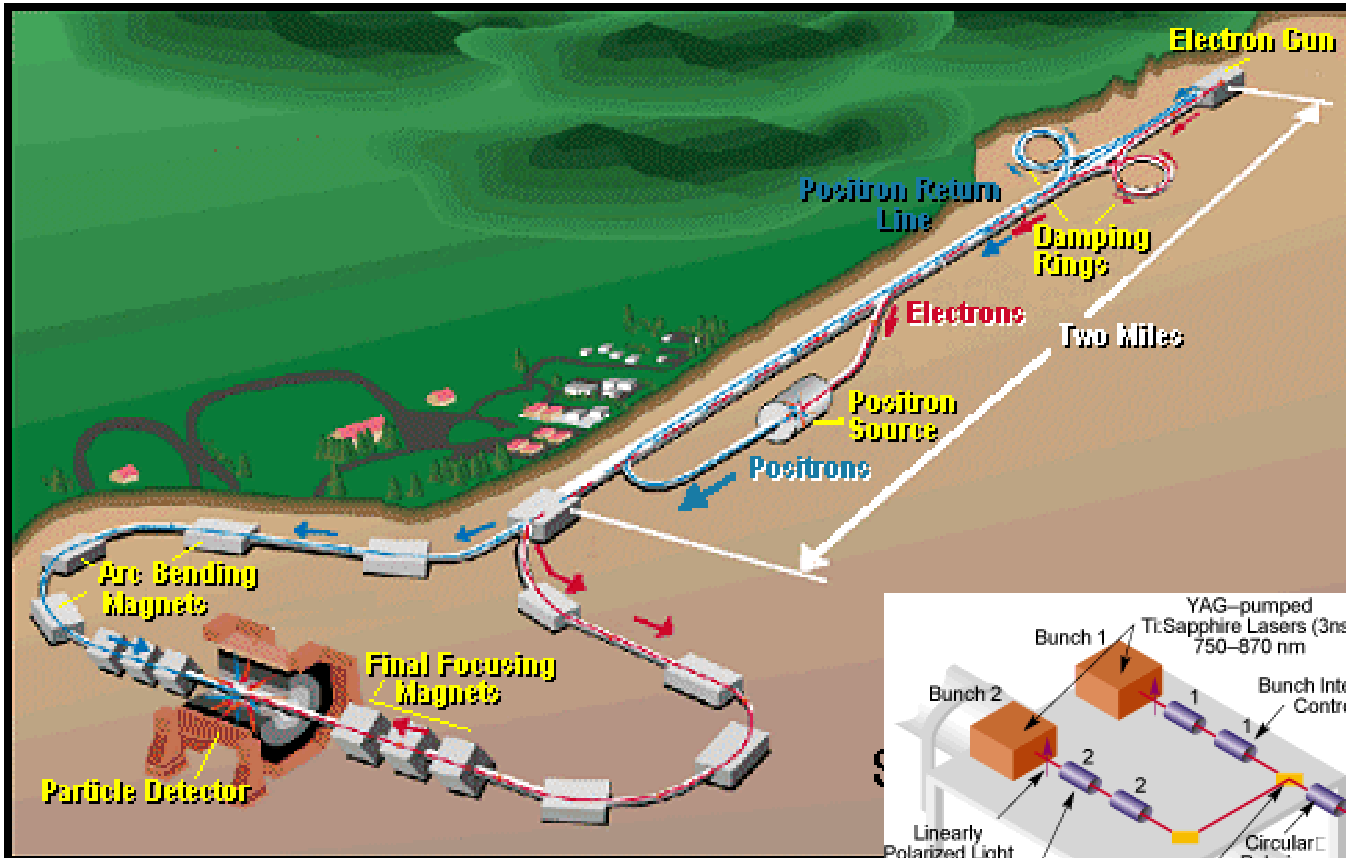
Polarization of electron beam: P~70 – 80%

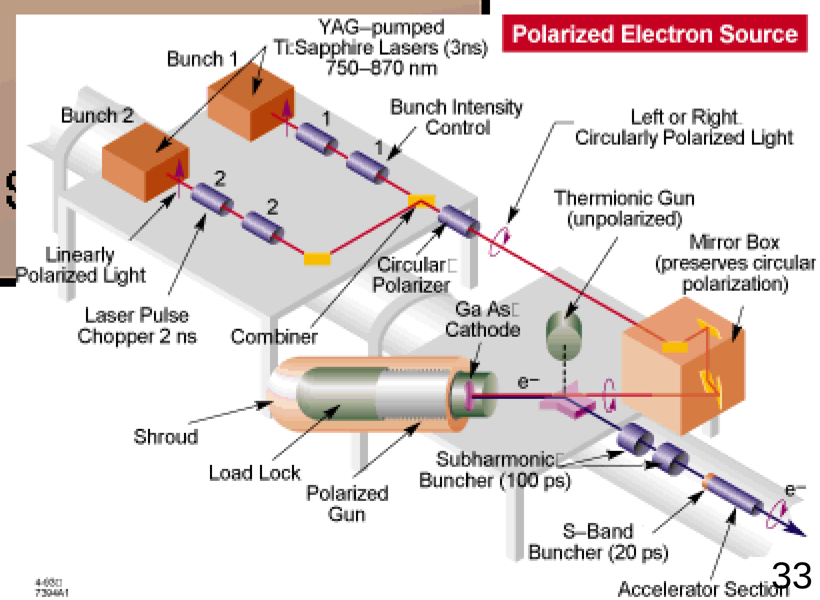
$$A_{LR} = \frac{1}{P}\frac{\sigma_L^f - \sigma_R^f}{\sigma_L^f + \sigma_R^f} = \frac{1}{P}\frac{2g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2}$$

$$= \frac{2(1-4\sin^2\theta_w)}{1+(1-4\sin^2\theta_w)^2}$$

Powerful determination of $\sin^2\theta_w$. Requires longuitudinal polarization of colliding beams: only possible in case of Linear Collider: SLC

SLAC Linear Collider





Typical beam polarization of 70%.

