

## Experimental tests of QED:

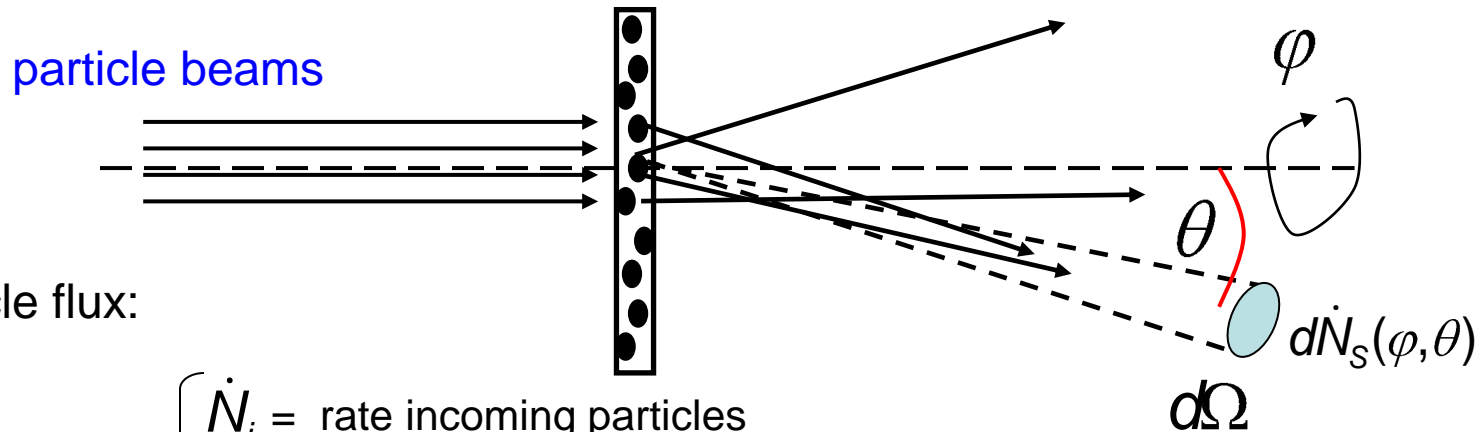
1. From the matrix element to the measurement
2.  $e^+e^-$  scattering experiments
3. Anomalous magnetic moment

# 1. From the matrix element to the measurement

## 1.1 Cross section – experimental definition

→ Most important observable to describe scattering processes.

Target:  
 $N_t$  scattering centers



Incident particle flux:

$$F = \frac{\dot{N}_i}{A} = n_i v_i \quad \left\{ \begin{array}{l} \dot{N}_i = \text{rate incoming particles} \\ n_i = \text{particle density in beam} \\ v_i = \text{velocity of particles} \end{array} \right.$$

Particles rate scattered  
into  $d\Omega(\varphi, \theta)$ :  $d\dot{N}_s(\varphi, \theta)$

$$d\sigma = \frac{d\dot{N}_s(\varphi, \theta)}{F \cdot N_t} = \frac{d\dot{N}_s(\varphi, \theta)}{\Phi}$$

Differential cross section:

$$\frac{d\sigma(\varphi, \theta)}{d\Omega} = \frac{d\dot{N}_s(\varphi, \theta)}{F N_t \cdot d\Omega} = \frac{d\dot{N}_s(\varphi, \theta)}{\Phi \cdot d\Omega}$$

↑  
 $N_t=1$

Total cross section:

The total cross section is obtained from the total rate of scattered particles:.

$$\sigma_{tot} = \frac{\dot{N}_s}{\Phi}$$

respectively:

$$\sigma_{tot} = \int \left( \frac{d\sigma}{d\Omega} \right) d\Omega = \frac{1}{\Phi} \int \left( \frac{d\dot{N}_s}{d\Omega} \right) d\Omega$$

↑  
 $d \cos \theta d\varphi$   
of scattered particle

Dimension  $\sigma = \frac{\text{time}^{-1}}{(\text{time}^{-1} \times \text{area}^{-1})} = \text{area}$

Units:  $[\sigma] = 1 \text{ b} = 10^{-28} \text{ m}^2$   
 $1 \text{ b} = 1 \text{ barn}$

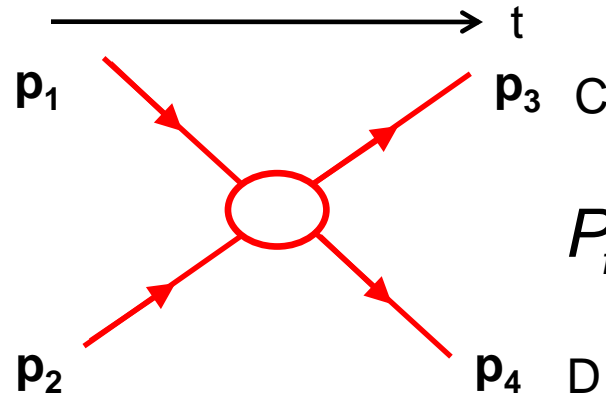
# 1.2 Scattering matrix and transition amplitude

Scattering process:

$$1 + 2 \rightarrow 3 + 4$$

$$P_i = p_1 + p_2$$

$$P_f = p_3 + p_4$$



Initial and final states:

$$|i\rangle \rightarrow |t\rangle$$

Scattering operator (S matrix):

$$\lim_{t \rightarrow +\infty} |t\rangle = \mathbf{S} |i\rangle$$

Measurement selects specific state  $f$ .

Probability  $S_{fi}$  to find  $f$ :

$$\langle f | t \rangle = \langle f | \mathbf{S} | i \rangle = S_{fi}$$

$$\mathbf{S}_{fi} = \delta_{fi} + i(2\pi)^4 \delta^4(P_f - P_i) T_{fi} \quad T_{fi} = \langle f | \mathbf{T} | i \rangle$$

Probability that collection of states **i** will make the transition to a final state **f**:

$$P_{fi} = |S_{fi}|^2$$

$$P_{fi} = (2\pi)^8 \left[ \frac{1}{4} (P_f - P_i) \right]^2 |T_{fi}|^2$$

... or to all possible final states **f**:

$$P_{fi} = \sum_f |S_{fi}|^2$$

### Final-state phase-space:

The calculation of the transition probability has to consider the number of possible states for each of the out-going particles: → phase-space factor

$dN_f$  = number of states  
within  $\vec{p}$  and  $\vec{p} + d\vec{p}$ :

$$dN_f = \frac{d^3 p_C}{2E_C (2\pi)^3} \cdot \frac{d^3 p_D}{2E_D (2\pi)^3} \cdot \dots$$

# 1.3 Amplitude, cross section and phase space

$$d\sigma = \frac{\text{transition rate per unit volume}}{\text{incident flux}}$$

$$d\sigma = \underbrace{\frac{|S_{fi}|^2 dN_f}{VT}}_{w_{fi}} \frac{1}{\Phi}$$

$w_{fi}$  = Transition rate / V

$$\begin{aligned} w_{fi} &= \frac{|S_{fi}|^2 dN_f}{VT} = \frac{(2\pi)^8 \delta^4(P_f - P_i) |T_{fi}|^2}{VT} dN_f \\ &= \frac{(2\pi)^8 \delta^4(P_f - P_i) \delta^4(0) |T_{fi}|^2}{VT} dN_f \\ &= (2\pi)^4 \delta^4(P_f - P_i) |T_{fi}|^2 dN_f \end{aligned}$$

## Incident flux $\Phi$ :

Lab: 1  $\longrightarrow$  2

CMS: 1  $\longrightarrow$   $\longleftarrow$  2  $\vec{p}_1 = -\vec{p}_2$

$$\text{Lab: } \Phi|_{\text{unitV}} = \rho_1 v_1 N_2 / V = 2E_1 2E_2 \left( \frac{|\vec{p}_1|}{E_1} \right)$$

$$\begin{aligned} \text{CMS: } \Phi|_{\text{unitV}} &= 2E_1 2E_2 |\vec{v}_1 - \vec{v}_2| = 2E_1 2E_2 \left| \frac{\vec{p}_1}{E_1} - \frac{\vec{p}_2}{E_2} \right| = 4|\vec{p}_1|(E_1 + E_2) \\ &= 4 \left( (\vec{p}_1 \cdot \vec{p}_2)^2 - m_1^2 m_2^2 \right)^{1/2} \end{aligned}$$

### Reminder: Normalization of states

$$\langle (\vec{p}, s) | (\vec{p}', s') \rangle = \delta_{ss'} (2\pi)^3 2E_p \delta(\vec{p} - \vec{p}')$$

States are delta-function in momentum space.

$$= \delta_{ss'} 2E_p \int d^3x e^{i(\vec{p} - \vec{p}') \cdot \vec{x}} = 2E_p V \Big|_{s=s'}$$

Differential cross section:

$$d\sigma = \frac{|T_{fi}|^2}{4 \left[ (\mathbf{p}_1 \cdot \mathbf{p}_2)^2 - m_1^2 m_2^2 \right]^{1/2}} \overbrace{(2\pi)^4 \delta^4(P_f - P_i) \frac{d^3 p_C}{2E_C (2\pi)^3} \cdot \frac{d^3 p_D}{2E_D (2\pi)^3}}^{d\Phi_2(P_i, p_C, p_D)}$$

Lorentz invariant phase-space factor for n particles:

$$d\Phi_n(P, \underbrace{p_1, p_2, \dots, p_n}_{\text{Final state}}) = (2\pi)^4 \delta^4(P - (p_1 + p_2 + \dots + p_n)) \prod_{\text{final}} \frac{d^3 p_f}{(2\pi)^3 2E_f}$$

See also PDG

<http://pdg.lbl.gov/2009/reviews/rpp2009-rev-kinematics.pdf>

(uses unfortunately different normalization)

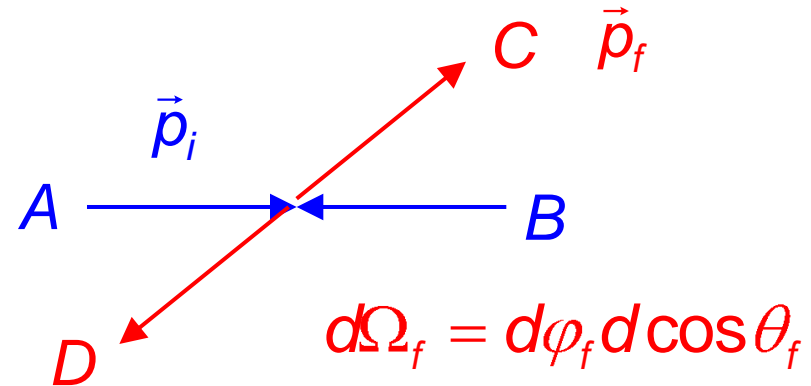


## Phase space integration for two-particles final-state (CMS)

Center of Mass System :

$$\vec{p}_i = \vec{p}_A = -\vec{p}_B \quad \vec{p}_f = \vec{p}_C = -\vec{p}_D$$

$$s = (E_A + E_B)^2$$



$$d\Phi_2 \xrightarrow{\int}$$

$$\int d\Phi_2 = \frac{1}{4\pi^2} \int \delta^3(\vec{p}_C + \vec{p}_D) \delta(E_A + E_B - E_C - E_D) \frac{d^3 p_C}{2E_C} \frac{d^3 p_D}{2E_D}$$

$$\int d\Phi_2 = \frac{1}{4\pi^2} \int \delta^3(\vec{p}_C + \vec{p}_D) \delta(E_A + E_B - E_C - E_D) \frac{d^3 p_C}{2E_C} \frac{d^3 p_D}{2E_D}$$

In the CMS need to integrate only over  $p_c$ :  $d^3p = d\Omega p^2 dp$

$$\int d\Phi_2 = d\Omega_C \frac{1}{16\pi^2} \int \delta(E_A + E_B - E_C - E_D) \frac{|\vec{p}_C|^2 d|\vec{p}_C|}{E_C E_D} \quad \boxed{\vec{p}_f = \vec{p}_C = -\vec{p}_D}$$

With

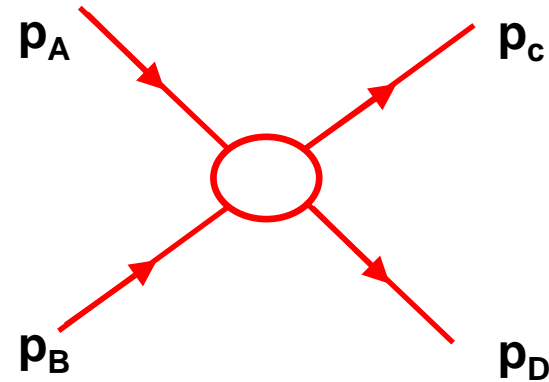
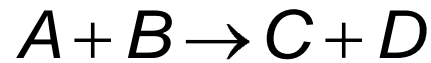
$$\int \delta[h(\omega)] g(\omega) d\omega = \left( g \left| \frac{dh}{d\omega} \right|^{-1} \right)_{f=0} \quad \text{and} \quad h = \sqrt{s} - \sqrt{\vec{p}_C^2 + m_C^2} - \sqrt{\vec{p}_C^2 + m_C^2}$$

$$\rightarrow \left. \frac{dh}{d|\vec{p}_C|} \right|_{h=0} = \frac{|\vec{p}_C| \sqrt{s}}{E_C E_D}$$



$$\int d\Phi_2 = \frac{1}{16\pi^2} \int \frac{|\vec{p}_f|}{\sqrt{s}} d\Omega_f$$

## 1.4 Differential cross section ...putting everything together



CMS

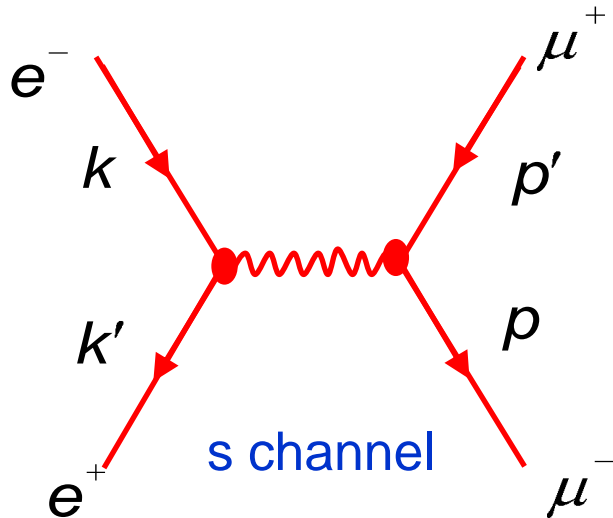
$$d\sigma = \frac{|T_{fi}|^2}{\Phi} d\Phi_2 = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|} \cdot |T_{fi}|^2 d\Omega_f$$
$$\frac{d\sigma}{d\Omega_f} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|} \cdot |T_{fi}|^2$$

