3. Anomalous magnetic moment

3.1 Magnetic moment of the electron:

Dirac equation with electron coupling to electro-magnetic field:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu \quad \Rightarrow \quad \boxed{(i\gamma^\mu D_\mu - m)\psi = 0}$$

$$\vec{p} \rightarrow \vec{\pi} = \vec{p} - e\vec{A} \qquad \text{(canonical momentum)}$$

⇒ Ansatz for the solution as for free particle: $\begin{pmatrix} \mathrm{X} \\ \Phi \end{pmatrix} = \begin{pmatrix} \chi\, e^{-ipx} \\ \varphi\, e^{-ipx} \end{pmatrix}$

⇒

$$i\frac{\partial}{\partial t}\mathrm{X} = \vec{\sigma}\vec{\pi}\,\Phi + (eA^0 + m)\mathrm{X}$$

$$i\frac{\partial}{\partial t}\Phi = \vec{\sigma}\vec{\pi}\,\mathrm{X} + (eA^0 - m)\Phi = 0$$

Non-relativistic limit: $E \approx m, \quad eA^0 << 2m, \quad e^{-ipx} \to e^{-imt}$ ← Driving term

For this limit it makes sense to separate interaction via charge and magnetic moment

$$i\frac{\partial}{\partial t}\chi = \vec{\sigma}\vec{\pi}\,\varphi + eA^0\chi \qquad (1)$$

$$i\frac{\partial}{\partial t}\varphi = \vec{\sigma}\vec{\pi}\,\chi + (eA^0 - 2m)\varphi \qquad (2)$$

from (2) $\varphi = \frac{\vec{\sigma}\vec{\pi}}{2m}\chi$ inserted in (1):

$$i\frac{\partial}{\partial t}\chi = \left[\frac{(\vec{\sigma}\vec{\pi})^2}{2m} + eA^0\right]\chi$$

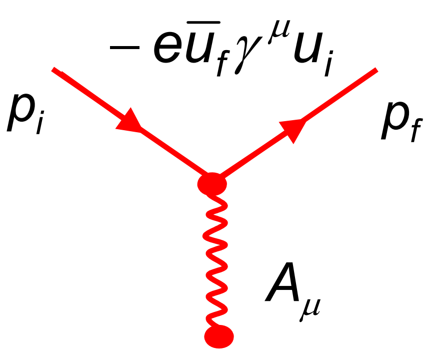
Pauli equation.

$$(\vec{\sigma}\vec{\pi})^2 = \sigma_i \sigma_j \pi^i \pi^j = \pi^2 + \frac{1}{4}[\sigma_i, \sigma_j][\pi^i, \pi^j] = \pi^2 + e\vec{\sigma}\vec{B}$$

$$i\frac{\partial}{\partial t}\chi = \left[\frac{(\vec{p} - e\vec{A})^2}{2m} + \underbrace{\frac{e}{2m}\vec{\sigma}\vec{B}} + eA^0\right]\chi$$

$$= g\frac{e}{2m}\frac{\vec{\sigma}}{2}\vec{B} = g\frac{e}{2m}\vec{S}\vec{B} \quad \text{with} \quad g = 2$$

Gordon decomposition for electron current:



Interaction due to spin

$$= \frac{e}{2m}\bar{u}_f\Big((p_f + p_i)^\mu + i\sigma^{\mu\nu}(p_f - p_i)^\nu\Big)u_i\, A_\mu$$

spinless charge

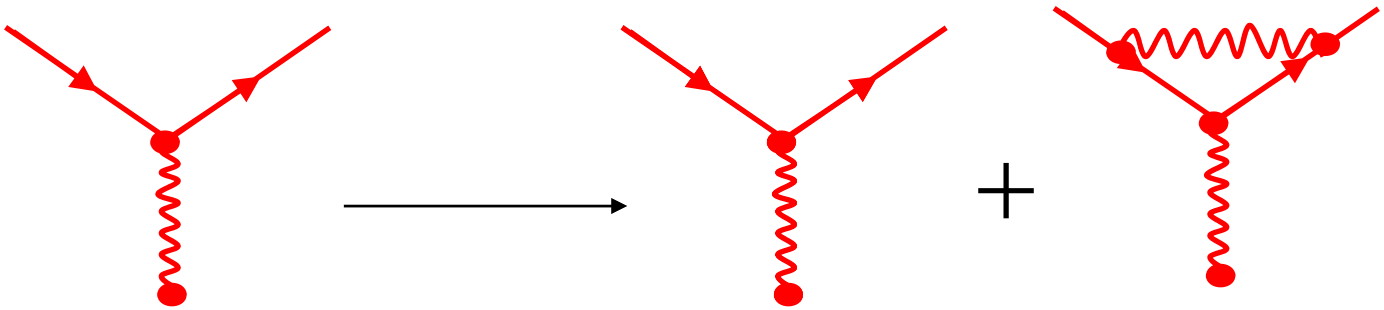
$$\sigma^{\mu\nu} = \frac{i}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)$$