Precise determination of beam polarization using a Compton Polarimeter

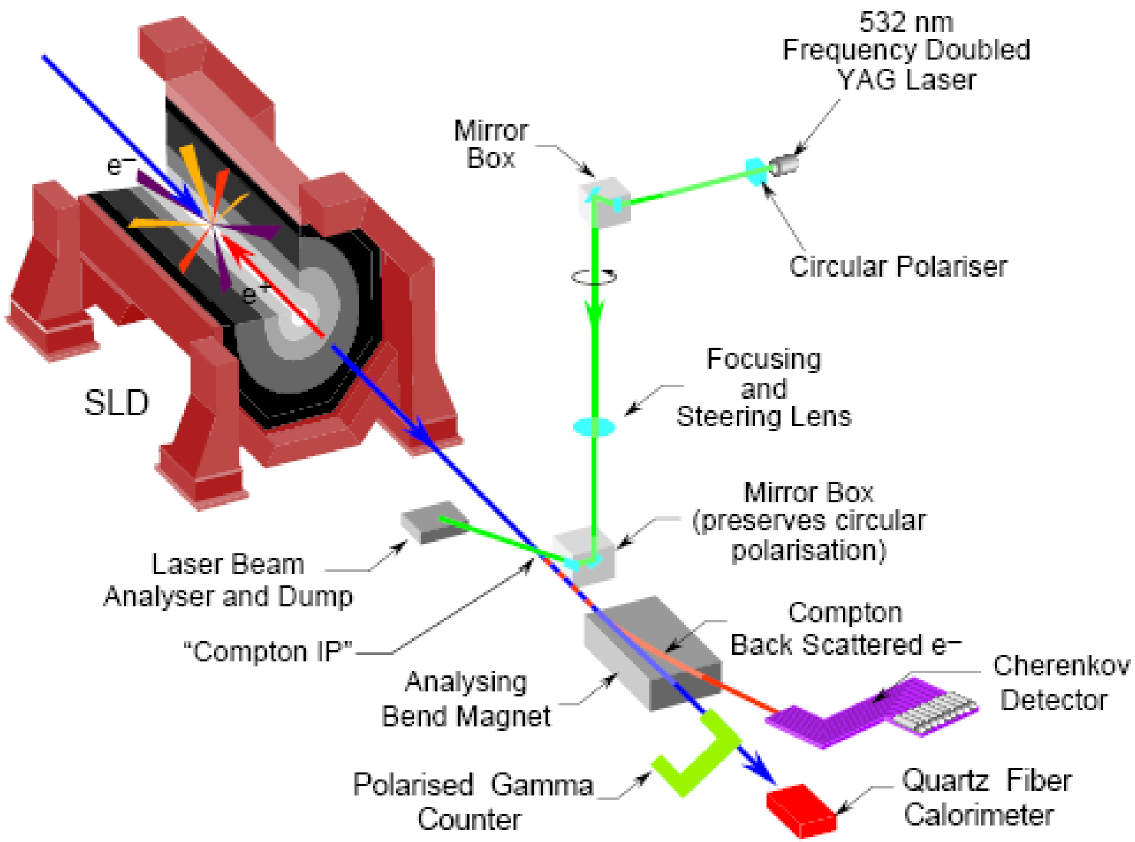
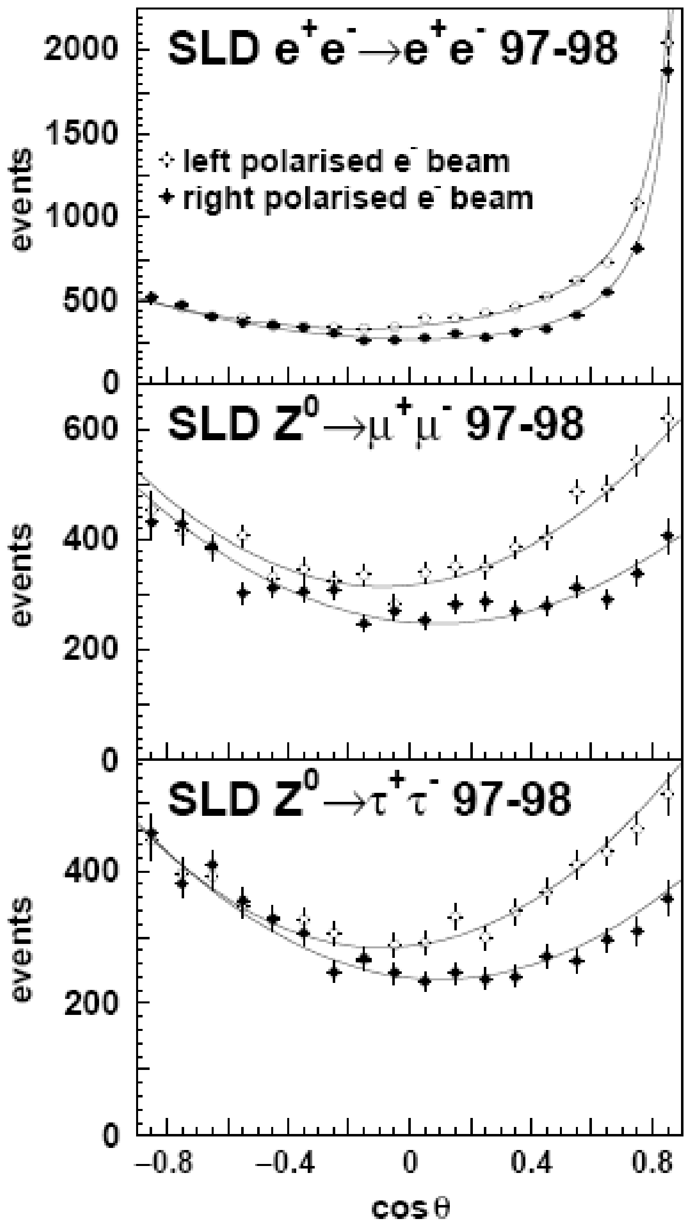


Figure 3.1: A conceptual diagram of the SLD Compton Polarimeter. The laser beam, consisting of 532 nm wavelength 8 ns pulses produced at 17 Hz and a peak power of typically 25 MW, were circularly polarised and transported into collision with the electron beam at a crossing angle of 10 mrad approximately 30 meters from the IP. Following the laser/electron-beam collision, the electrons and Compton-scattered photons, which are strongly boosted along the electron beam direction, continue downstream until analysing bend magnets deflect the Compton-scattered electrons into a transversely-segmented Cherenkov detector. The photons continue undeflected and are detected by a gamma counter (PGC) and a calorimeter (QFC) which are used to cross-check the polarimeter calibration.

Leptonic final states:



SLD

Asymmetry clearly seen for LH and RH cross section.