- The dynamics of the scattering process is contained in the matrix element  $M_{fi}$  which can be calculated using Feynman rules
- $1/s$  dependence of the cross section because of initial/final state kinematics

## 2. $e^+e^-$ scattering experiments

### 2.1 Myon pair production

$$e^+ e^- \rightarrow \mu^+ \mu^-$$



$$T_{fi} \sim -\frac{e^2}{q^2} \underbrace{\bar{v}_e(k') \gamma_\mu u_e(k) \cdot \bar{u}_\mu(p) \gamma^\mu v_\mu(p')}_{\text{Spinors describe a specific spin state of the fermions}}$$

Spinors describe a specific spin state of the fermions

For non-polarized ingoing particles and for non-observation of final state spin one observes unpolarized cross sections  $\Rightarrow$  need to **average over possible initial spin states** and **sum over all final spin states**.

$$\overline{|T_{fi}|^2} = \frac{1}{4} \cdot \sum_{s_e, s'_e} \sum_{s_\mu, s'_\mu} |T_{fi}|^2$$

Averaging and summing over spins of initial/final state:

$$\begin{aligned}
 |\overline{M}_{fi}|^2 &= \frac{1}{4} \frac{e^4}{s^2} \sum_{s,s',r,r'} |\bar{u}_{\mu,s}(p_3) \gamma^\nu v_{\mu,s'}(p_4) \bar{v}_{e,r}(p_2) \gamma_\nu u_{e,r'}(p_1)|^2 \\
 &= 8 \frac{e^4}{s^2} \left[ (p_1 p_4)(p_2 p_3) + (p_2 p_4)(p_1 p_3) \right]
 \end{aligned}$$

neglect masses

Lorentz invariant!

By using the Mandelstam variables in the relativistic limit

$$\begin{aligned}
 s &= (p_1 + p_2)^2 = m^2 + m^2 + 2p_1 p_2 \approx 2p_1 p_2 \approx 2p_3 p_4 \\
 t &= (p_1 - p_3)^2 = m^2 + M^2 - 2p_1 p_3 \approx -2p_1 p_3 \approx -2p_2 p_4 \\
 u &= (p_1 - p_4)^2 = m^2 + M^2 - 2p_1 p_4 \approx -2p_1 p_4 \approx -2p_2 p_3
 \end{aligned}$$

if masses neglected

$$\overline{|M_{fi}|^2} = 2 e^4 \frac{t^2 + u^2}{s^2}$$

Remember: matrix element squared can be expressed in s, u, t!

$$\overline{|T_{fi}|^2}_{e^+e^- \rightarrow \mu^+\mu^-}(s, t, u) = 2e^4 \frac{t^2 + u^2}{s^2}$$

$$\downarrow \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot |T_{fi}|^2$$

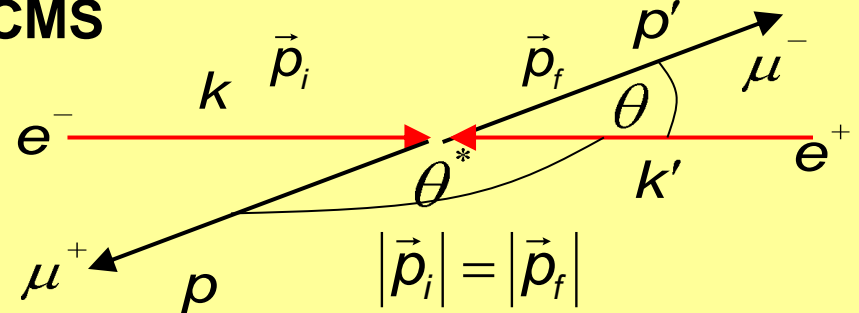
$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{e^4}{32\pi^2} \cdot \frac{1}{s} \cdot \frac{t^2 + u^2}{s^2} \\ &= \frac{e^4}{64\pi^2} \cdot \frac{1}{s} \cdot (1 + \cos^2 \theta) \end{aligned}$$

$$\downarrow e^2 = 4\pi\alpha$$

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{CMS}} = \frac{\alpha^2}{4s} \cdot (1 + \cos^2 \theta)$$

## Kinematics for high-relativistic particles

**CMS**



$$p^2 = p'^2 = k^2 = k'^2 = 0$$

$$s = (k + k')^2 \approx 4E_i^2$$

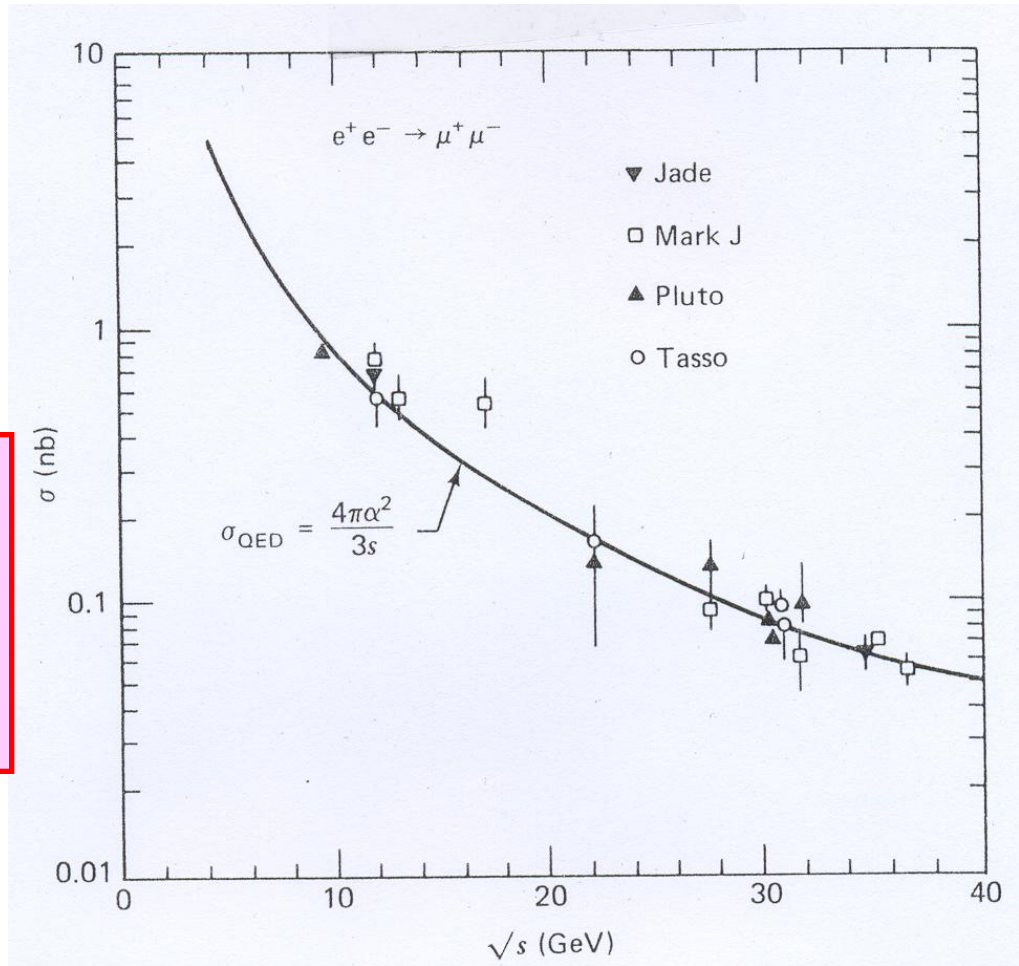
$$\begin{aligned} t = (k - p)^2 &\approx -2kp \approx -2E_i^2(1 - \cos\theta^*) \\ &\approx -\frac{s}{2}(1 + \cos\theta) \end{aligned}$$

$$\begin{aligned} u = (k - p')^2 &\approx -2kp' \approx -2E_i^2(1 - \cos\theta) \\ &\approx -\frac{s}{2}(1 - \cos\theta) \end{aligned}$$

← 1/s dependence from flux factor

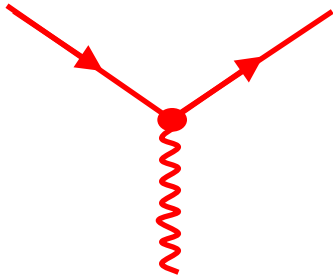
$$\left. \frac{d\sigma}{d\Omega} \right|_{CMS} = \frac{\alpha^2}{4s} \cdot (1 + \cos^2 \theta)$$

$$\sigma_{tot} = \frac{4\pi\alpha^2}{3s} = \frac{86.86 \text{ nb GeV}^2}{s}$$



**Fig. 6.6** The total cross section for  $e^-e^+ \rightarrow \mu^- \mu^+$  measured at PETRA versus the center-of-mass energy.

## Comments about the angular distribution



Decomposition of the fermion current:

$$\begin{aligned}\bar{u}\gamma^\mu u &= (\bar{u}_R + \bar{u}_L)\gamma^\mu (u_R + u_L) \\ &= \bar{u}_R\gamma^\mu u_R + \bar{u}_L\gamma^\mu u_L\end{aligned}$$

$$\bar{u}_L = u_L^\dagger \gamma^0 = u^\dagger \frac{1}{2}(1 - \gamma^5)\gamma^0 = \bar{u} \frac{1}{2}(1 + \gamma^5)$$

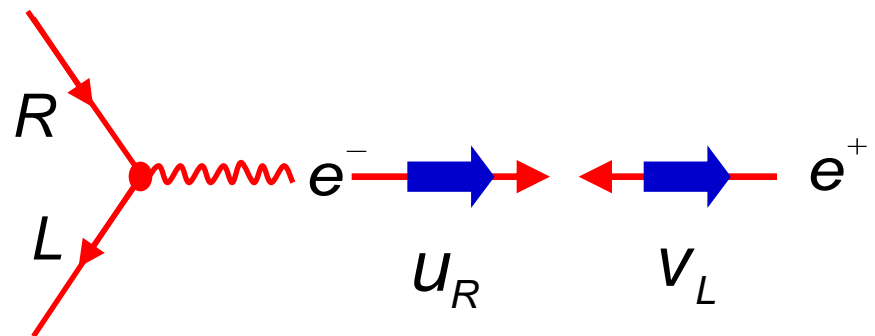
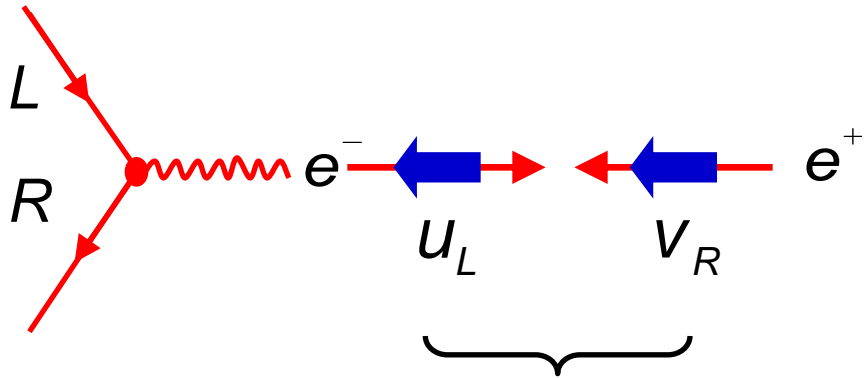
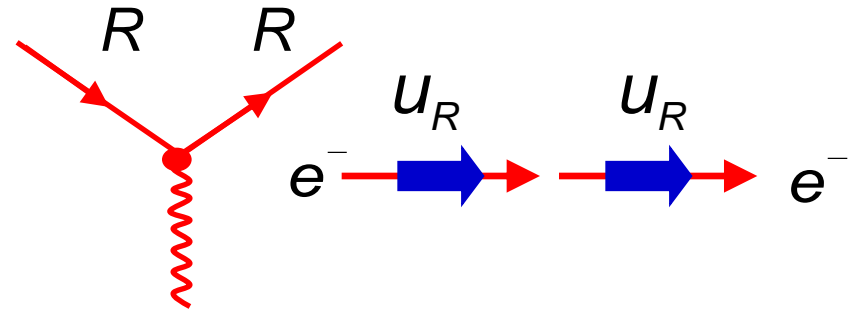
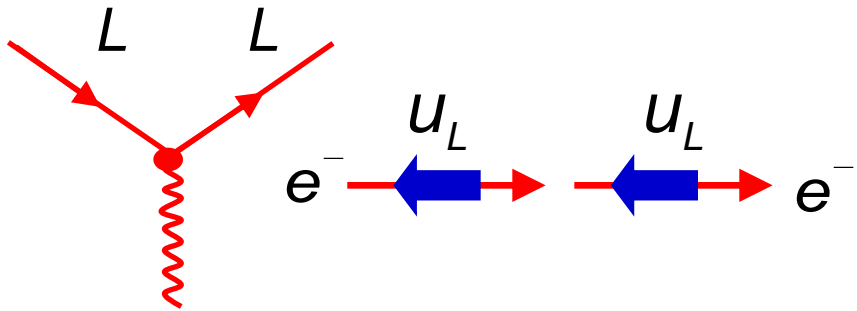
$$\gamma^5 \gamma^0 = -\gamma^0 \gamma^5 \quad (\gamma^5)^2 = 1$$



