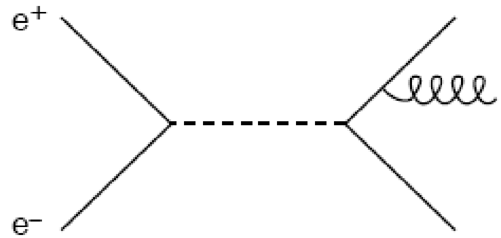
Experimental Tests of QCD

1. Test of QCD in e+e- annihilation
2. Running of the strong coupling constant
3. Study of QCD in deep inelastic scattering

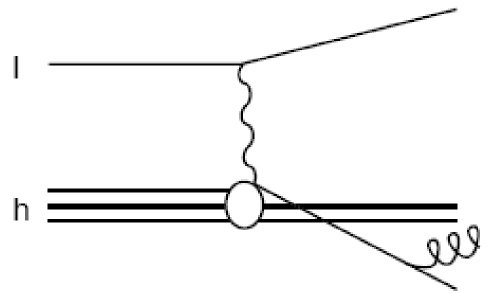
Disclaimer:
Due to the lack of time I have selected only a few items!

Test of QCD in different processes

e^+e^-

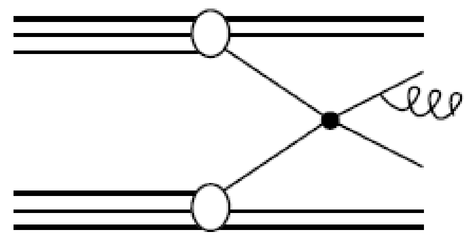


DIS



Discussed in
Section 1 and 3

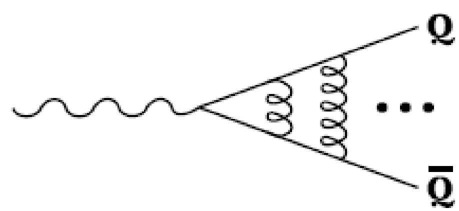
hh



SPS/Tevatron /
LHC

(not discussed)

Heavy quarkonia

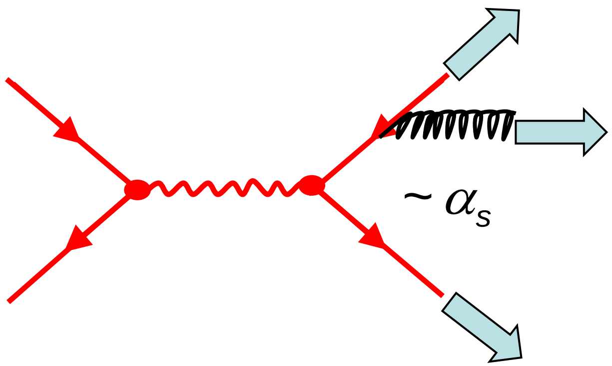


not discussed

1. Test of QCD in e^+e^- annihilation

1.1 Discovery of the gluon

Discovery of 3-jet events by the TASSO collaboration (PETRA) in 1977:



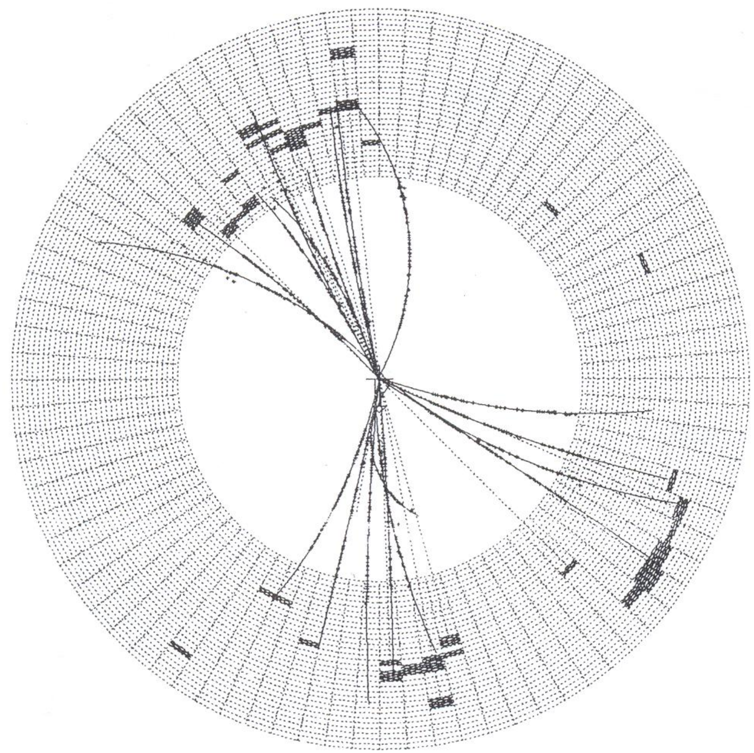


Fig. 11.12 A three-jet event observed by the JADE detector at PETRA.

3-jet events are interpreted as quark pairs with an additional hard gluon.

$$\frac{\#3-\text{jet events}}{\#2-\text{jet events}} \approx 0.15 \sim \alpha_s$$

α_s is large

at $\sqrt{s}$=20 GeV

1.2 Spin of the gluon

Angular distribution of jets depend on gluon spin:

Ordering of 3 jets: $E_1 > E_2 > E_3$

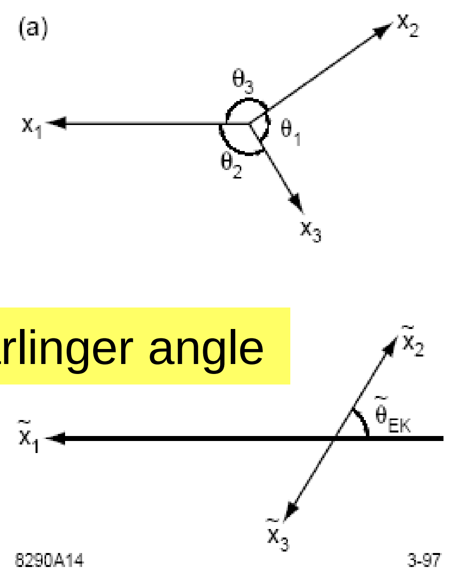


Figure 8: (a) Representation of the momentum vectors in a three-jet event, an (b) definition of the Ellis-Karliner angle.