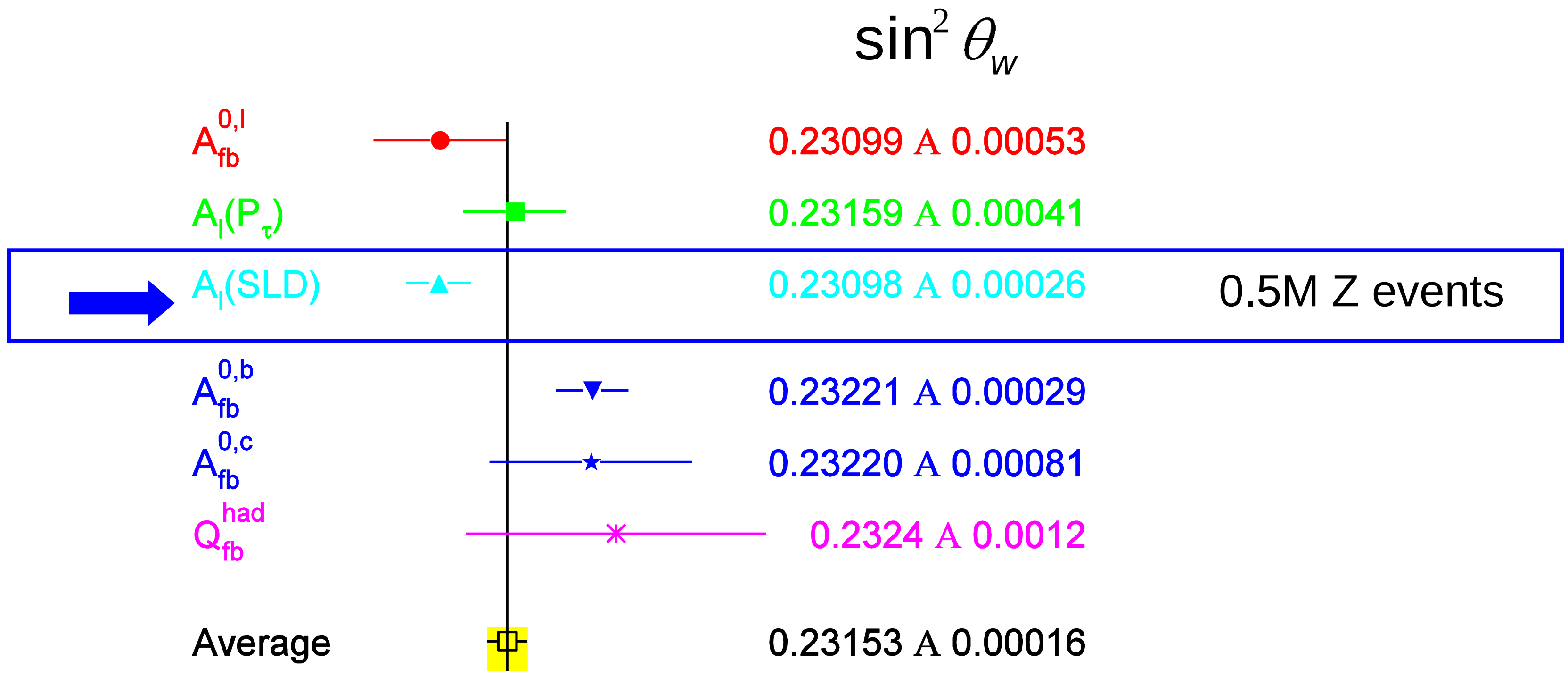
SLD All data:

$$A_{LR} = 0.1513 \pm 0.0021$$

$$\sin^2\theta_W = 0.23098 \pm 0.00026$$

With 0.5×10^6 Z-decays

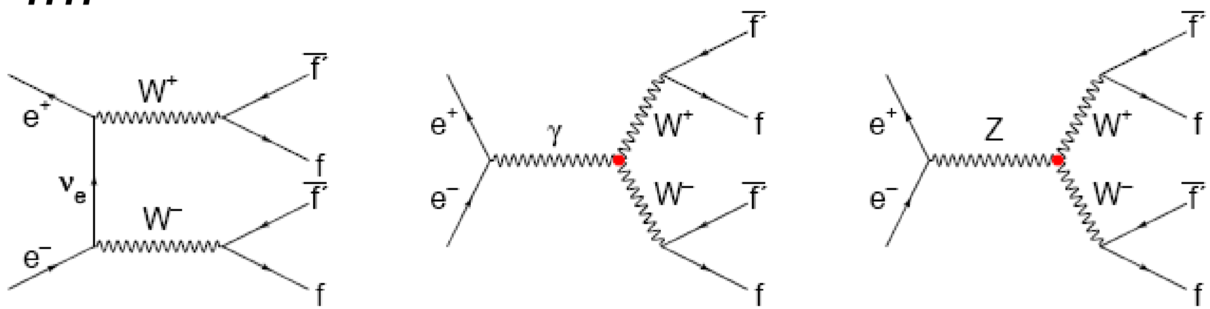
SLD versus 4×4.5 ×10^6 Z-decays at LEP



3. Precision tests of the W sector (LEP2 and Tevatron)

↑ ~10K WW events / experiment

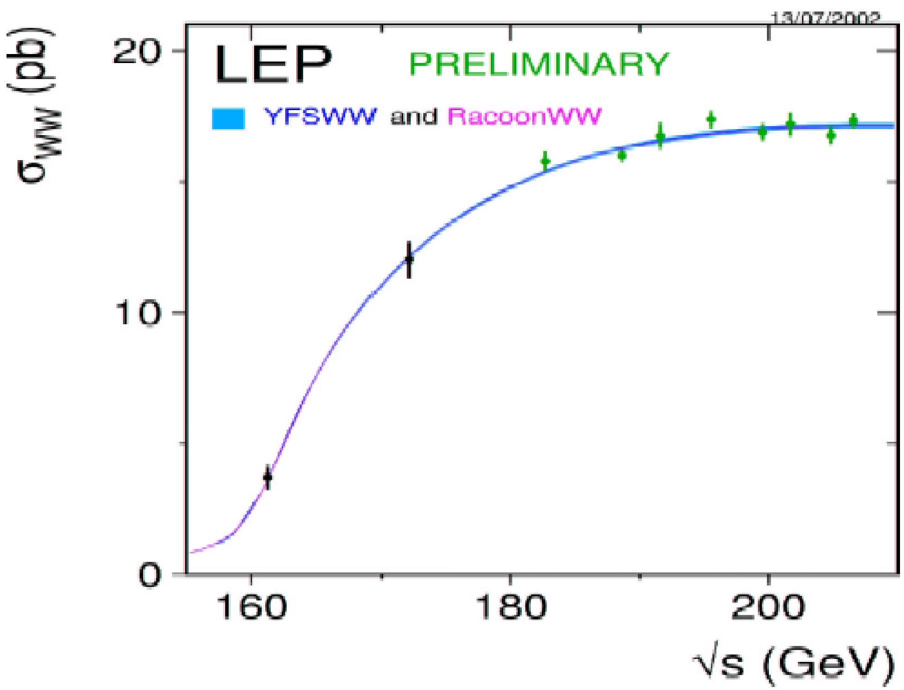
$$e^+e^- \to WW \to f\bar{f}f\bar{f}$$



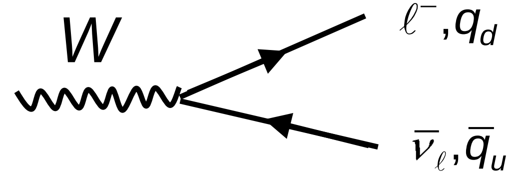
Threshold behavior of the cross section (kinematics, phase space) for ee→WW production:

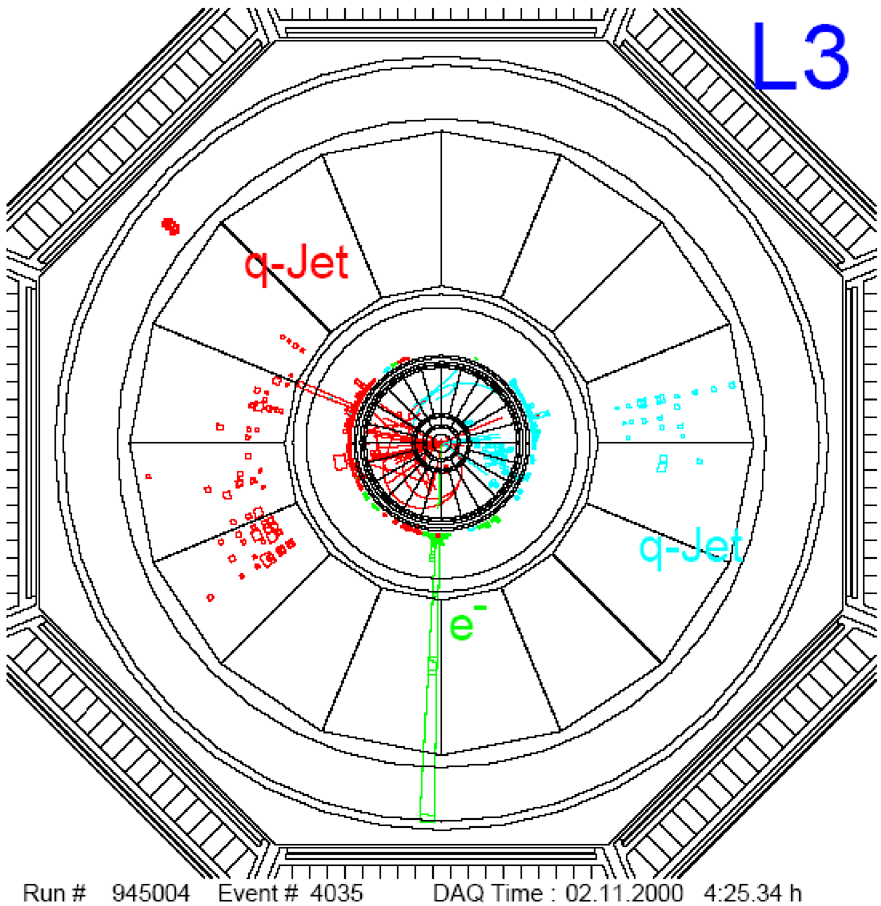
Phase space factor = f(M_W, √s):