Symbolically – correct only  
for massless fermions

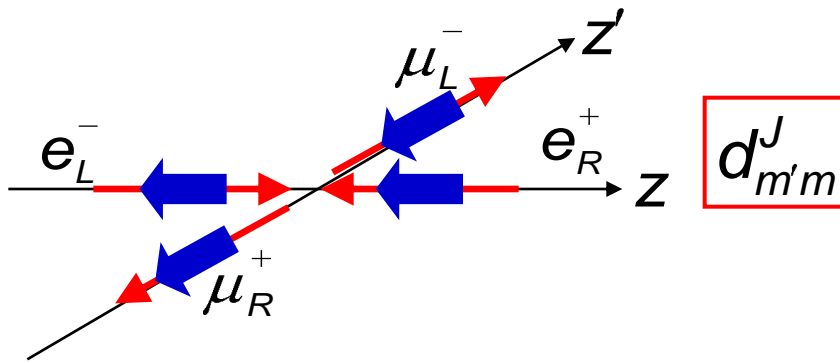


# Vector current $ie\gamma^\mu$ :



Photon spin = 1

Angular distribution  $e^+ e^- \rightarrow \mu^+ \mu^-$

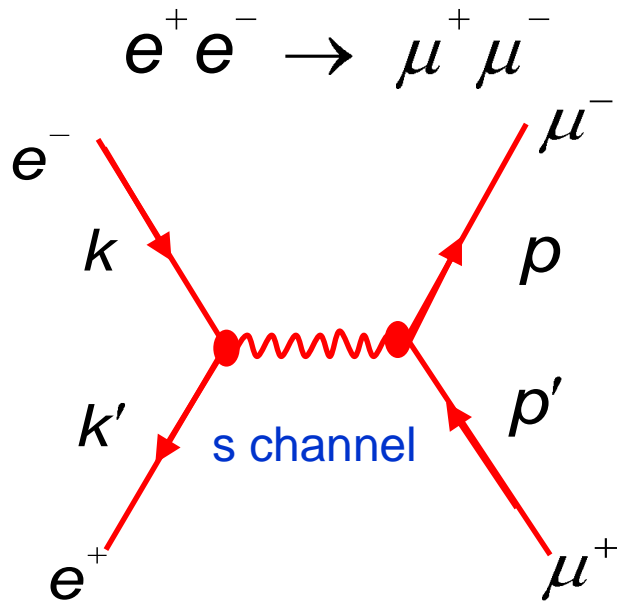


Axis z	rotation	Axis z'
$J=1$	$\xrightarrow{d_{-1-1}^1}$	$J=1$
$m_z = -1$		$m_{z'} = -1$
$J=1$	$\xrightarrow{d_{+1-1}^1}$	$J=1$
$m_z = -1$		$m_{z'} = +1$

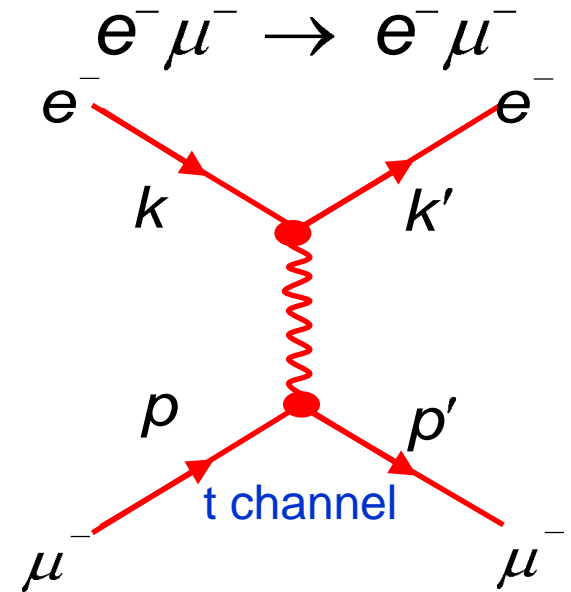
Change of quantization axis

$$\frac{d\sigma}{d\Omega} \sim \left| \mathcal{Y}_{-1-1}^1 \right|^2 + \left| \mathcal{Y}_{+1-1}^1 \right|^2 \sim \frac{1}{4} (1 + \cos\theta)^2 + \frac{1}{4} (1 - \cos\theta)^2 \sim 1 + \cos^2 \theta$$

Angular distribution is an effect of vector  $ie\gamma^\mu$  coupling!



Crossing  
 $k' \rightarrow -k' \quad p \rightarrow -p$



$$s = (k + k')^2 \quad \rightarrow \quad \tilde{t} = (k - k')^2$$

$$t = (k - p)^2 \quad \rightarrow \quad \tilde{s} = (k + p)^2$$

$$u = (k - p')^2 \quad \rightarrow \quad \tilde{u} = (k - p')^2 = u$$

$$\overline{|M|^2}_{e^+ e^- \rightarrow \mu^+ \mu^-}(s, t, u) = \overline{|M|^2}_{e^- \mu^- \rightarrow e^- \mu^-}(\tilde{t}, \tilde{s}, \tilde{u})$$

$$\overline{|M|^2}_{e^+ e^- \rightarrow \mu^+ \mu^-}(s, t, u) = 2e^4 \frac{t^2 + u^2}{s^2} \quad \Rightarrow \quad \overline{|M|^2}_{e^- \mu^- \rightarrow e^- \mu^-}(\tilde{t}, \tilde{s}, \tilde{u}) = 2e^4 \frac{\tilde{s}^2 + \tilde{u}^2}{\tilde{t}^2}$$

# Fermion scattering - Summary

## Feynman Diagrams

$$\overline{|\mathcal{M}|^2}/2e^4$$

	Forward peak	Backward peak	Forward	Interference	Backward
Møller scattering $e^-e^- \rightarrow e^-e^-$ (Crossing $s \leftrightarrow u$ )					
				$\frac{s^2 + u^2}{t^2} + \frac{2s^2}{tu} + \frac{s^2 + t^2}{u^2}$ ( $u \leftrightarrow t$ symmetric )	
Bhabha scattering $e^-e^+ \rightarrow e^-e^+$			Forward	Interference	Time-like
				$\frac{s^2 + u^2}{t^2} + \frac{2u^2}{ts} + \frac{u^2 + t^2}{s^2}$	
$e^-e^+ \rightarrow \mu^-\mu^+$ (Crossing $s \leftrightarrow t$ )					
				$\frac{s^2 + u^2}{t^2}$ "Rutherford"	
				$\frac{u^2 + t^2}{s^2}$	



## 2.2 Experimental methods

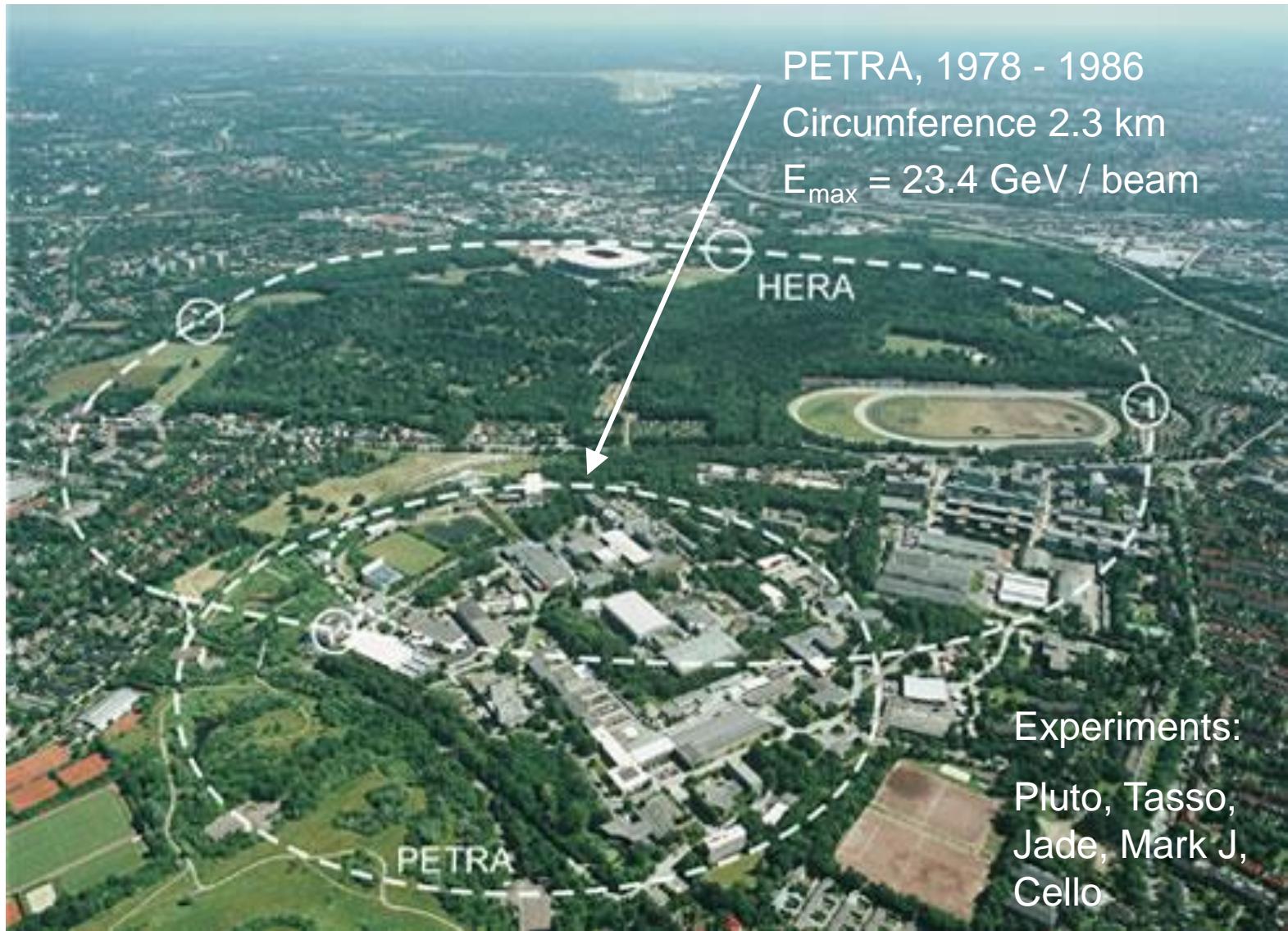
### e<sup>+</sup>e<sup>-</sup> accelerator (selection)

Accelerator	Lab	$\sqrt{s}$	$L_{\text{int}} / \text{Exper.}$	
SPEAR	SLAC	2 – 8 GeV		
PEP	SLAC	→29 GeV	220 - 300 pb <sup>-1</sup>	
PETRA	DESY	12 - 47 GeV	~20 pb <sup>-1</sup>	today
TRISTAN	KEK	50 – 60 GeV	~20 pb <sup>-1</sup>	
LEP	CERN	90 GeV	~200 pb <sup>-1</sup>	Z physics

### Cross section (experimental definition)

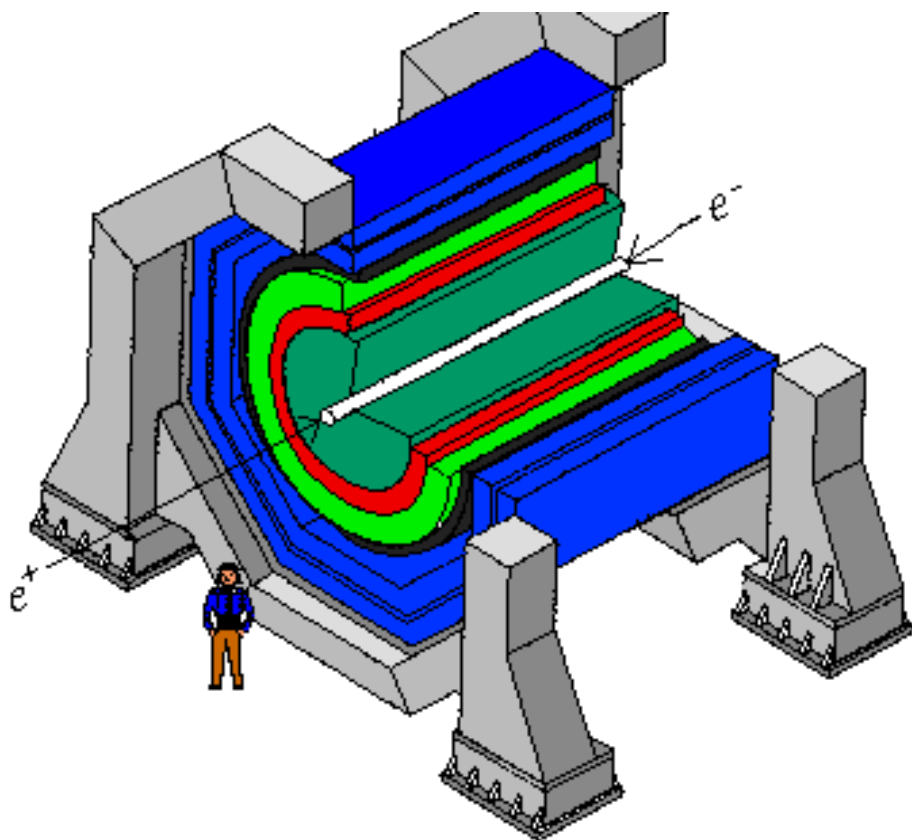
$$\sigma(e^+e^- \rightarrow f\bar{f}) = \frac{N_{ff}(1-b)}{\varepsilon L_{\text{int}}}$$