Non-relativistic limit $\varphi^+\left(\frac{e}{2m}\vec{\sigma}\vec{B}\right)\varphi$ wg. $u = \begin{pmatrix}\varphi \\ \not{\chi}\end{pmatrix}$

3.2 Effect of higher order corrections

$$\frac{e}{2m}\bar{u}_f\Big((p_f+p_i)^\mu + i\sigma^{\mu\nu}(p_f-p_i)^\nu\Big)u_i\ A_\mu$$

$$\frac{e}{2m}\bar{u}_f\left((p_f+p_i)^\mu + (1+\frac{\alpha}{2\pi})i\sigma^{\mu\nu}(p_f-p_i)^\nu\right)u_i\ A_\mu$$



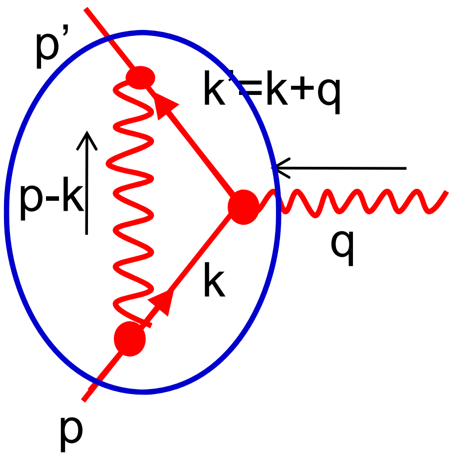
$g = 2$ $\qquad\qquad$ $g = 2 \quad + \quad \dfrac{\alpha}{\pi}$

1st order: $\langle\vec{\mu}_e\rangle = -\dfrac{e}{2m}(2+\dfrac{\alpha}{\pi})\cdot\dfrac{1}{2}\cdot\langle\vec{\sigma}\rangle$

$$g = 2 + \frac{\alpha}{\pi}$$

$$a = \frac{g-2}{2} = \frac{\alpha}{2\pi}$$

Comments about higher order corrections:



$$-ie\bar{u}(p')\gamma^{\mu}u(p) \quad \rightarrow \quad -ie\bar{u}(p')\Gamma^{\mu}u(p)$$

$$\Gamma^{\mu} = \gamma^{\mu} + \delta\Gamma^{\mu}$$

$$\bar{u}(p')\delta\Gamma^{\mu}(p',p)u(p)$$

$$= \int \frac{d^4k}{(2\pi)^4} \frac{-ig_{\nu\rho}}{(k-p)^2 + i\varepsilon} \bar{u}(p')(-ie\gamma^{\nu}) \frac{i(\not{k}'+m)}{k'^2 - m^2 + i\varepsilon} \gamma^{\mu} \frac{i(\not{k}+m)}{k^2 - m^2 + i\varepsilon} (-ie\gamma^{\rho})u(p)$$

Problem: Integral diverges for large as well as for small loop momenta (UV and infra-red divergent).

We will discuss later how to deal with the divergent parts. The remaining non-divergent part modifies the couplings.

Higher order corrections to g-2

Radiative corrections g-2 are calculated to the 4-loop level:

Feynman Graphs	
O(α)	1
O(α^2)	7
O(α^3)	72
O(α^4)	891
til O(α^4)	971

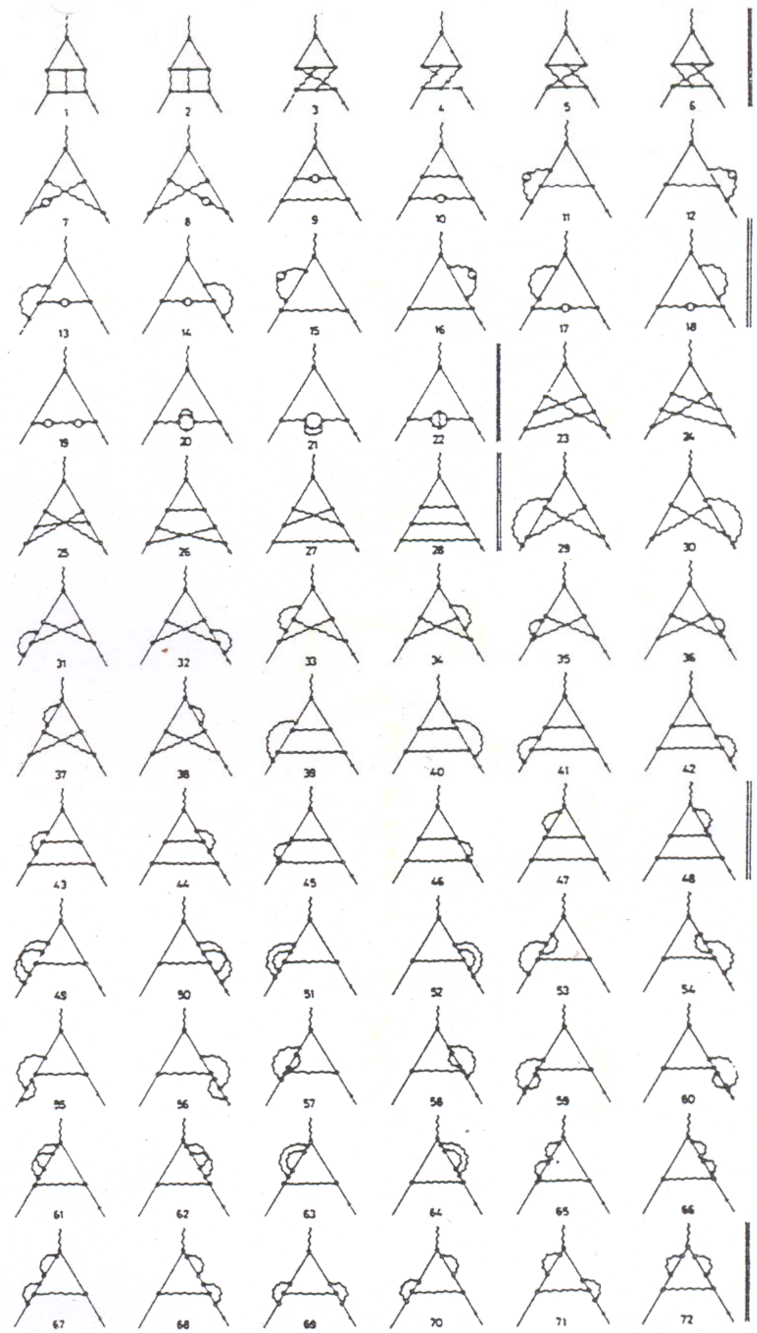


Fig. 8.2 The Feynman graphs which have to be evaluated in computing the α^3 corrections the lepton magnetic moments (after Lautrup *et al.* 1972).

Most precise QED prediction.

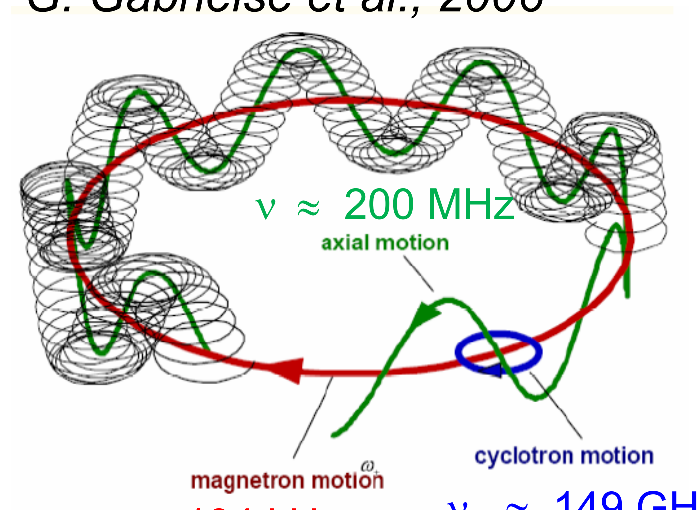
T. Kinoshita et al.

Kinoshita 2006 $$a_e = \frac{\alpha}{2\pi} - 0.328...\left(\frac{\alpha}{\pi}\right)^2 + 1.182....\left(\frac{\alpha}{\pi}\right)^3 - 1.505....\left(\frac{\alpha}{\pi}\right)^4$$

Kinoshita 2007 $$a_e = \frac{\alpha}{2\pi} - 0.328...\left(\frac{\alpha}{\pi}\right)^2 + 1.182....\left(\frac{\alpha}{\pi}\right)^3 - 1.9144....\left(\frac{\alpha}{\pi}\right)^4$$

3.3 Electron g-2 measurement

H. Dehmelt et al., 1987
G. Gabrielse et al., 2006

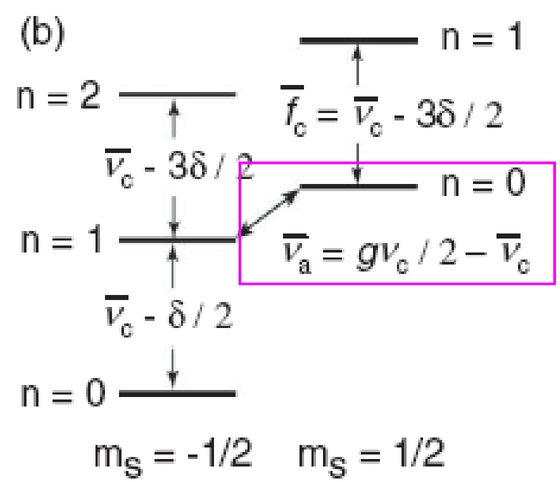


<u>Experimental method:</u>
Storage of **single** electrons in a Penning trap (electrical quadrupole + axial B field)
$\Rightarrow$ complicated electron movement (cyclotron and magnetron precessions).