→ Allows determination of M_W

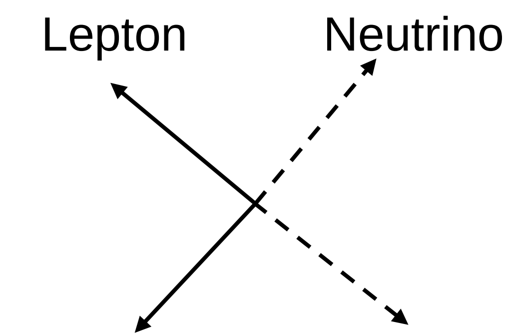


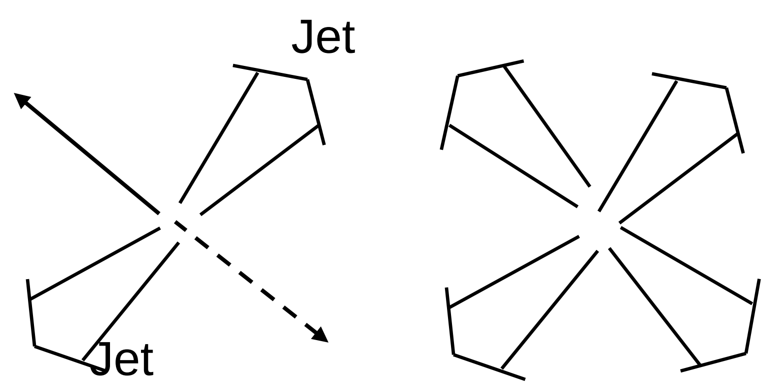
W decays





WW → { qq$\ell\nu$ 44%, qqqq 45%, $\ell\nu\ell\nu$ 11% }

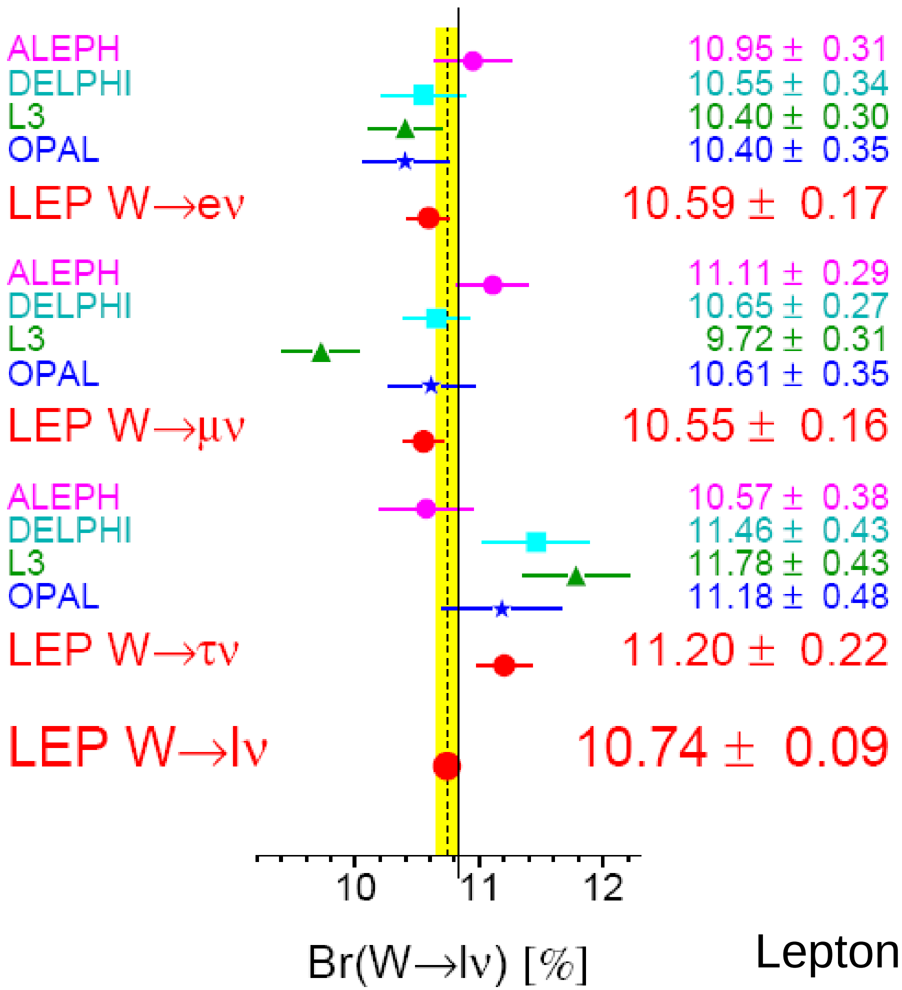




Easiest signature for a mass measurement:

$W_1 \rightarrow l\nu$ $W_2 \rightarrow$JetJet: use JetJet invariant mass

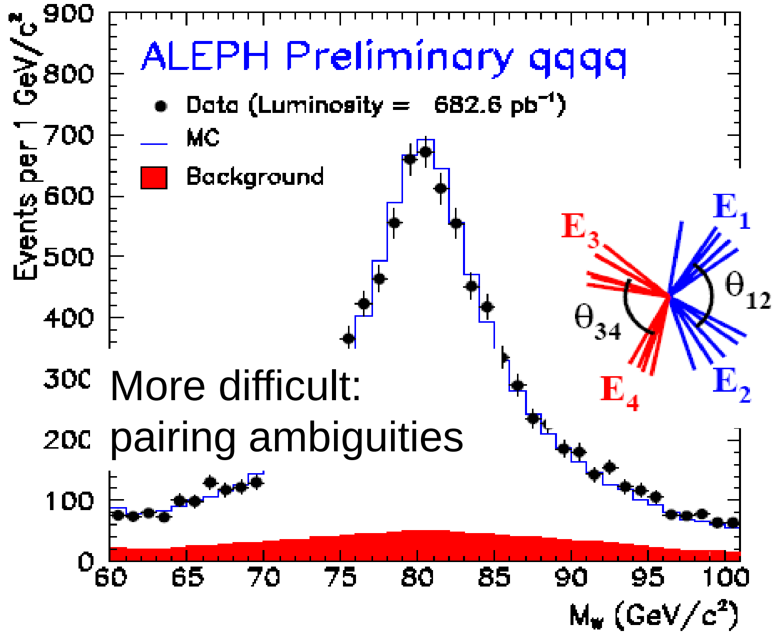
W branching ratios



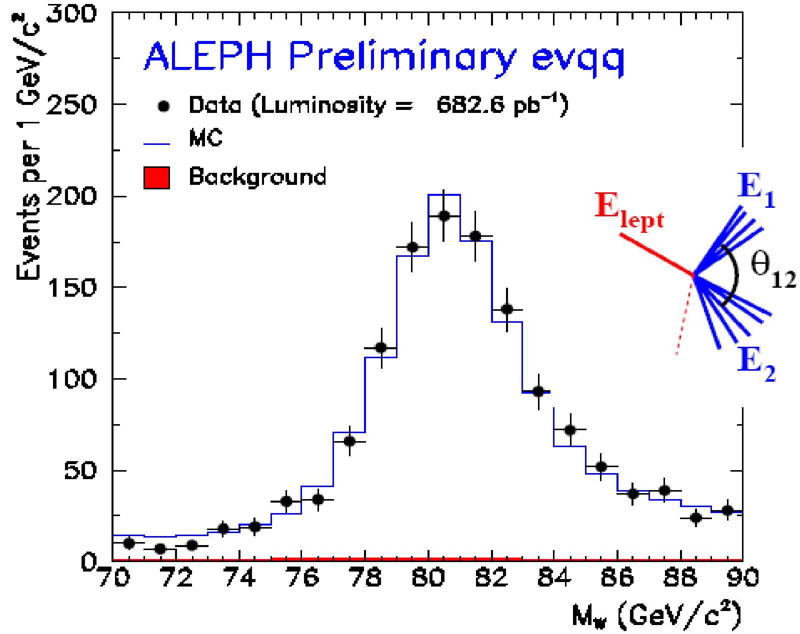
$$Br(W \to q\bar{q}) = (67.77 \pm 0.28)\%$$