- $N_{ff}$  number of detected  $e^+e^- \rightarrow ff$  events
- $b$  background fraction
- $\varepsilon$  acceptance / efficiency
- $L_{\text{int}}$  integrated luminosity of collider



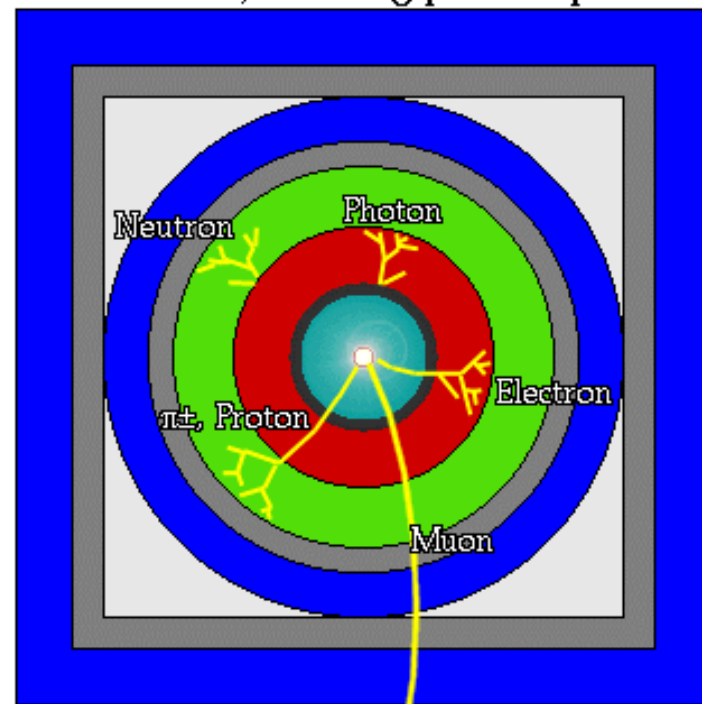


# Particle detectors



A detector cross-section, showing particle paths

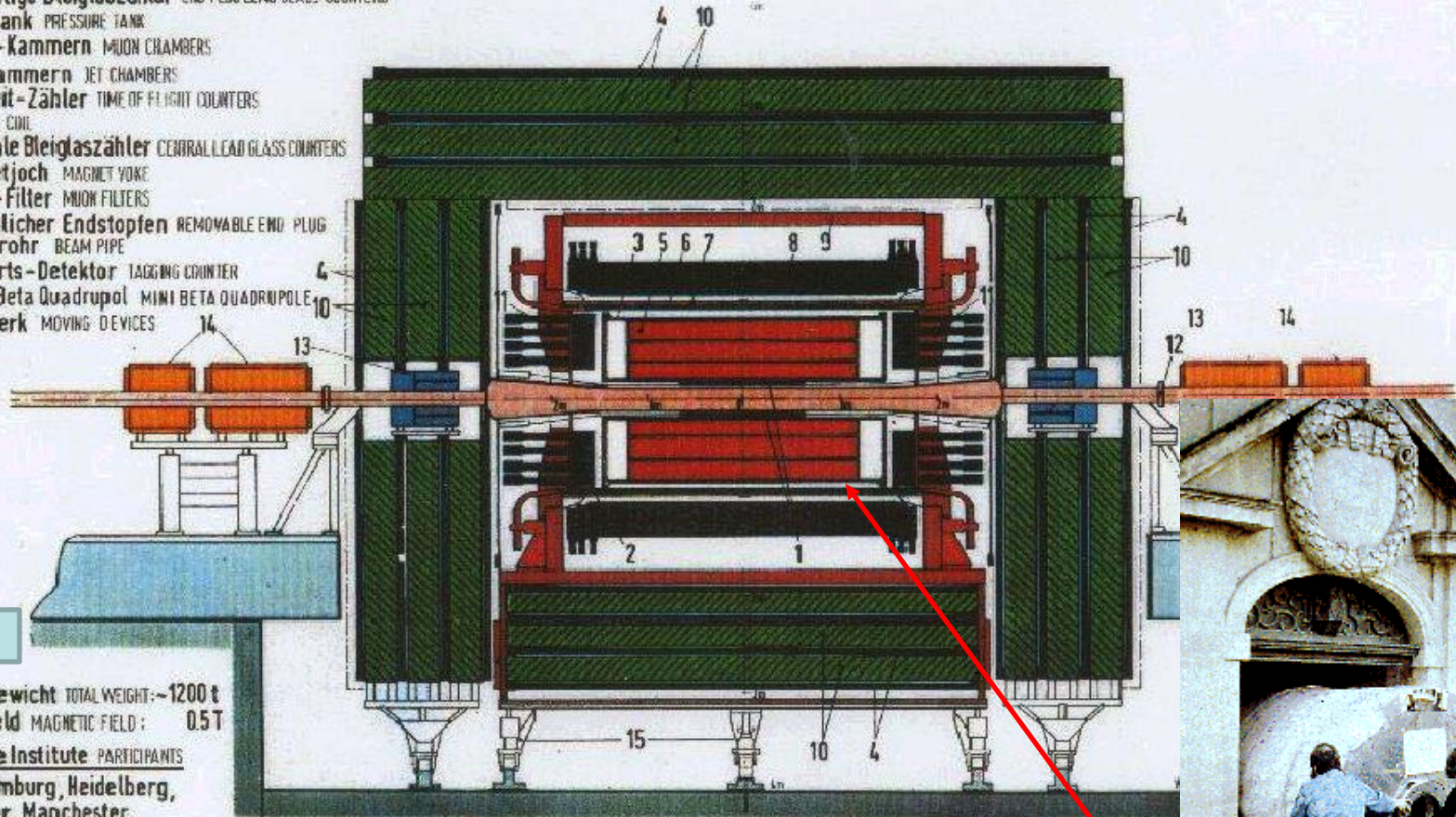
- Beam Pipe (center)
- Tracking Chamber
- Magnet Coil
- E-M Calorimeter
- Hadron Calorimeter
- Magnetized Iron
- Muon Chambers



# Japan – Deutschland – England

## MAGNETDETEKTOR **JADE** MAGNET DETECTOR

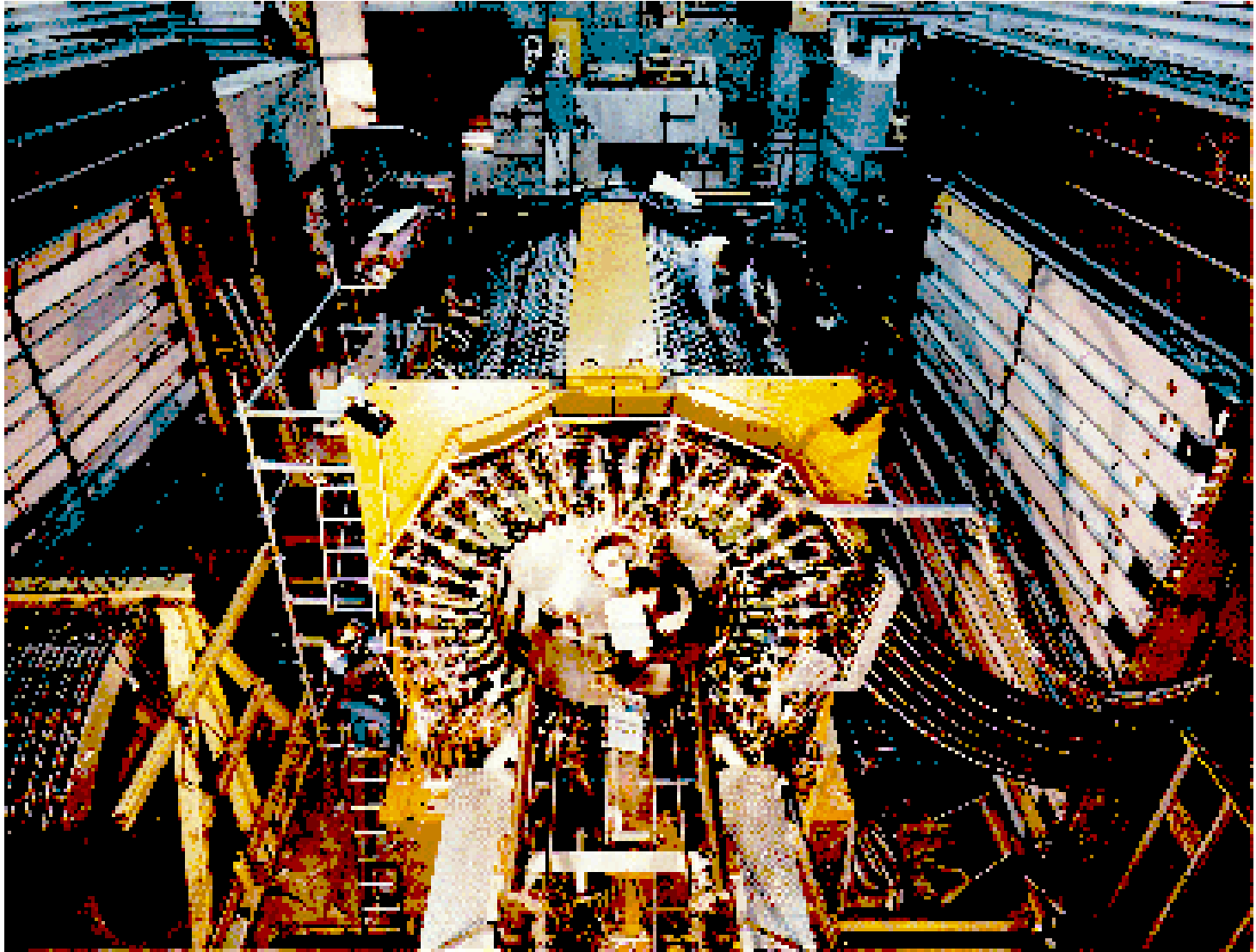
- 1 Strahlrohrzähler BEAM PIPE COUNTERS
- 2 Endseitige Bleiglaszähler END PLUG LEAD GLASS COUNTERS
- 3 Drucktank PRESSURE TANK
- 4 Myon-Kammern MUON CHAMBERS
- 5 Jet-Kammern JET CHAMBERS
- 6 Flugzeit-Zähler TIME OF FLIGHT COUNTERS
- 7 Spule COIL
- 8 Zentrale Bleiglaszähler CENTRAL LEAD GLASS COUNTERS
- 9 Magnetjoch MAGNET YOKE
- 10 Myon-Filter MUON FILTERS
- 11 Beweglicher Endstopfen REMOVABLE END PLUG
- 12 Strahlrohr BEAM PIPE
- 13 Vorwärts-Detektor TAGGING COUNTER
- 14 Mini-Beta Quadrupol MINI BETA QUADRUPOLE
- 15 Fahrwerk MOVING DEVICES



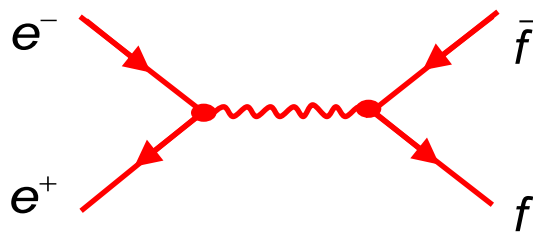
Gesamtgewicht TOTAL WEIGHT: ~1200 t  
 Magnetfeld MAGNETIC FIELD: 0.5 T  
 Beteiligte Institute PARTICIPANTS  
 DESY, Hamburg, Heidelberg,  
 Lancaster, Manchester,  
 Rutherford Lab., Tokio