Measure direction of jet-1 in the rest frame of jet-2 and jet-3: θ_{EK}

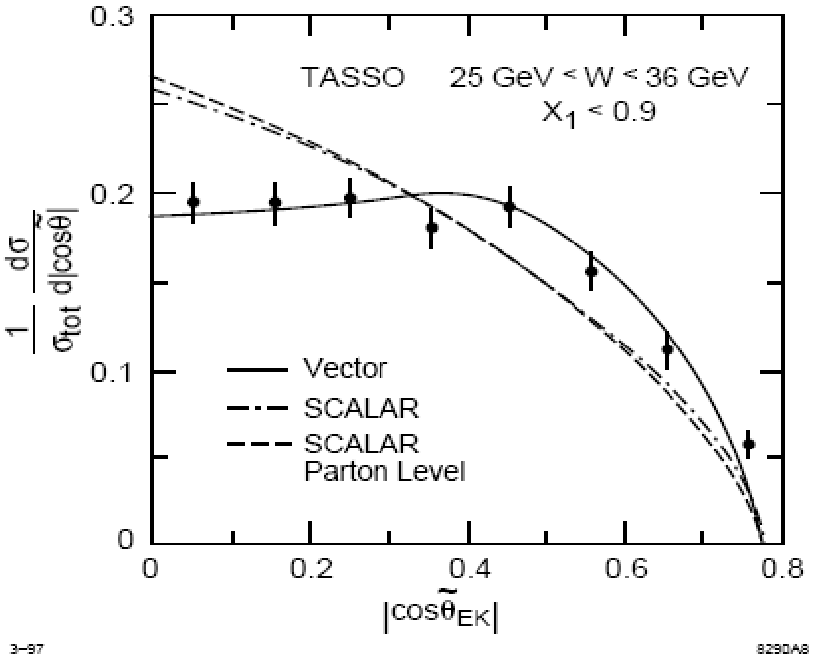


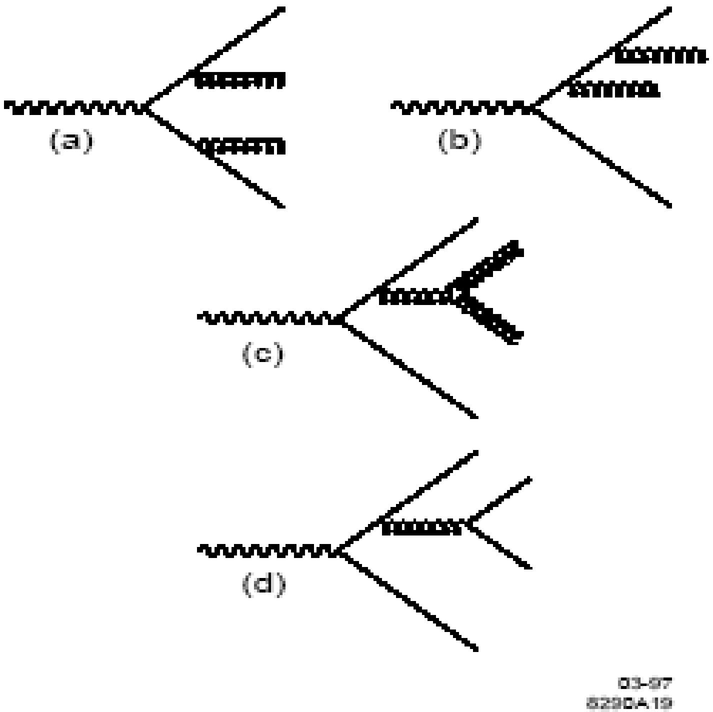
Figure 9: The Ellis-Karliner angle distribution of three-jet events recorded by TASSO at $Q \sim 30$ GeV [18]; the data favour spin-1 (vector) gluons.

Gluon spin J=1

1.3 Multi-jet events and gluon self coupling

$\underbrace{\text{gluon self coupling}}_{\text{Non-abelian gauge theory (SU(3))}}$

4-jet events



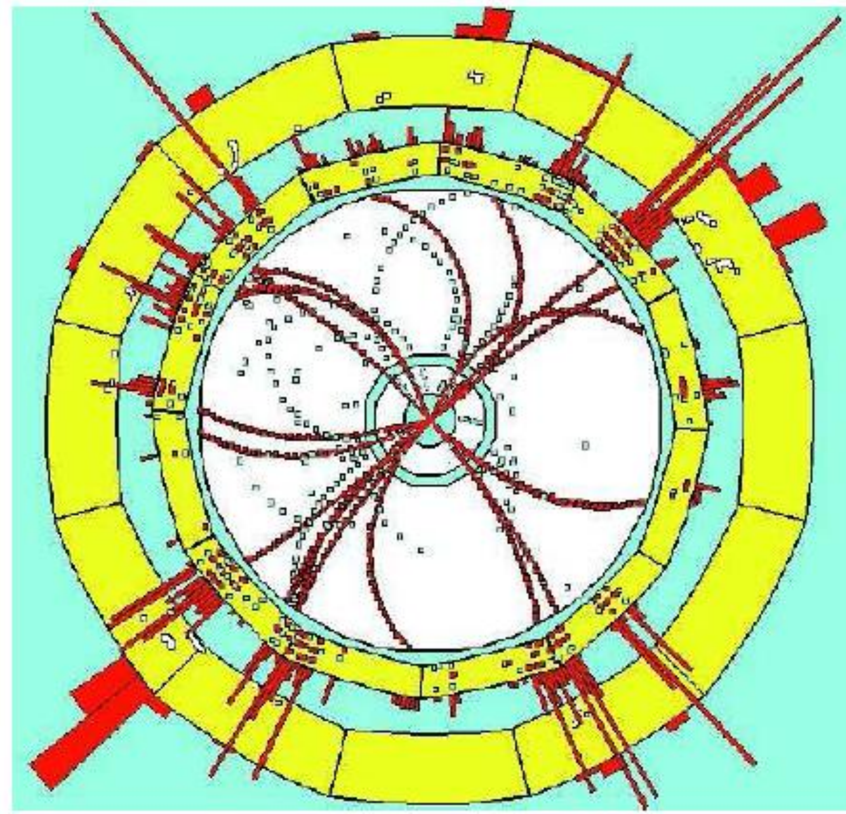


Figure 1: Hadronic event of the type $e^+e^- \to 4$ jets recorded with the ALEPH detector at LEP-I.