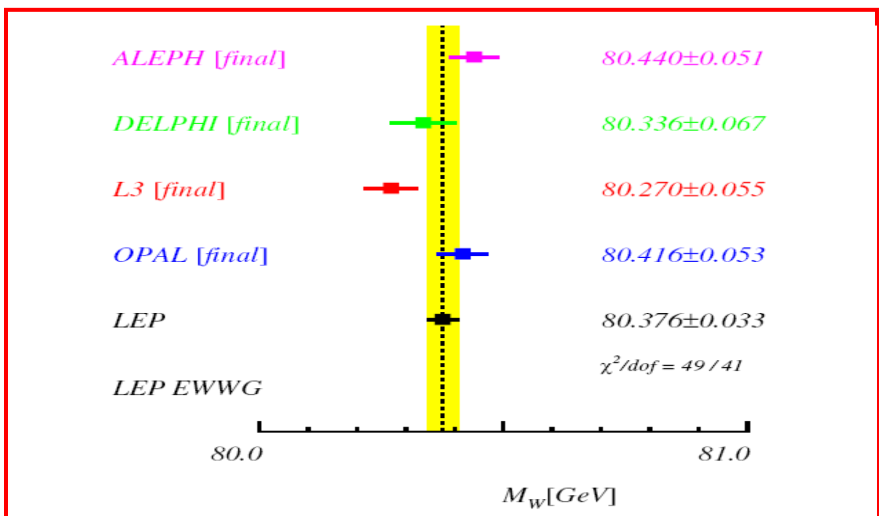
Lepton universality tested to 2%

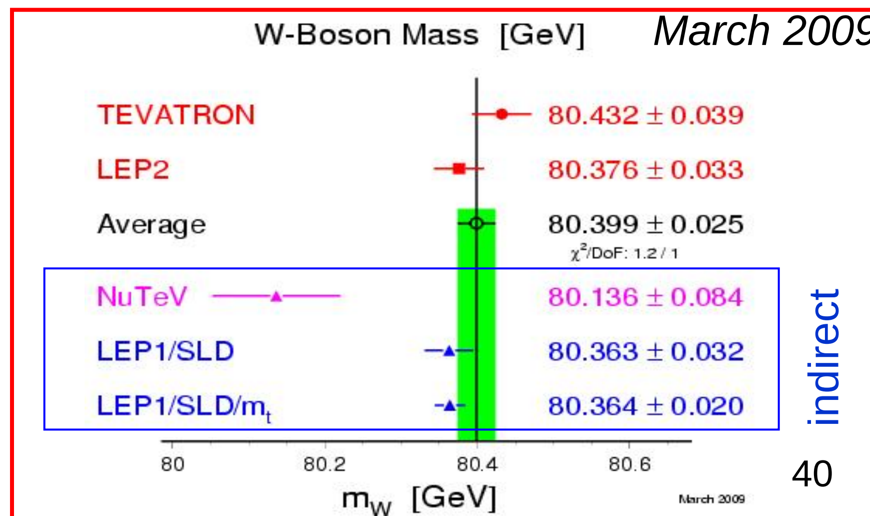
Invariant W mass recontruction



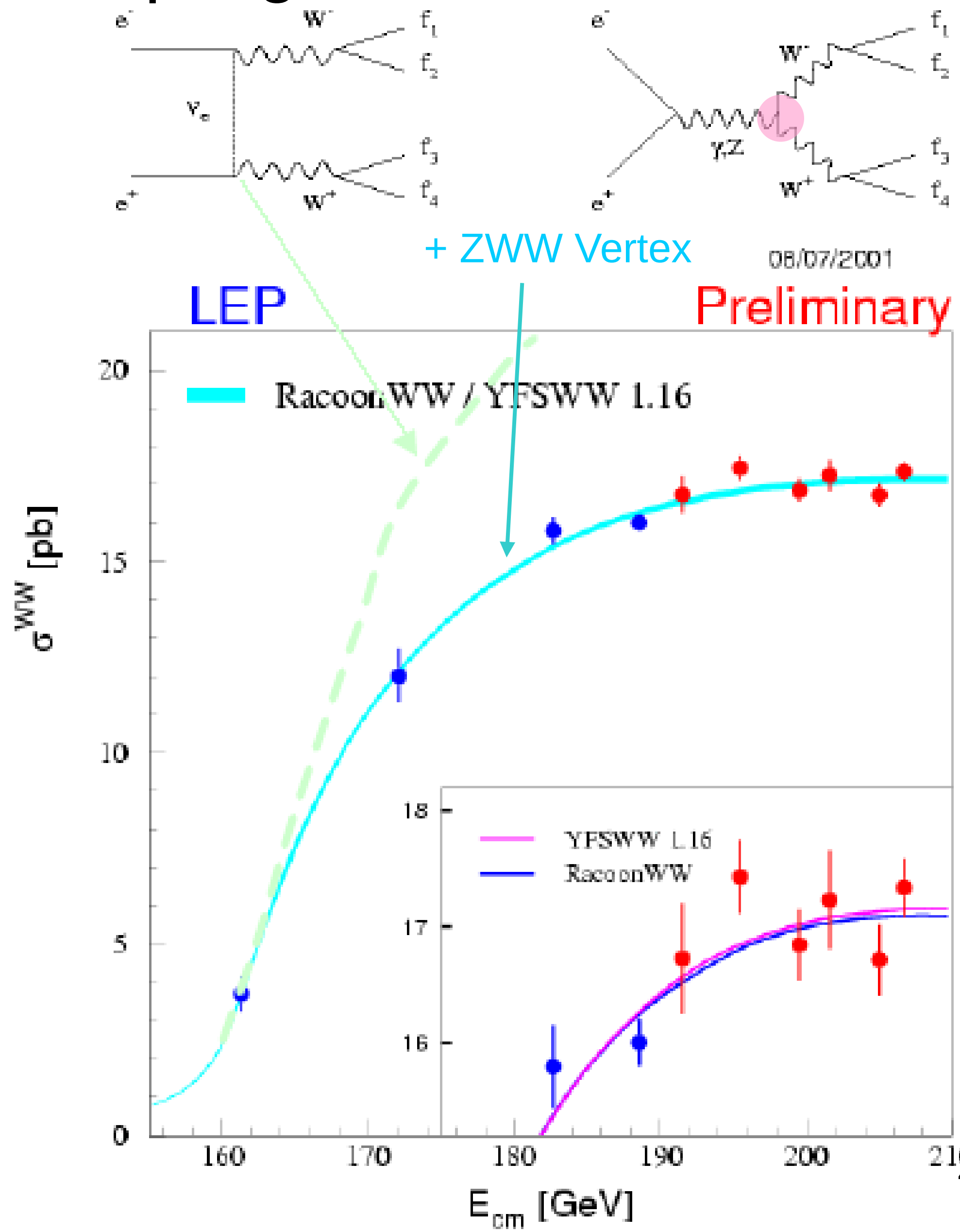
More difficult:
pairing ambiguities







Effect of triple gauge coupling



Data confirms the existence of the γ/ZWW triple gauge boson vertex

4. Higher order corrections and the Higgs mass

$$\sin^2\theta_w = 1 - \frac{M_W^2}{M_Z^2} \qquad \sin\theta_w = \frac{e}{g}$$

Lowest order SM predictions

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \quad \Longrightarrow \quad \bar{\rho} = 1 + \Delta\rho$$