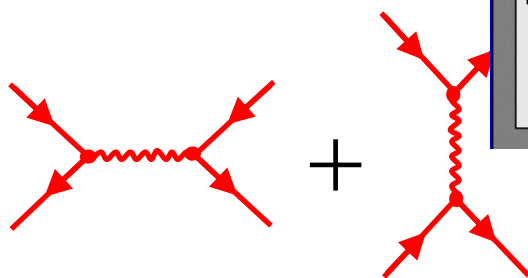




# Experimental Signatures:

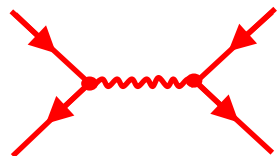


$$f \bar{f} = e^- e^+$$



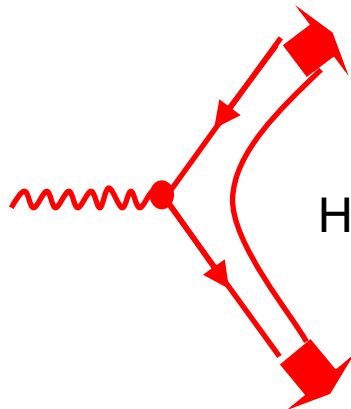
$$\mu^- \mu^+$$

$$\tau^- \tau^+$$



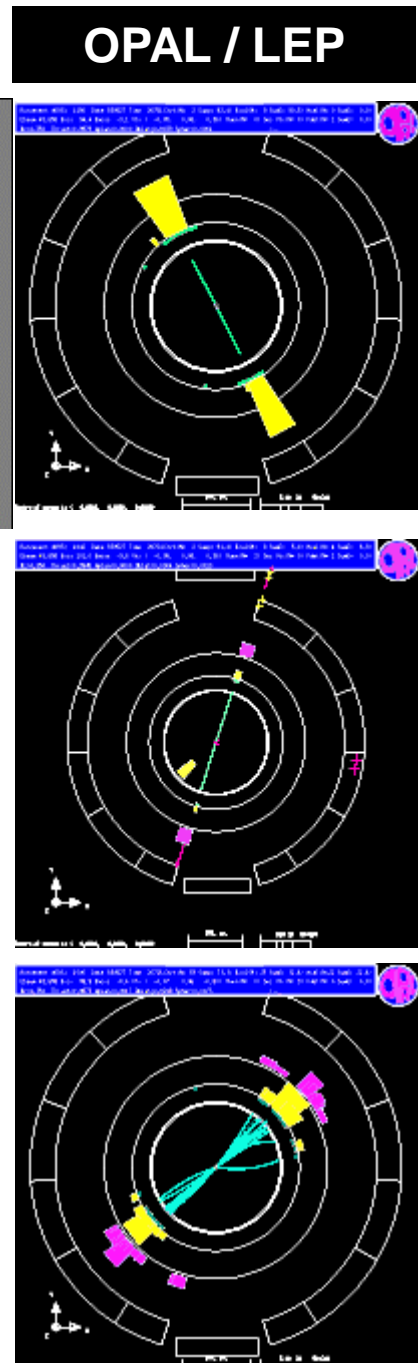
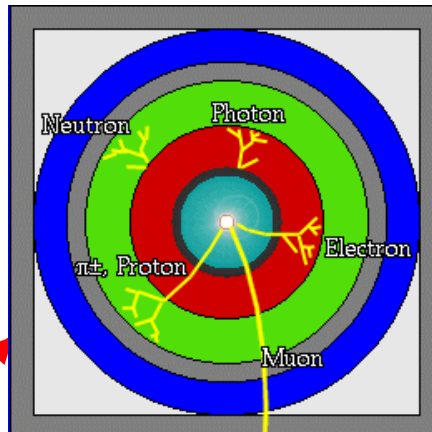
$$\mu^- \mu^+$$

$$q \bar{q} \text{ mit } q = u, d, s, c, b, (t)$$



Hadron jets

$$q \bar{q}$$

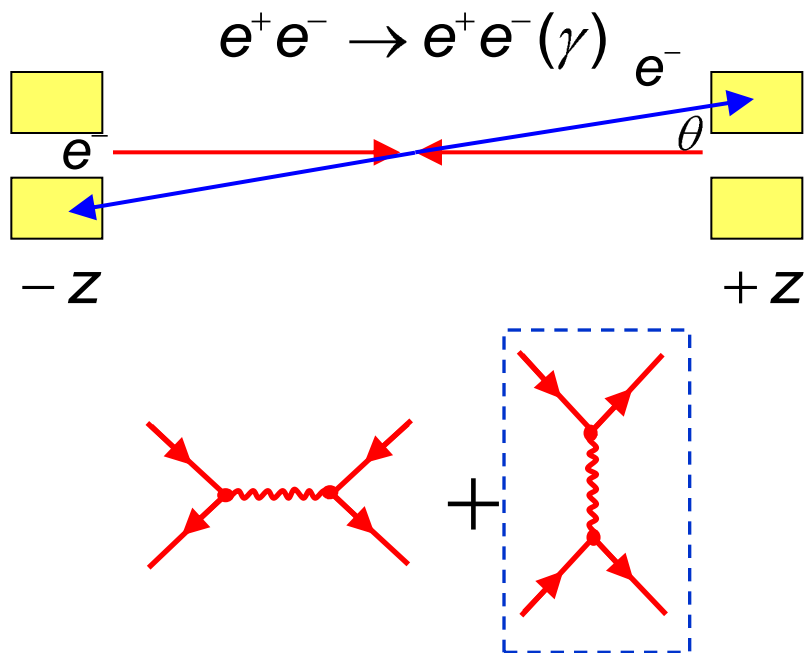


$$e^- e^+$$

# Determination of integrated luminosity

$$L_{\text{int}} = \int L_{ee}(t) dt$$

small angle Bhabha scattering  
(low momentum transfer):



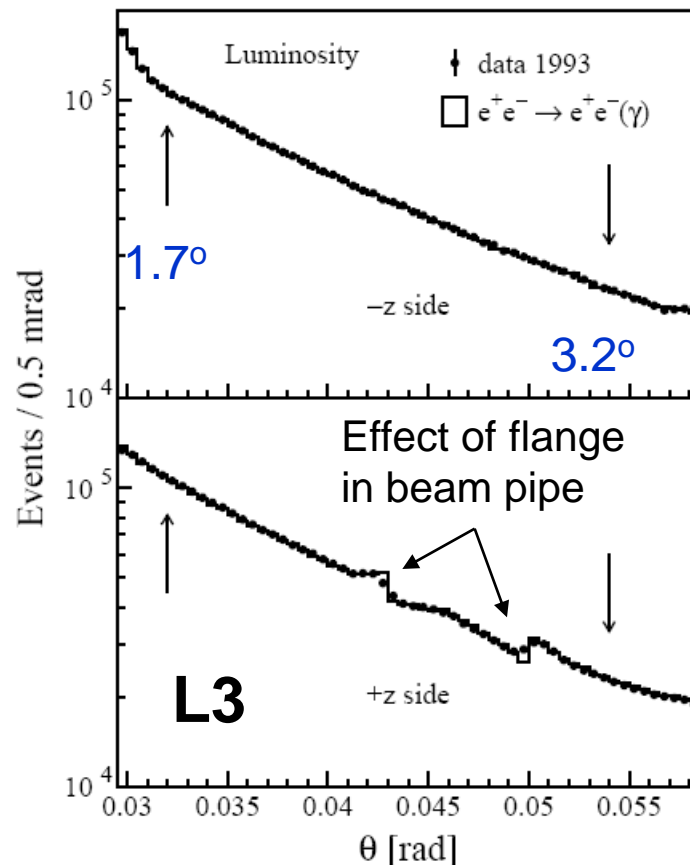
Small angle Bhabha scattering is t channel dominated: theoretical cross section  $\sigma_{\text{theo}}$  well known.



$$L_{\text{int}} = \frac{N_{ee}}{\sigma_{\text{theo}} \mathcal{E}}$$

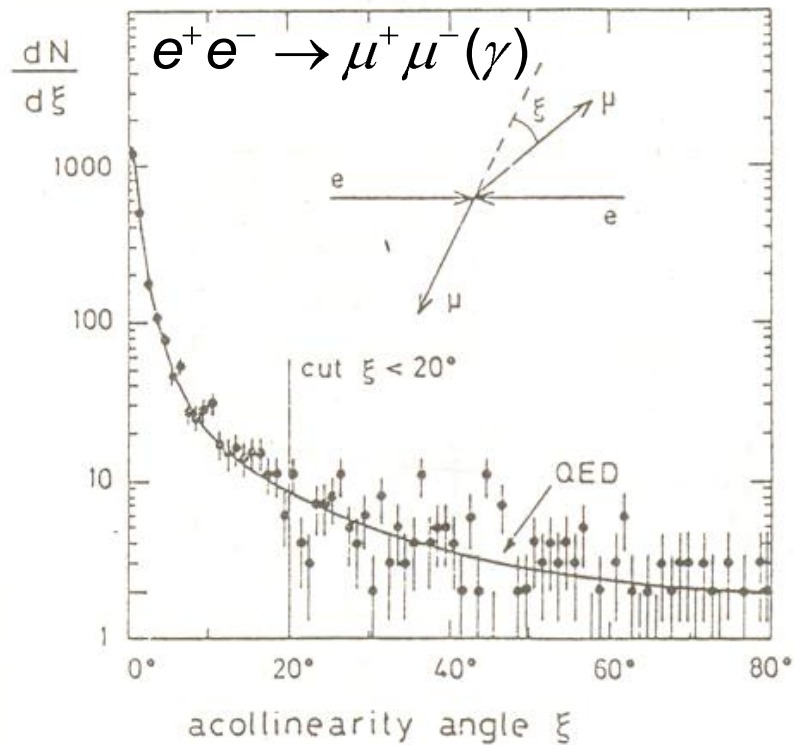
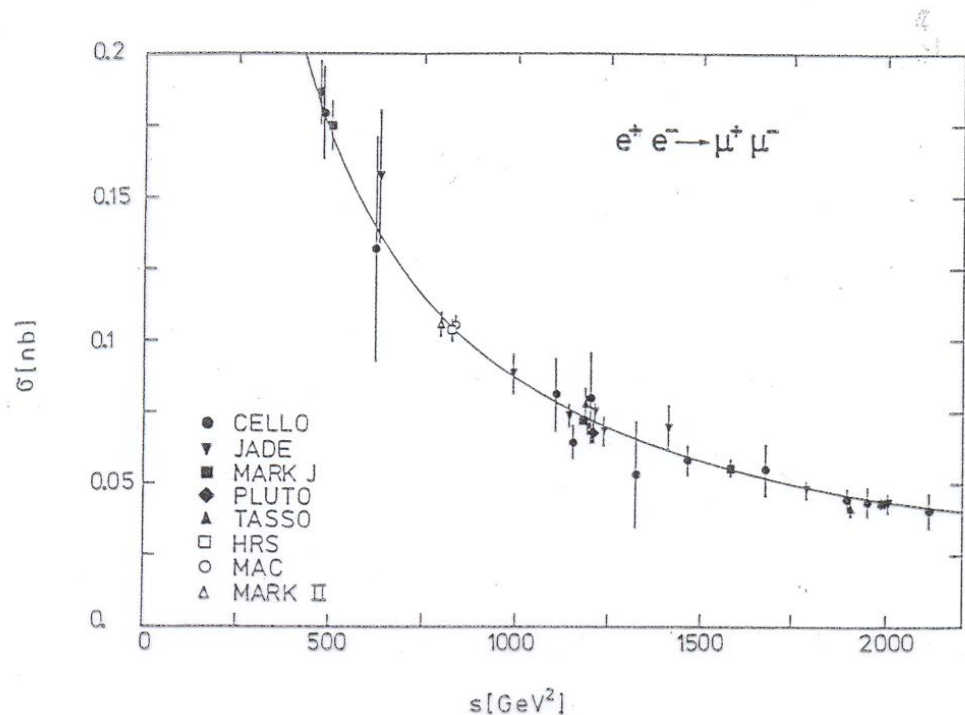
At LEP:  
typ. errors < 0.5%

$$\sigma(e^+e^- \rightarrow f \bar{f}) = \frac{N_{ff}(1-b)}{\mathcal{E} L_{\text{int}}}$$



# 2.3 $e^+e^- \rightarrow \mu^+\mu^-$

Acollinearity

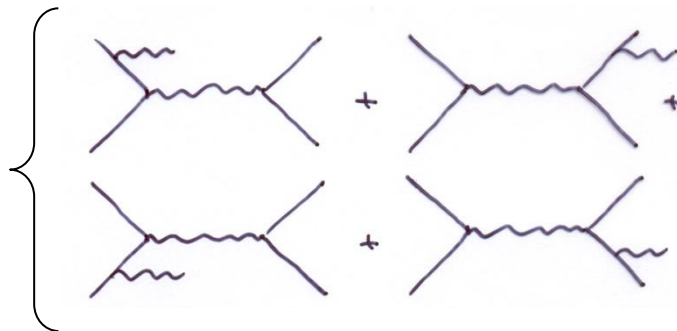


Good agreement with QED!

Quantitative limit for new physics ?