Cyclotron frequency $\omega_C = 2\frac{eB}{2mc}$

Spin precession frequency $\omega_s = g\frac{eB}{2mc}$

Energy levels single electron:



Idea: bound electron:

$$E(n, m_s) = \frac{g}{2}h\nu_c m_s + \left(n + \frac{1}{2}\right)h\bar{\nu}_c - \frac{1}{2}h\delta\left(n + \frac{1}{2} + m_s\right)^2$$

Trigger RF induced transitions (ω_a) between different n states or spin flips:

$$\omega_a = \omega_s - \omega_c = (g-2)\mu_B B$$
$$a = \frac{g-2}{2} = \frac{\omega_s - \omega_c}{\omega_c}$$

$$a_{e^-} = 0.001\,159\,652\,188\,4\,(43)$$
$$a_{e^+} = 0.001\,159\,652\,187\,9\,(43)$$

H. Dehmelt et al. 1987

$$a_e = 0.001\,159\,652\,180\,85\,(76)$$

G. Gabrielse et al. 2006

$\Rightarrow$ most precise value of α:

$$\alpha^{-1}(a_e) = 137.035\,999\,710\,(96)$$

For comparison α from Quanten Hall

$$\alpha^{-1}(qH) = 137.036\,003\,00\,(270)$$

$$a_e = \frac{\alpha}{2\pi} - 0.328\ldots\left(\frac{\alpha}{\pi}\right)^2 + 1.182\ldots\left(\frac{\alpha}{\pi}\right)^3 - 1.505\ldots\left(\frac{\alpha}{\pi}\right)^4$$

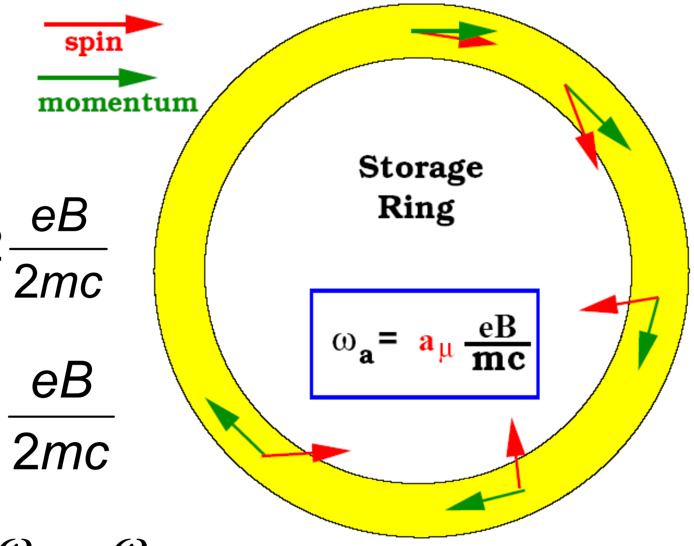
Theory

$$a_e = 0.001\,159\,652\,133\,(290)$$

$$a_e = 0.001\,159\,652\,180\,85\,(76)$$

Phys. Rev. Lett. ***97****, 030801 (2006)*
Phys. Rev. Lett. ***97****, 030802 (2006)*

3.4 Experimental determination of muon g-2



(exaggerated ~20x)

Principle:

- store polarized muons in a storage ring; revolution with cyclotron frequency ω_c
- measure spin precession around the magnetic dipole field relative to the direction of cyclotron motion

$$\omega_C = 2\frac{eB}{2mc}$$

$$\omega_L = g\frac{eB}{2mc}$$

$$\omega_a = \omega_L - \omega_C$$

Precession:

$$\vec{\omega}_a = -\frac{e}{m_\mu c}\left[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})\vec{\beta} \times \vec{E}\right]$$

Difference between Lamor and cyclotron frequency

Effect of electrical focussing fields (relativistic effect).

$= 0$ for $\gamma = 29.3$

$\Leftrightarrow p_\mu = 3.094$ GeV/c

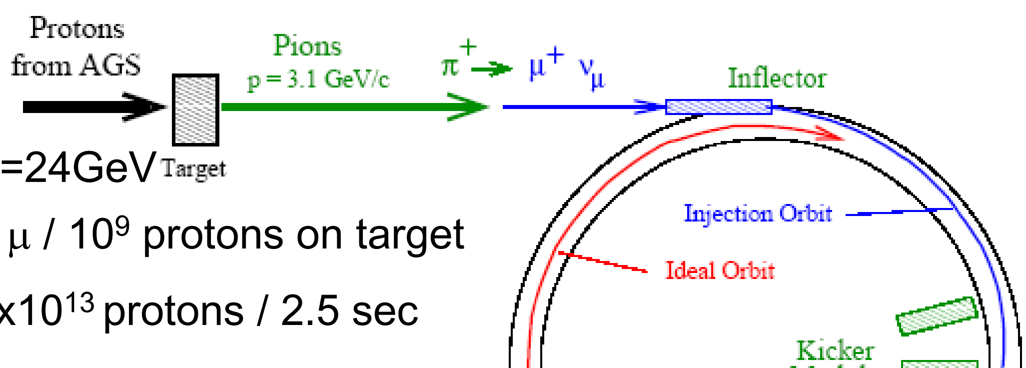
First measurements:

CERN 70s

$$a_{\mu^-} = 0.001165937(12)$$

$$a_{\mu^+} = 0.001165911(11)$$

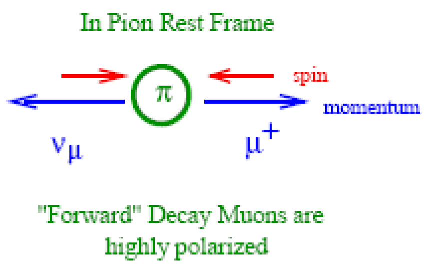
$(g\text{-}2)_\mu$ Experiment at BNL

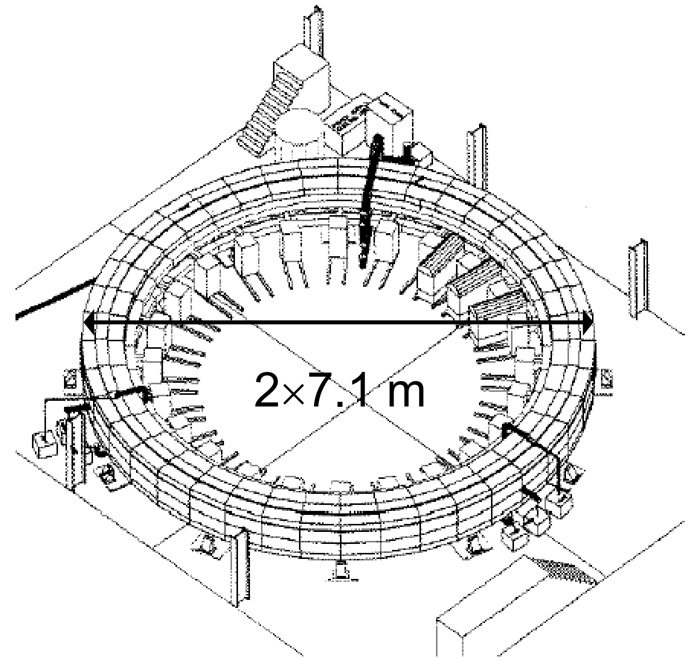


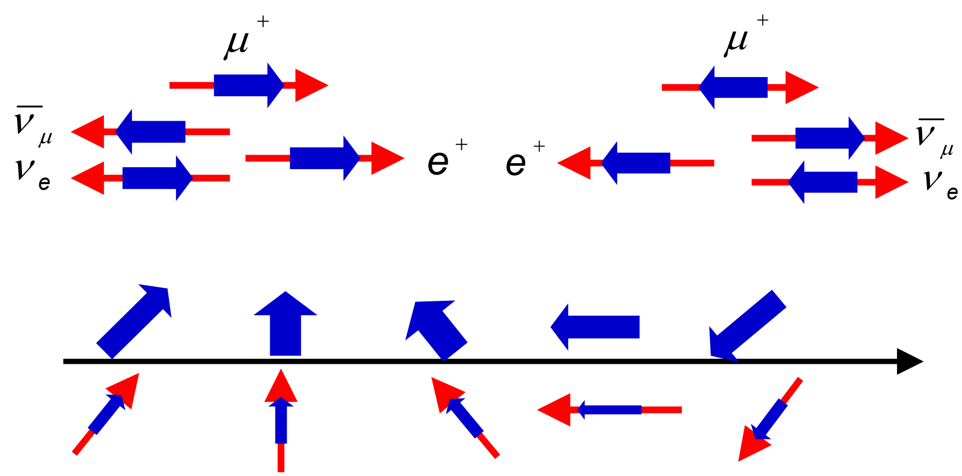
E=24GeV

1 μ / 10^9 protons on target

6×10^{13} protons / 2.5 sec



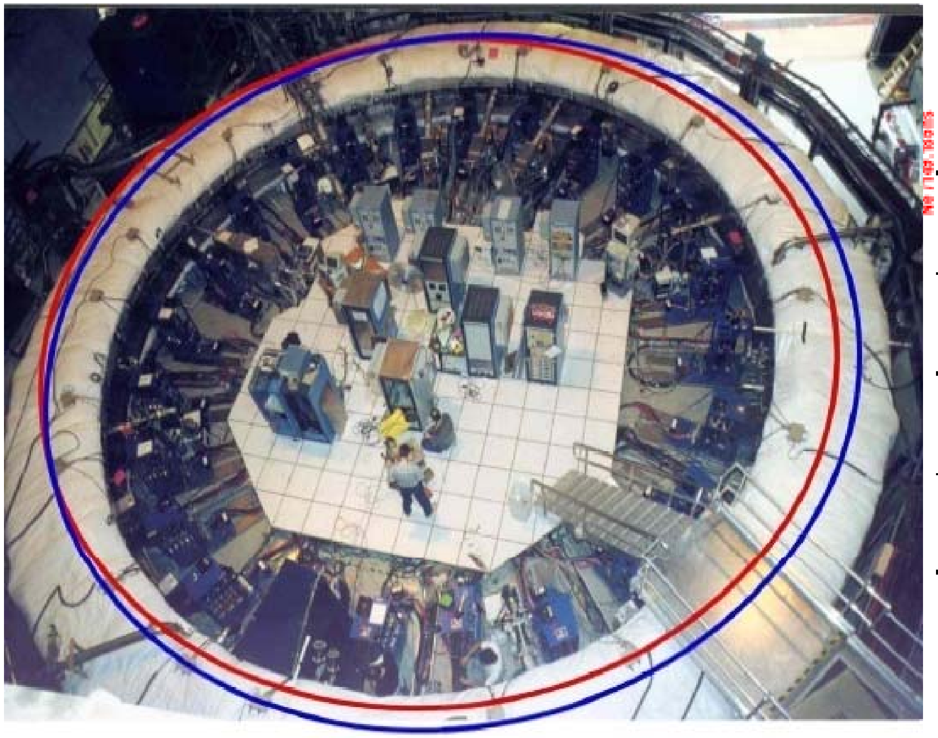




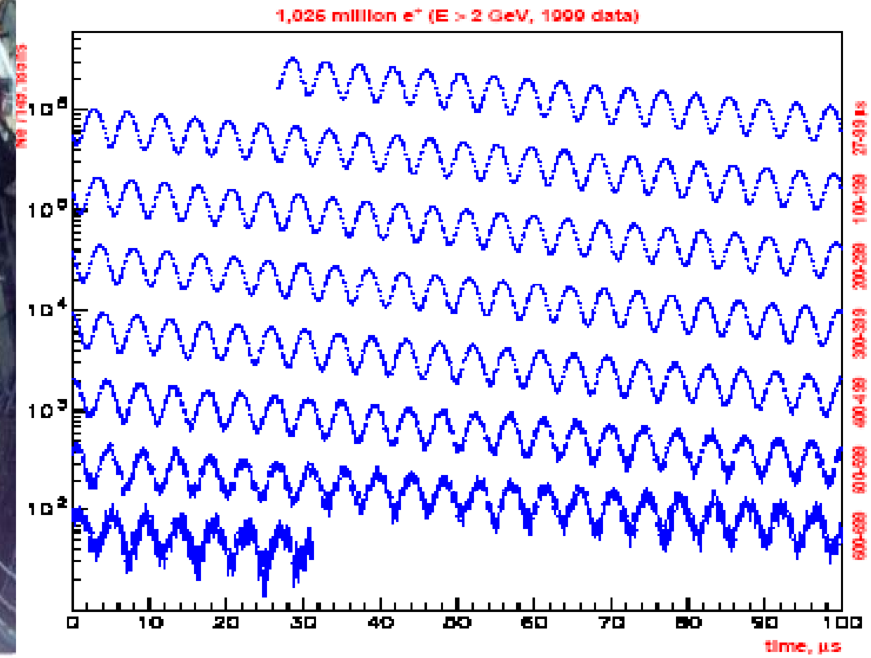
<u>"V-A" structure of weak decay:</u>

Use high-energy e^+ from muon decay to measure the muon polarization

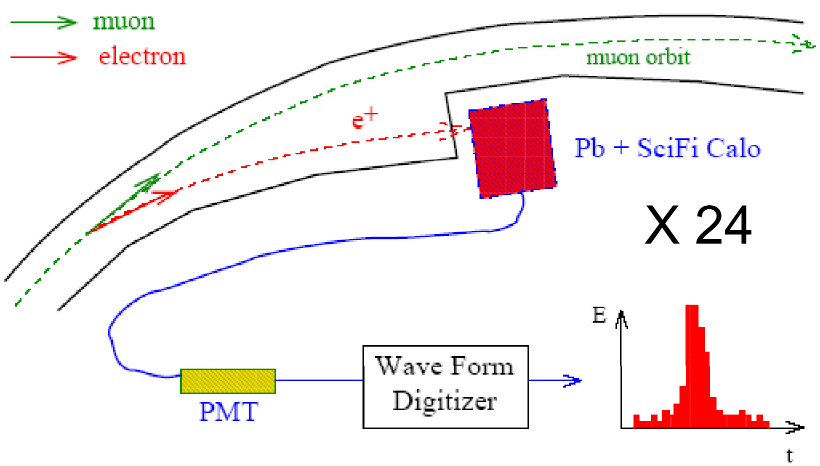
Weak charged current couples to LH fermions (RH anti-fermions)



Measure electron rate:



$$N(t) = N_0\, e^{-\lambda t}\left[1 + A\cos(\omega_a t + \varphi)\right]$$



$$\frac{\omega_a}{2\pi} = 229023.59(16)\,\text{Hz}$$

(0.7ppm)

$$a_\mu = \frac{\omega_a}{\frac{e}{m_\mu c}\langle B\rangle}\ ?$$

From ω_a to a_μ - How to measure the B field

$<B>$ is determined by measuring the proton nuclear magnetic resonance (NMR) frequency ω_p in the magnetic field.

$$a_\mu = \frac{\omega_a}{\frac{e}{m_\mu c}\langle B\rangle} = \frac{\omega_a}{\frac{e}{m_\mu c}\frac{\hbar\tilde{\omega}_p}{2\mu_p}} = \frac{\omega_a}{\frac{4\mu_\mu}{\hbar g_\mu}\frac{\hbar\tilde{\omega}_p}{2\mu_p}} = \frac{\omega_a/\tilde{\omega}_p}{\mu_\mu/\mu_p}(1+a_\mu)$$