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} \quad \Longrightarrow \quad \sin^2\theta_{eff} = (1 + \Delta\kappa)\sin^2\theta_W$$

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_W G_F} \quad \Longrightarrow \quad m_W^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_W G_F}(1 + \Delta r)$$

$$\alpha(0) \quad \Longrightarrow \quad \alpha(m_Z^2) = \frac{\alpha(0)}{1 - \Delta\alpha}$$

with : $\Delta\alpha = \Delta\alpha_{lept} + \Delta\alpha_{top} + \Delta\alpha_{had}^{(5)}$

Including radiative corrections

$$\Delta\rho, \Delta\kappa, \Delta r = f\left(m_t^2, \log(m_H), \ldots\right)$$

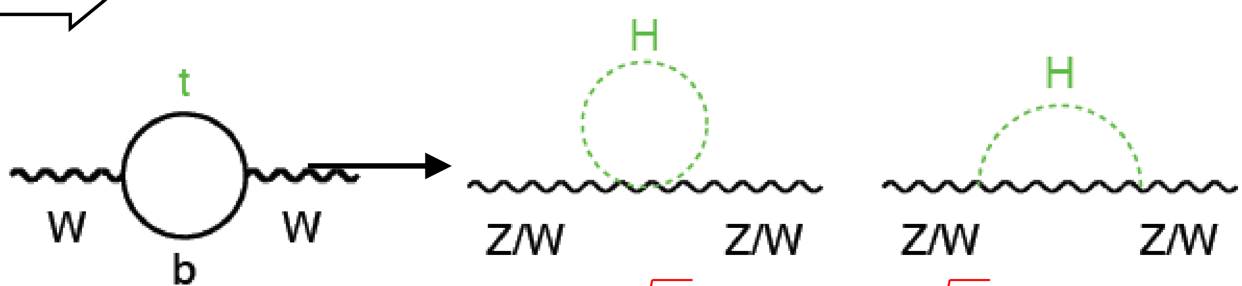
$\sin^2\theta_w$

g_A, g_V

$\Longrightarrow$

$\sin^2\theta_{eff}$

$\bar{g}_A, \bar{g}_V$



$$\bar{g}_A = \sqrt{\bar{\rho}}\, T^3 \qquad \bar{g}_V = \sqrt{\bar{\rho}}\,(T^3 - 2Q\sin^2\theta_{eff})$$

Top mass prediction from radiative corrections

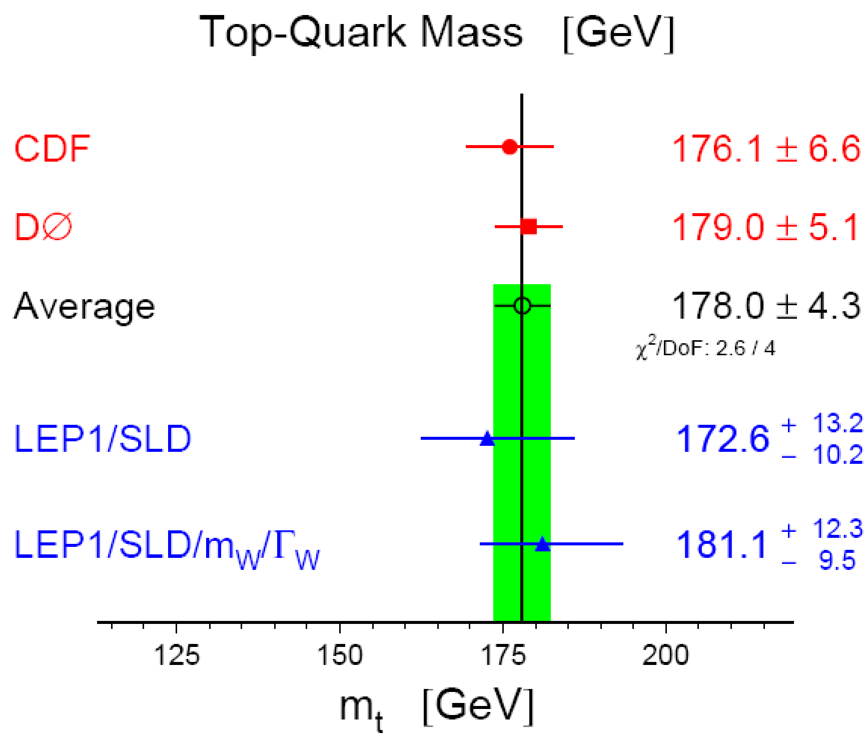
$$\Delta r(m_t, M_H) = -\frac{3\alpha\cos^2\theta_w}{16\pi\sin^4\theta_w}\frac{m_t^2}{M_W^2} - \frac{11\alpha}{48\pi\sin^2\theta_w}\ln\frac{M_H^2}{M_W^2} + \ldots$$

The measurement of the radiative corrections:

$$\sin^2\theta_{eff} \equiv \frac{1}{4}(1-\bar{g}_V/\bar{g}_A)$$

$$\sin^2\theta_{eff} = (1+\Delta\kappa)\sin^2\theta_w$$

Allows the indirect determination of the unknown parameters m_t and M_H.



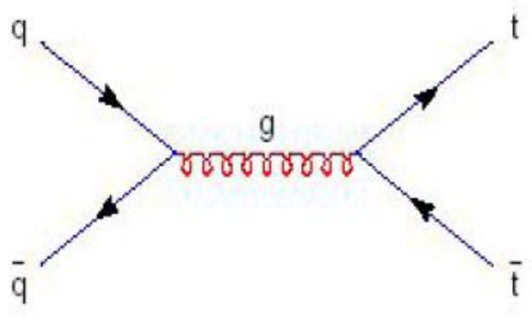
Direct measurement of m_t

Prediction of m_t by LEP before the discovery of the top at TEVATRON.