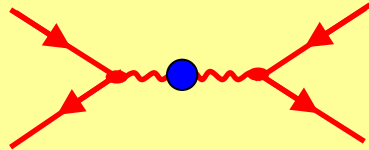
Effect of bremsstrahlung:

There will always be additional photons



## Possible deviation from QED:

- additional heavy photon



Modifies propagator

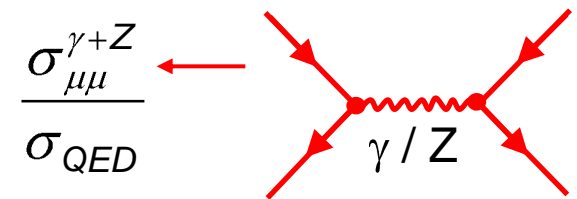
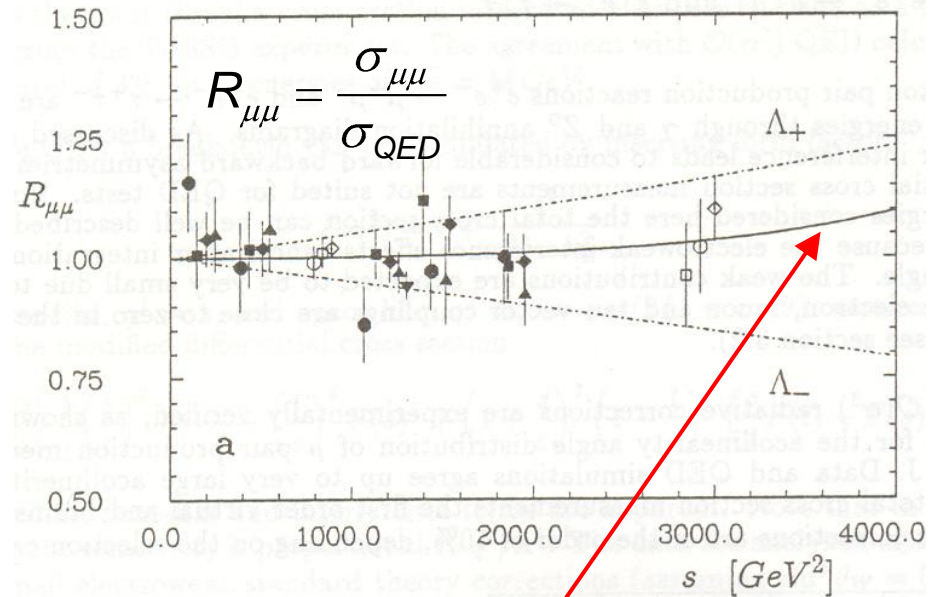
$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} - \frac{1}{q^2 - \Lambda^2} = \frac{1}{q^2} \left(1 - \frac{q^2}{q^2 - \Lambda^2}\right)$$

$$\approx \frac{1}{q^2} \left(1 + \frac{q^2}{\Lambda^2}\right)$$

$\Lambda$  corresponds to the mass of new photon

To also account for possible lower cross sections:

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} \left(1 \mp \frac{q^2}{q^2 - \Lambda_{\pm}^2}\right)$$



Additional heavy photon:

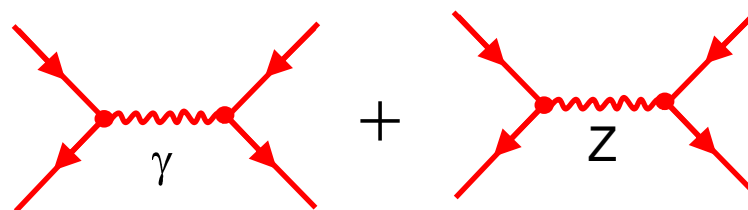
$$\sigma_{\mu\mu} = \frac{4\pi\alpha^2}{s} \left(1 \mp \frac{s}{s - \Lambda_{\pm}^2}\right)^2$$

$\rightarrow \Lambda_{\pm} > 200 \text{ GeV}$

Confirms "Coulomb law" down to  $10^{-18} \text{ m}$

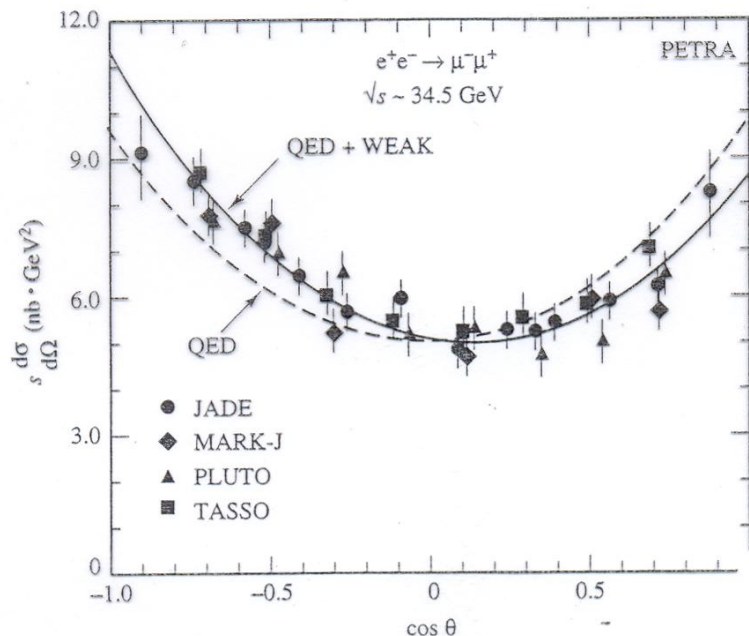
# Effect of Z boson exchange

„heavy photon w/ different couplings“



$$\left. \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \cdot (1 + \cos^2 \theta) \right|_{QED}$$

$$\left. \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \cdot (1 + \cos^2 \theta + A \cos \theta) \right|_{\gamma+Z}$$



The effect of the “heavy” Z boson is already seen at low energies!

Clear deviation from QED:

⇒ Effect of electro-weak  $\gamma/Z$  interference

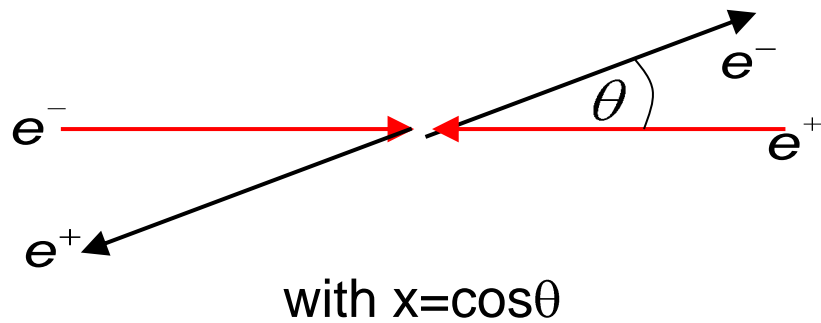
## 2.4 Bhabha scattering $e^+e^- \rightarrow e^+e^-$

$$M = \text{t channel} + \text{s channel}$$

$$|M|^2 = \underbrace{\left| \text{t channel} \right|^2}_{e^- \mu^- \rightarrow e^- \mu^-} + \text{interference} + \underbrace{\left| \text{s channel} \right|^2}_{e^+ e^- \rightarrow \mu^+ \mu^-}$$

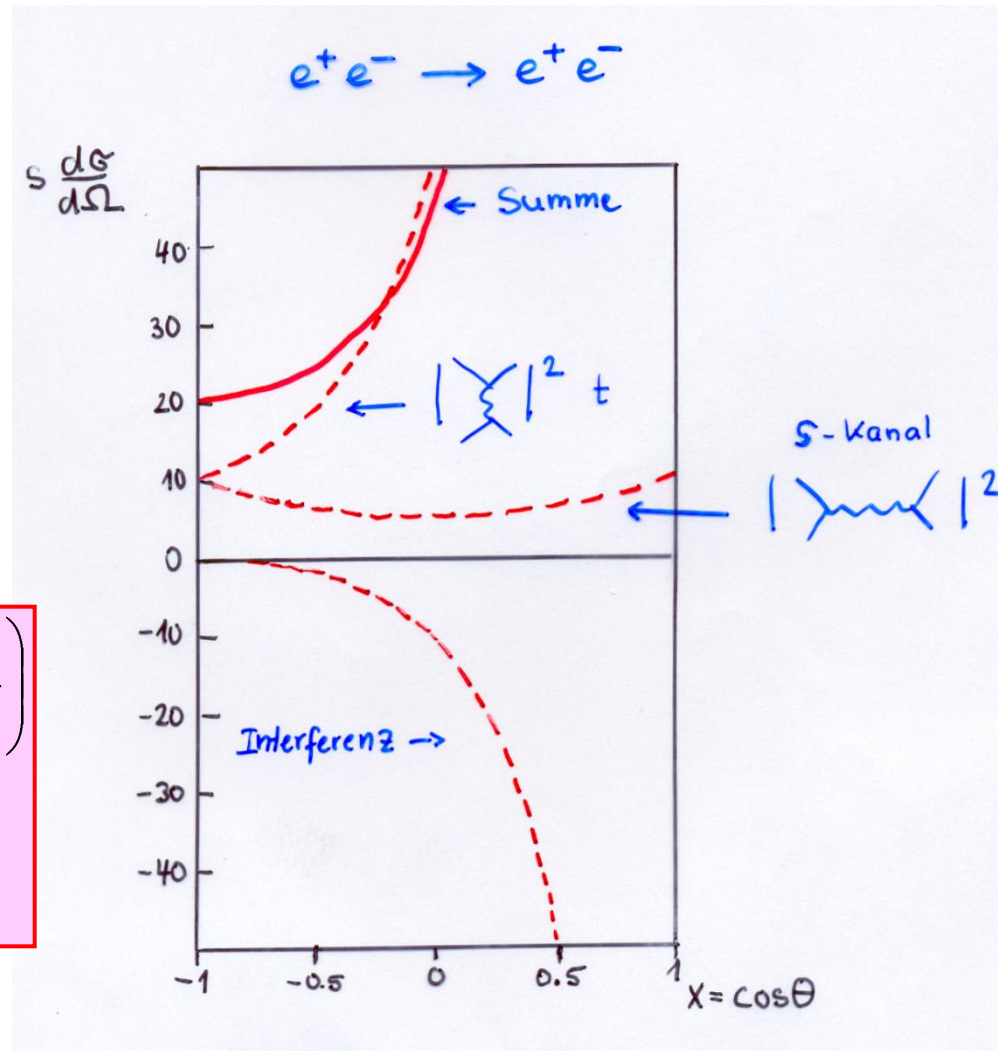
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s} \left( \frac{s^2 + u^2}{t^2} + \frac{2u^2}{ts} + \frac{t^2 + u^2}{s^2} \right)$$

CM system:



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s} \left( \frac{4 + (1+x)^2}{(1-x)^2} - \frac{(1+x)^2}{1-x} + \frac{1+x^2}{2} \right)$$

$$= \frac{\alpha^2}{2s} \left( \frac{3 + \cos^2\theta}{1 - \cos\theta} \right)^2$$





$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s} \left( \frac{3 + \cos^2 \theta}{1 - \cos \theta} \right)^2$$



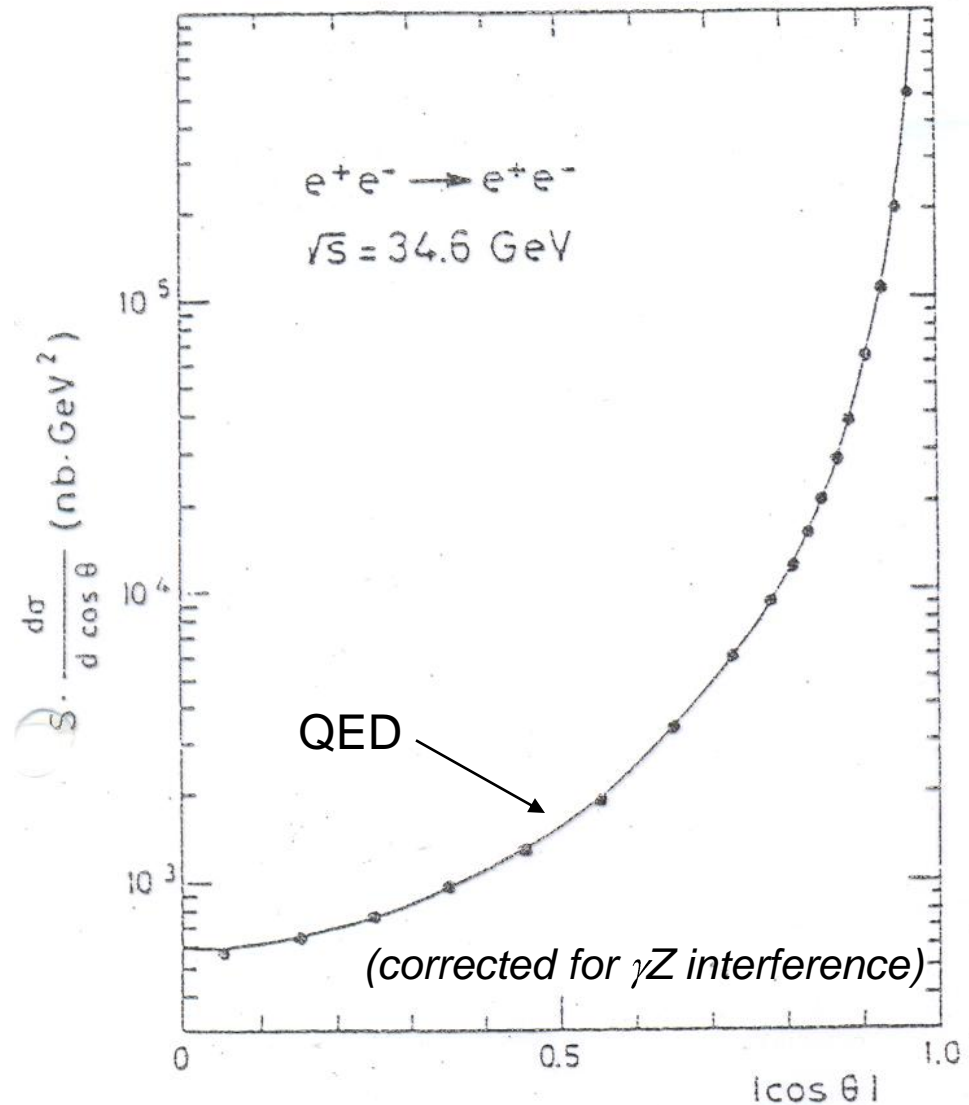
divergent for  $\cos\theta \rightarrow 1$

Additional „heavy photon“:

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} \left( 1 \mp \frac{q^2}{q^2 - \Lambda_{\pm}^2} \right)$$

⇒ Can be described by form factor:

$$F(q^2) = 1 \mp \frac{q^2}{q^2 - \Lambda_{\pm}^2}$$



Form factor modifies differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s} \left( \frac{u^2 + s^2}{t^2} |F(t)|^2 + \frac{2u^2}{ts} |F(t)F(s)| + \frac{u^2 + t^2}{s^2} |F(s)|^2 \right)$$

Fit to combined PETRA  $e^+e^-$  data:

$\Lambda_+ > 435$  GeV @ 95% CL

$\Lambda_- > 590$  GeV



In the “space picture” form factor corresponds to modified Coulomb potential at small distances:

$$\frac{1}{r} \rightarrow \frac{1}{r} (1 - e^{-\Lambda r})$$

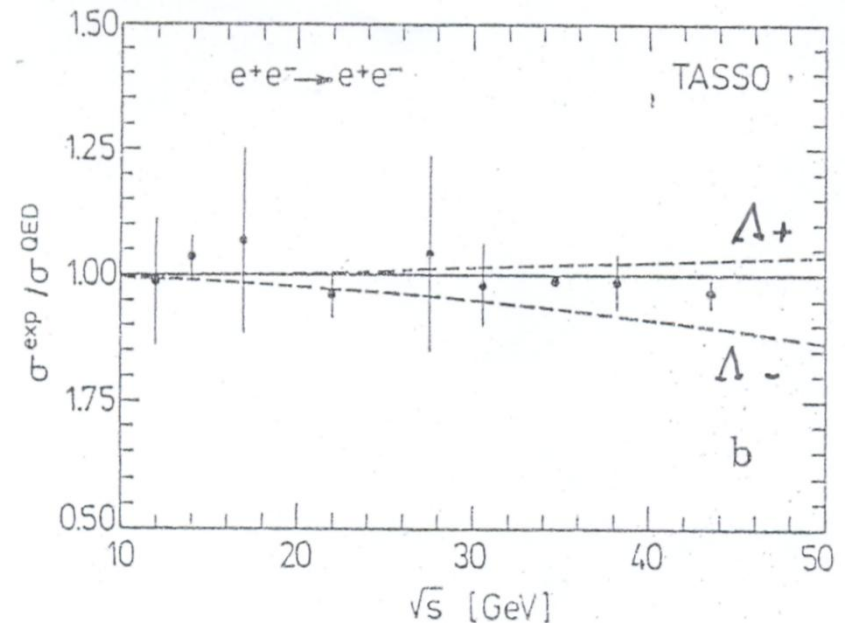
i.e.  $\Lambda$  measures point-like nature of  $e\gamma$  interaction (size of electron).

$\Lambda > \sim 500$  GeV  $\Leftrightarrow r_e < 0.197/500$  fm

Electr. substructure  $< 0.5 \times 10^{-18}$  m

Tasso:  $\Lambda_+ > 370$  GeV

$\Lambda_- > 190$  GeV



# 2.5 Discovery of the Tau-Lepton

MARK I (SLAC), 1975, M.Pert et al.

Nobel Prize 1995 for M.Pert

## Evidence for Anomalous Lepton Production in $e^+e^-$ Annihilation\*

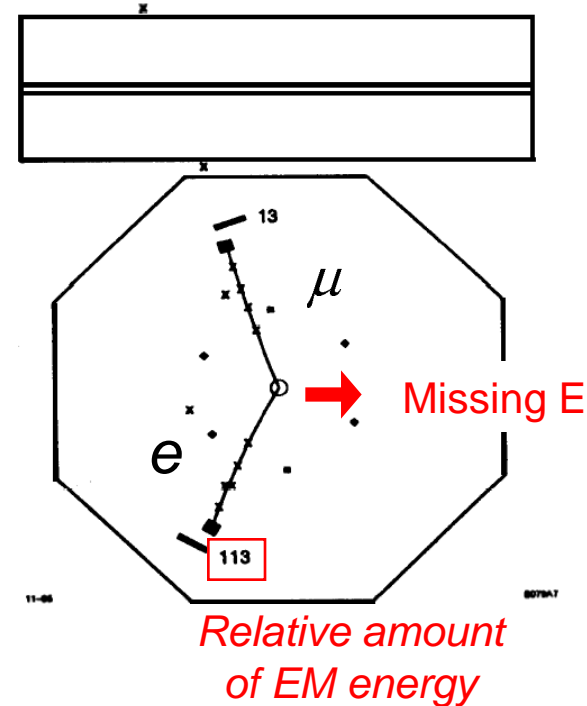
M. L. Perl, G. S. Abrams, A. M. Boyarski, M. Breidenbach, D. D. Briggs, F. Bulos, W. Chinowsky, J. T. Dakin,† G. J. Feldman, C. E. Friedberg, D. Fryberger, G. Goldhaber, G. Hanson, F. B. Heile, B. Jean-Marie, J. A. Kadyk, R. R. Larsen, A. M. Litke, D. Lüke,‡ B. A. Lulu, V. Lüth, D. Lyon, C. C. Morehouse, J. M. Paterson, F. M. Pierre,§ T. P. Pun, P. A. Rapidis, B. Richter, B. Sadoulet, R. F. Schwitters, W. Tanenbaum, G. H. Trilling, F. Vannucci,|| J. S. Whitaker, F. C. Winkelmann, and J. E. Wiss

Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720, and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305  
(Received 18 August 1975)

We have found events of the form  $e^+ + e^- \rightarrow e^\pm + \mu^\mp + \text{missing energy}$ , in which no other charged particles or photons are detected. Most of these events are detected at or above a center-of-mass energy of 4 GeV. The missing-energy and missing-momentum spectra require that at least two additional particles be produced in each event. We have no conventional explanation for these events.

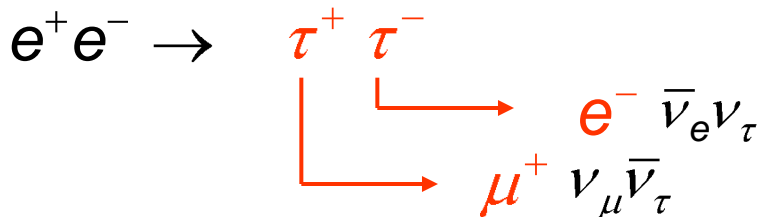
We have found 64 events of the form  $e^+ + e^- \rightarrow e^\pm + \mu^\mp + \geq 2$  undetected particles (1) for which we have no conventional explanation. The undetected particles are charged particles or photons which escape the  $2.6\pi$  sr solid angle

of the detector, or particles very difficult to detect such as neutrons,  $K_L^0$  mesons, or neutrinos. Most of these events are observed at center-of-mass energies at, or above, 4 GeV. These events were found using the Stanford Linear Accelerator Center-Lawrence Berkeley Laboratory (SLAC-

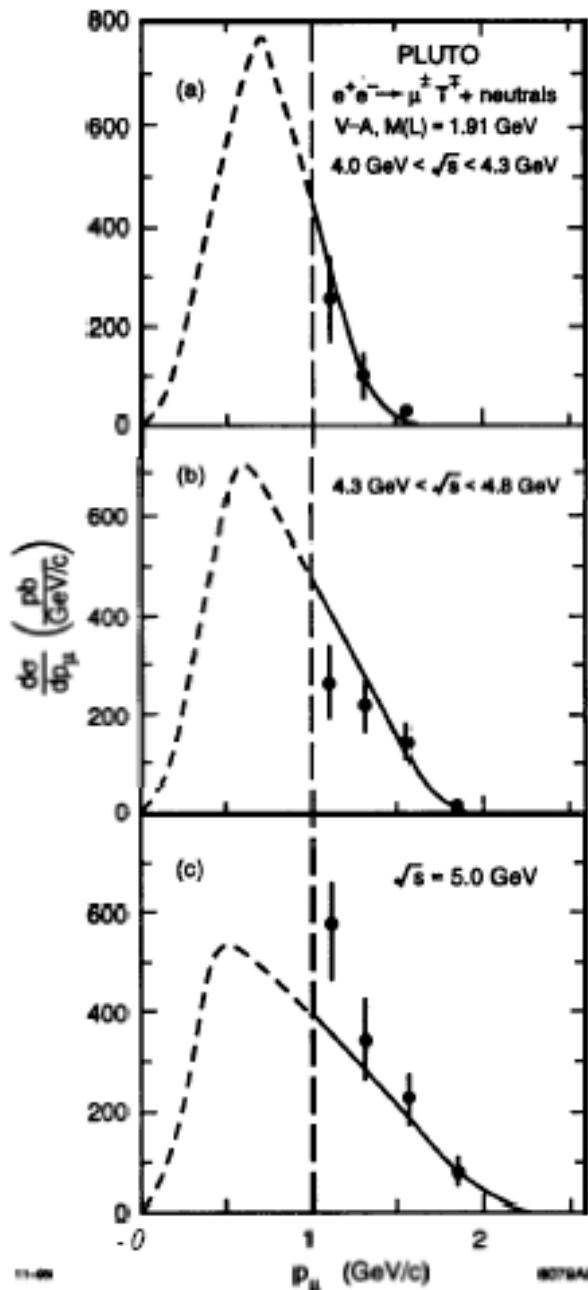


1489

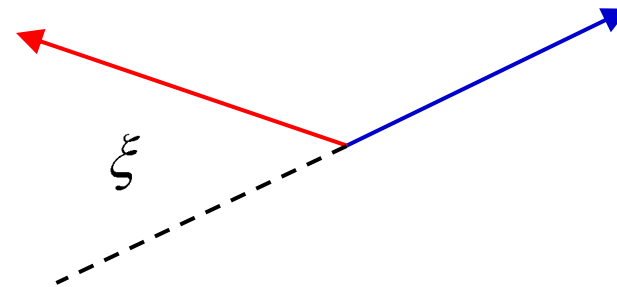
## Explanation:



A lot of Discussions in 1975:  
Are these events really decays of a new 3<sup>rd</sup> generation heavy lepton ?



1) Large acollinearity confirms tau hypothesis



2) Anomalous “single muon events” predicted:

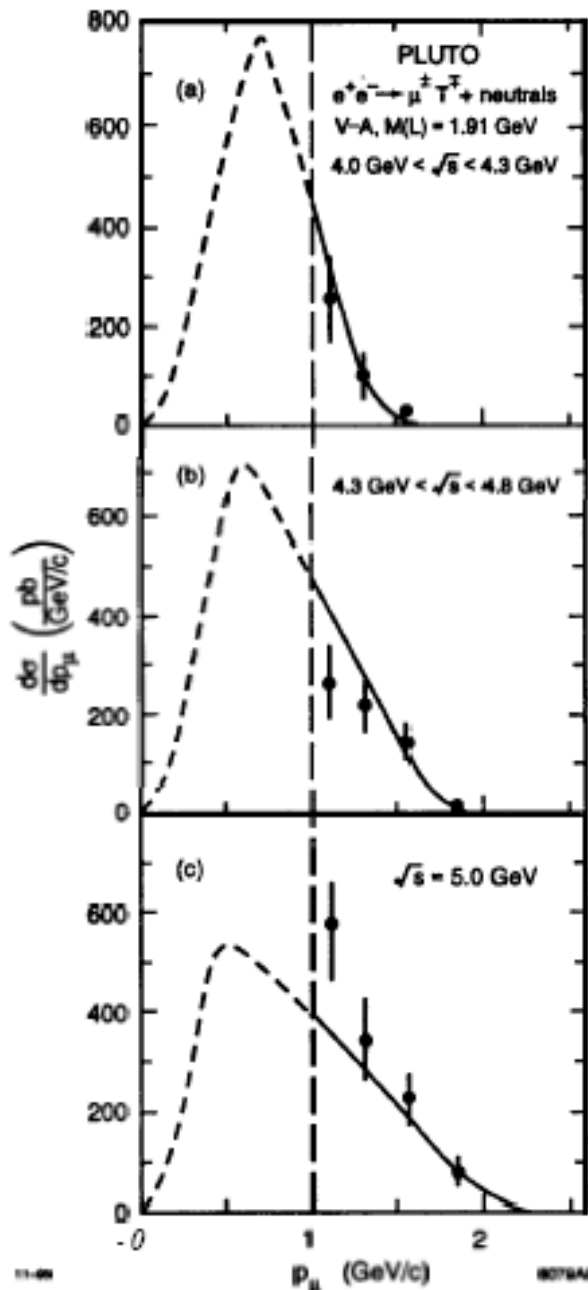
Expectation:  $BR(\tau \rightarrow e(\mu) \nu \bar{\nu}) \approx 20\%$

$BR(\tau \rightarrow h + \nu) \approx 60\%$

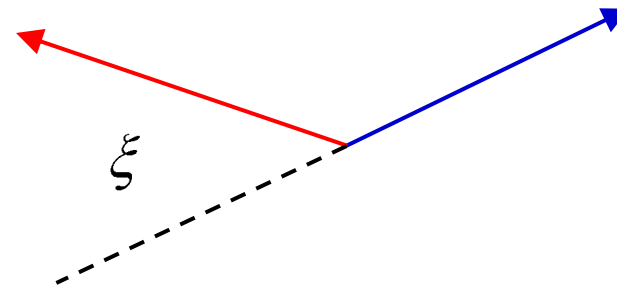
$\rightarrow e^+ + e^- \rightarrow \mu^\pm + h^\mp + \text{missing E}$



PLUTO (DESY, 1976) confirms the anomalous “single muon events”. Muon spectrum consistent with 3-body tau decay.



1) Large acollinearity confirms tau hypothesis



2) Anomalous “single muon events” predicted:

Expectation:  $BR(\tau \rightarrow e(\mu) \nu \bar{\nu}) \approx 20\%$

$BR(\tau \rightarrow h + \nu) \approx 60\%$

$\rightarrow e^+ + e^- \rightarrow \mu^\pm + h^\mp + \text{missing E}$

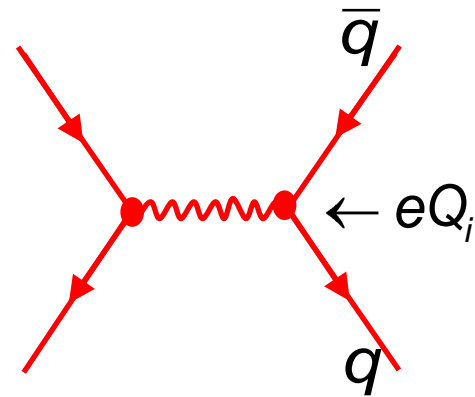


PLUTO (DESY, 1976) confirms the anomalous “single muon events”. Muon spectrum consistent with 3-body tau decay.