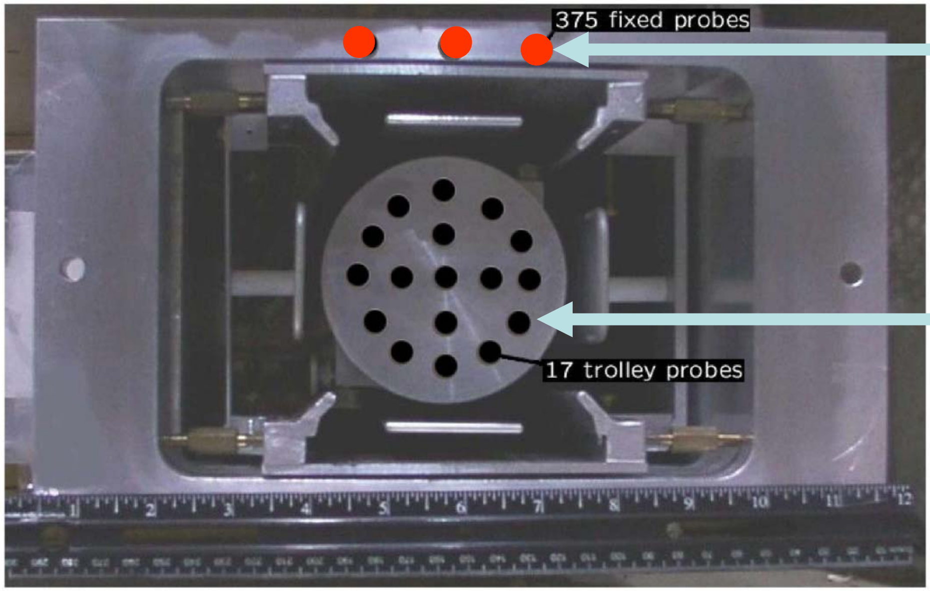
$$\Downarrow$$

$$a_\mu = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p}$$

$\mu_{\mu^+}/\mu_p = 3.183\,345\,39(10)$

W. Liu *et al.*, Phys. Rev. Lett. **82**, 711 (1999).

NMR trolley

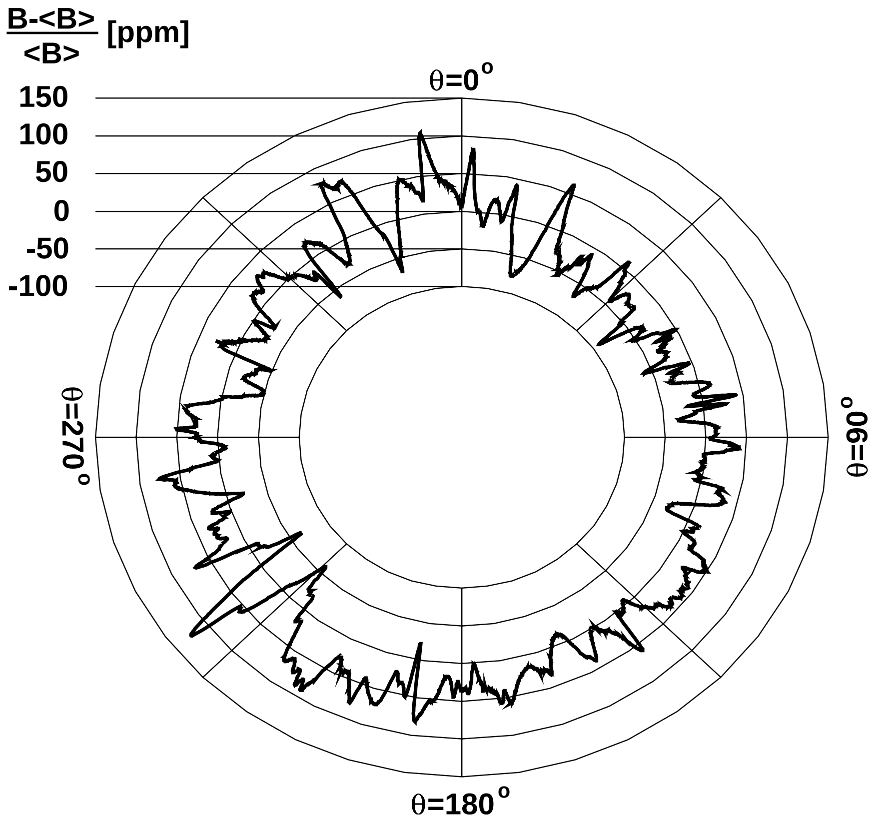


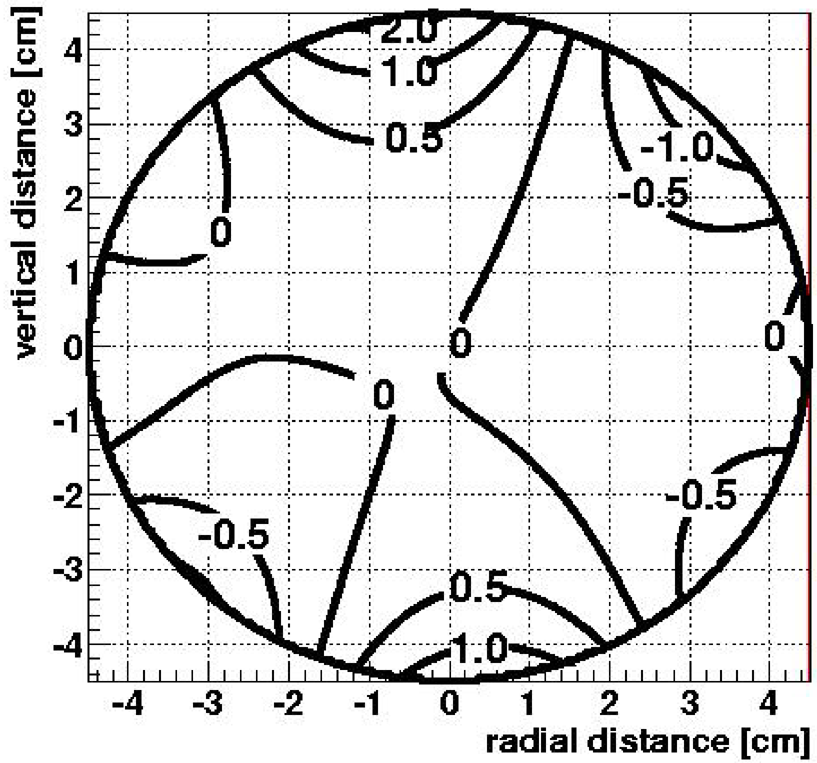
375 fixed NMR probes around the ring

17 trolley NMR probes

$\tilde{\omega}_p/2\pi$ = 61 791 400(11) Hz (0.2ppm)

B field determination

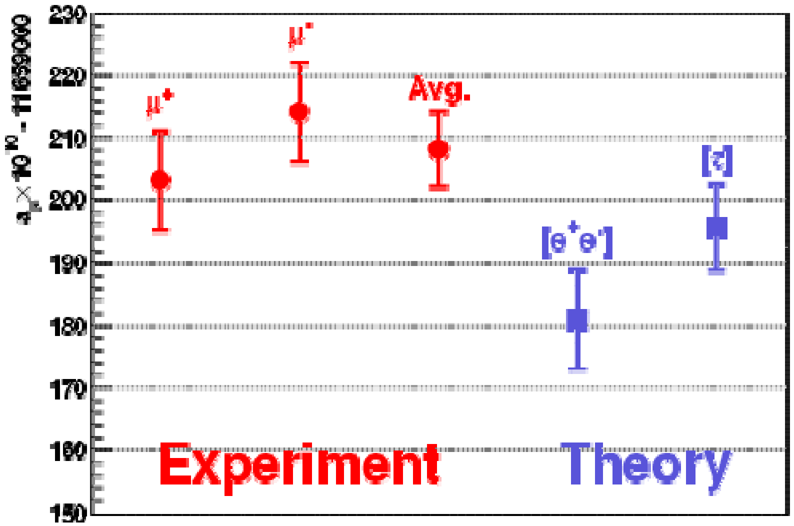




The *B* field variation at the center of the storage region.
<B>≈1.45 T

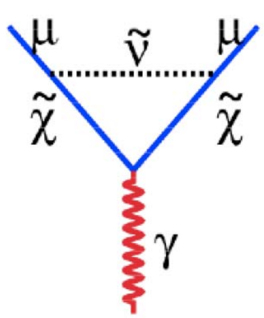
The *B* field averaged over azimuth.

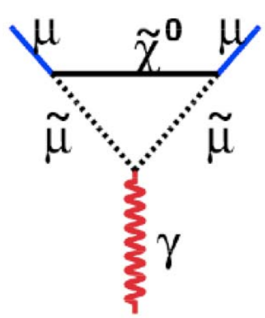
$$a_{\mu^+} = 11659203(8)\times10^{-10}(0.7ppm)$$
$$a_{\mu^-} = 11659214(8)\times10^{-10}(0.7ppm)$$
$$a_{\mu} = 11659208(6)\times10^{-10}(0.5ppm)$$



Up to a 2.6σ deviation:

- Often interpreted as sign of New Physics: SUSY contributions
- careful: “Theory” has uncertainties!





Potential SUSY contributions:

Remarks: Theoretical prediction of a_μ

Beside pure QED corrections there are weak corrections (W, Z) exchange and „hadronic corrections“

$$a_\mu = a_\mu^{QED} + a_\mu^{Had} + a_\mu^{EW}$$

(For the electron with much lower mass the hadronic and weak corrections are suppressed, and can be neglected.)

→ Determination of hadronic corrections is difficult and is in addition based on data: hot discussion amongst theoreticians how to correctly use the data.

Hadronic corrections