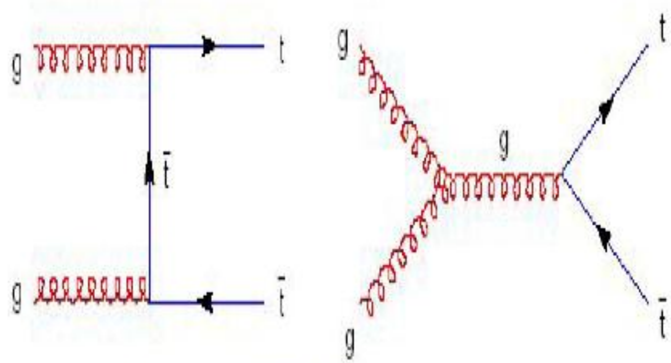
Good agreement between the indirect prediction of m_t and the value obtained in direct measurements confirm the radiative corrections of the SM

Observation of the top quark at TEVATRON (1995)

$p\bar{p}$ @ 2 TeV

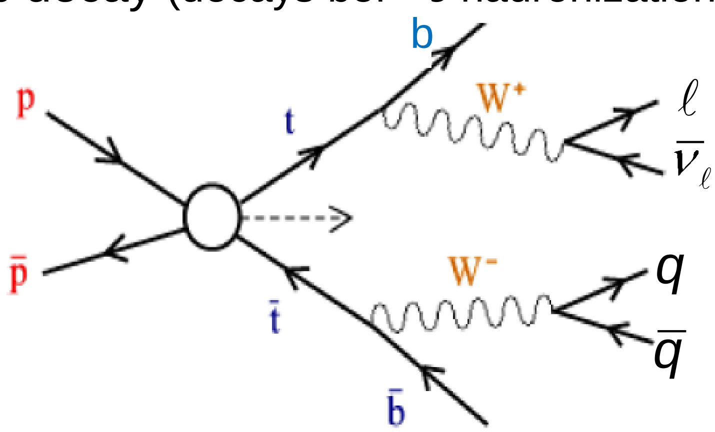


$q\bar{q}$ annihilation (85%)



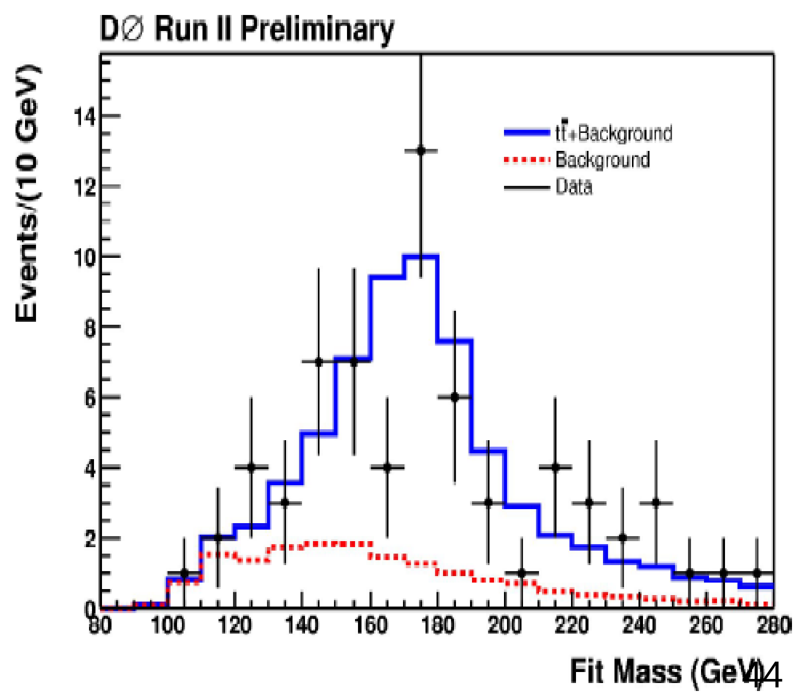
gluon fusion (15%)

Top decay (decays before hadronization)

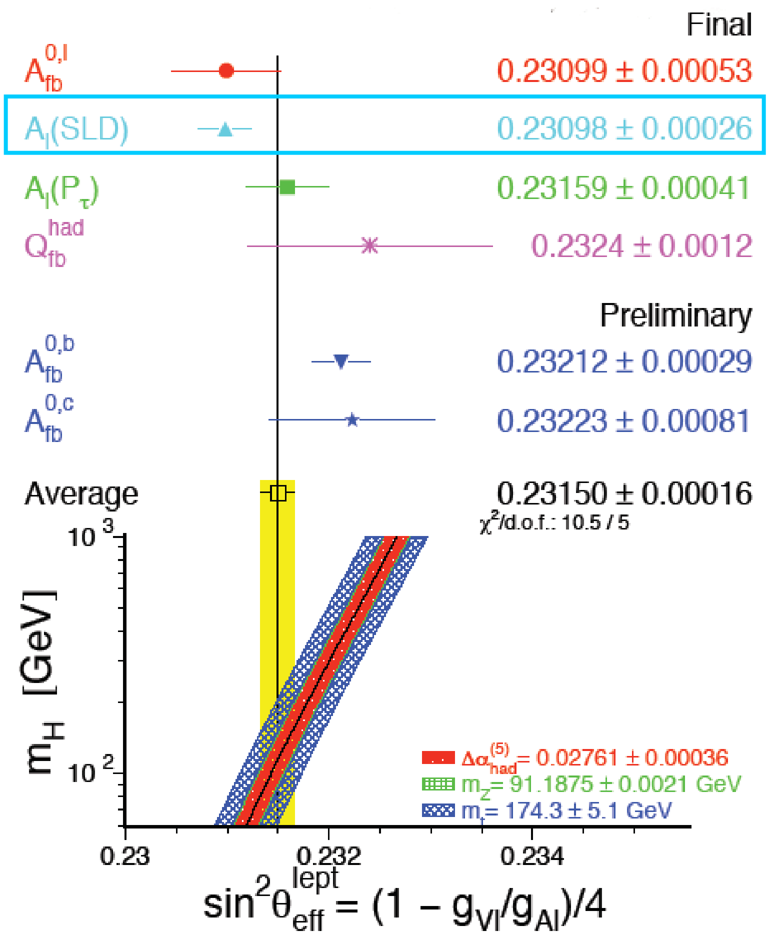


Channel used for mass reconstruction:

$$m_t = m_{inv}(b - jet, W \to jet + jet)$$



Higgs mass prediction from radiative corrections



$$\Delta r(m_t, M_H) = -\frac{3\alpha\cos^2\theta_W}{16\pi\sin^4\theta_W}\frac{m_t^2}{M_W^2} - \frac{11\alpha}{48\pi\sin^2\theta_W}\ln\frac{M_H^2}{M_W^2} + \dots$$

Fits to electro-weak data:

$m_H = 87^{+35}_{-26}$ GeV

$m_H < 157$ GeV (95% CL)

Assumption for fit:

- SM including Higgs
- No confirmation of Higgs mechanism

Higgs seems to be light!