## 2.6 $e^+e^- \rightarrow \text{hadrons}$

$e^+e^-$  annihilation to a pair of quarks with subsequent hadronization.

Quarks have fractional charges and carry “color” as additional quantum number.



$$Q_i = \begin{cases} +\frac{2}{3} \\ -\frac{1}{3} \end{cases}$$

Additional color factor  $N_C$

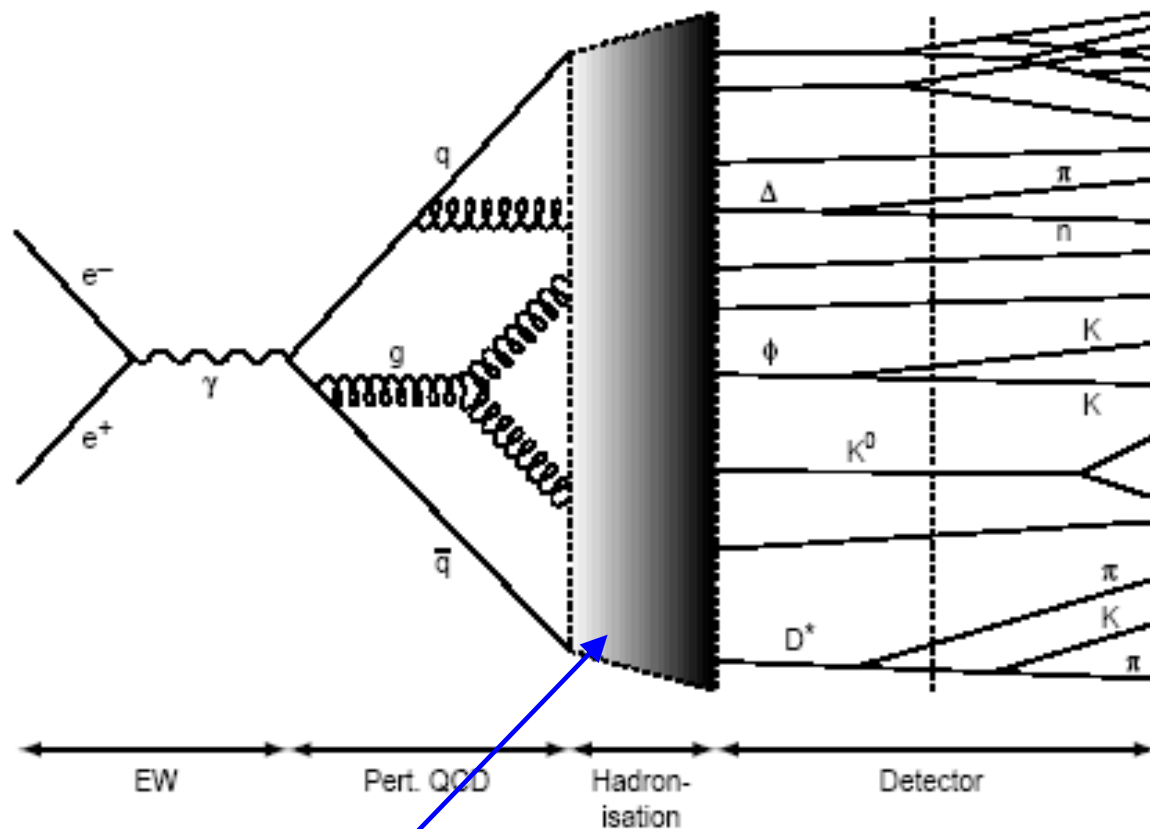


$$\left. \frac{d\sigma}{d\Omega} \right|_{ee \rightarrow \text{hadrons}} = \frac{\alpha^2}{4s} \cdot N_C \cdot \underbrace{\sum_{\text{quarks } i} Q_i^2}_{\text{Sum over kinematically possible quark flavors: } 4m_q^2 < s} (1 + \cos^2 \theta)$$

Sum over kinematically possible quark flavors:  
 $4m_q^2 < s$

$\sqrt{s}$	Quarks
$< \sim 3 \text{ GeV}$	uds
$< \sim 10 \text{ GeV}$	udsc
$< \sim 350 \text{ GeV}$	udscb
$> \sim 350 \text{ GeV}$	udscbt

# From Quarks to Jets



Described successfully by different phenomenological fragmentation models realized as Monte Carlo programs: **PHYTIA**, **HERWIG**

*~ 20 particles at 90 GeV*

# Quark jets and angular distribution

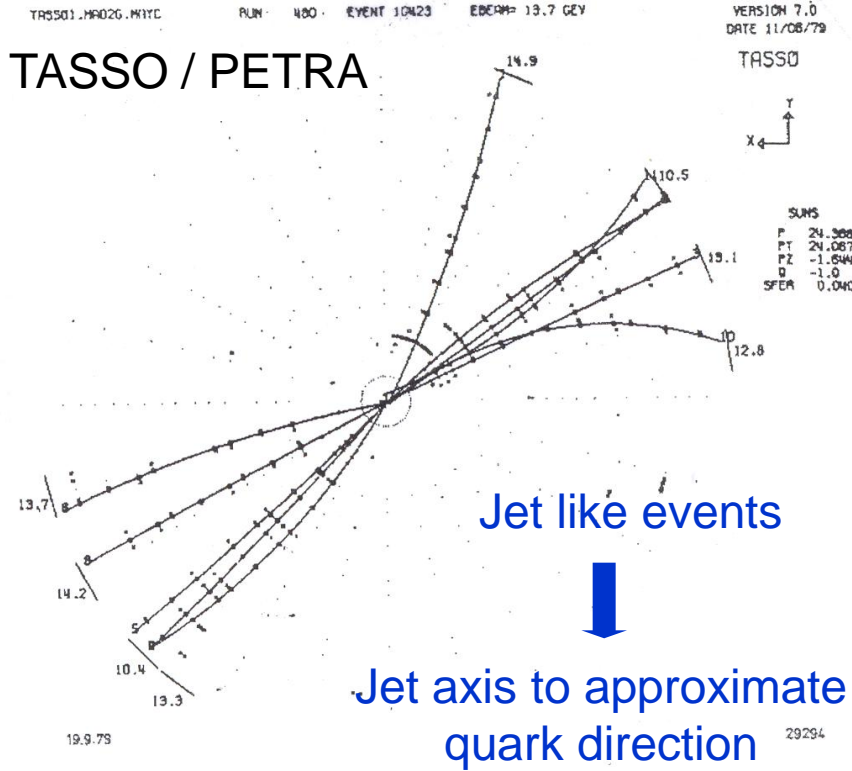


Fig.2 A typical multihadron event at 27.4 GeV recorded in the central detector. The inner 4 layers belong to the proportional chamber, the following 9 are zero degree layers of the drift chamber. The solid bars at the periphery mark time-of-flight counters.

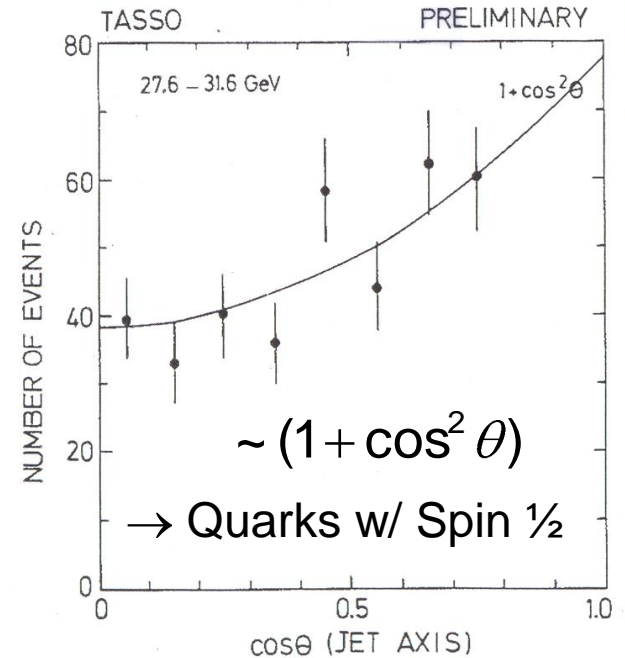
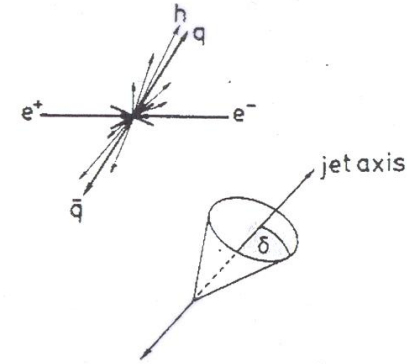


Fig.7 Angular distribution of the jet axis with respect to the beam.

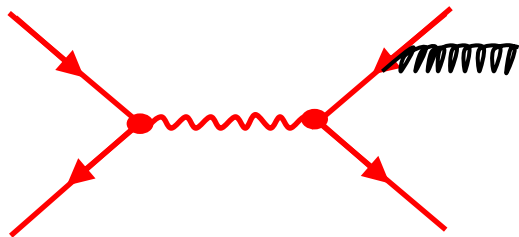
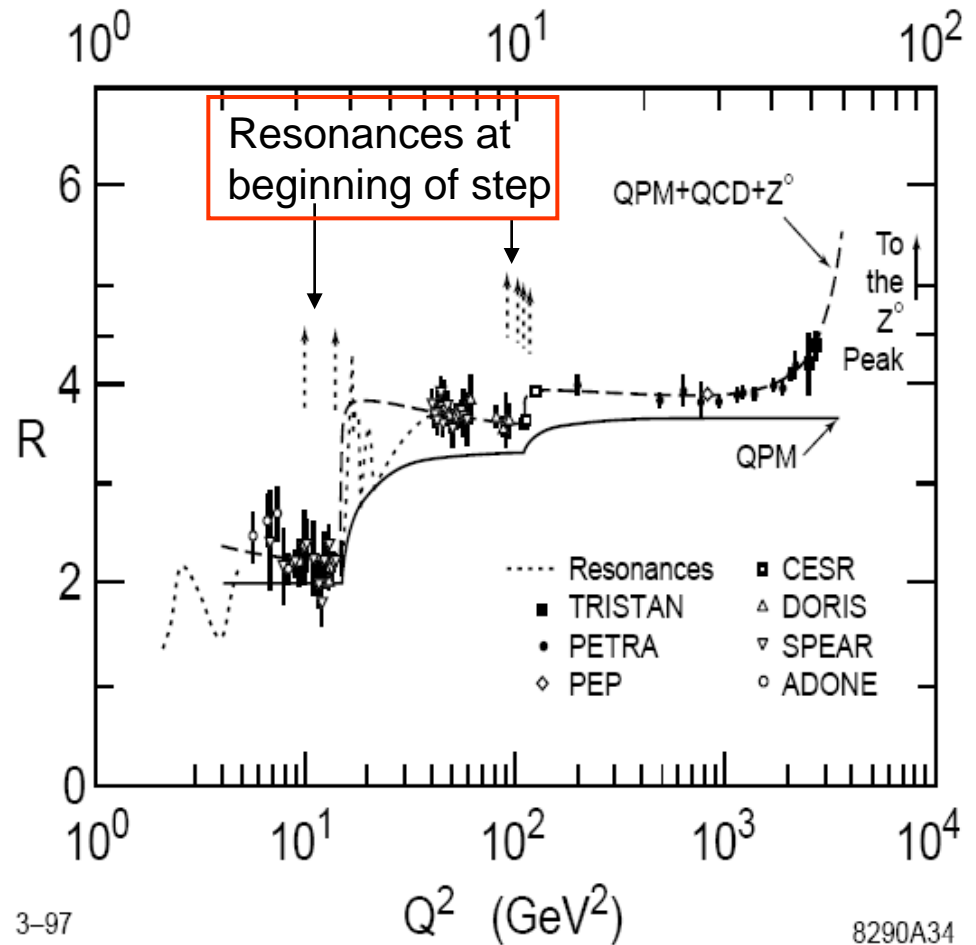


Definition:

$$R_{had} = \frac{\sigma(ee \rightarrow hadrons)}{\sigma(ee \rightarrow \mu\mu)} = 3 \cdot \sum_i Q_i^2$$

$\sqrt{s}$	Quarks	$R_{had} = 3 \cdot \sum_i Q_i^2$
$< \sim 3 \text{ GeV}$	uds	$3 \cdot 6/9 = 2.00$
$< \sim 10 \text{ GeV}$	udsc	$3 \cdot 10/9 = 3.33$
$< \sim 350 \text{ GeV}$	udscb	$3 \cdot 11/9 = 3.67$
$> \sim 350 \text{ GeV}$	udscbt	$3 \cdot 15/9 = 5.00$

Data lies systematically higher than the prediction from Quark Parton Model (QPM)  $\rightarrow$  gluon bremsstrahlung.



$$\sigma(s) = \sigma_{QED}(s) \left[ 1 + \underbrace{\frac{\alpha_s(s)}{\pi} + 1.411 \cdot \frac{\alpha_s(s)^2}{\pi^2} + \dots}_{\sim 7\%} \right]$$