Multi-jet event ALEPH exp (LEP)

➡ 4 jet events allow to test the existence of gluon self coupling.

Multiple jets and jet algorithm

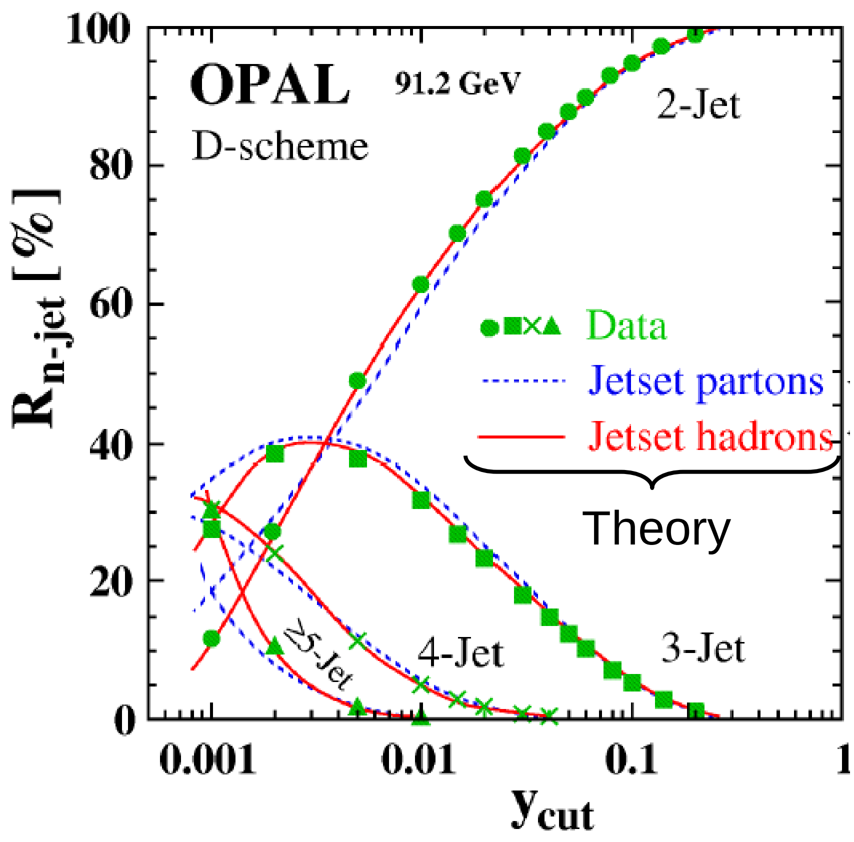
Jet Algorithm

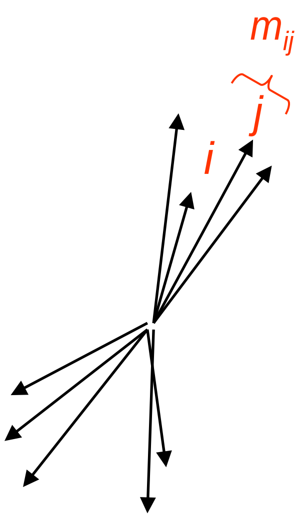
Hadronic particles i and j are grouped to a pseudo particle k as long as the invariant mass is smaller than the **jet resolution parameter**:

$$\frac{m_{ij}^2}{s} < y_{cut}$$

m_{ij} is the invariant mass of i and j.

Remaining pseudo particles are **jets.**





Remark: today, different jet algorithms are used.

Gauge group structure of the strong interaction

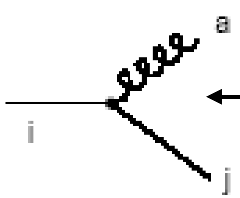
Dynamics of gauge theory defined by commutation relation of gauge group generators:

$$\left[T^a, T^b\right] = i \sum_c f^{abc} \cdot T^c \qquad T^a = \frac{\lambda^a}{2}$$

Structure constants

λ^a Gell-Mann matrices

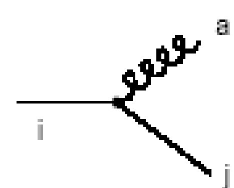
The generators and the structure constants appear in the vertex functions of the Feynman graphs:



vertex factor

$$-i g_s (T^a)_{ij} \gamma^\mu$$

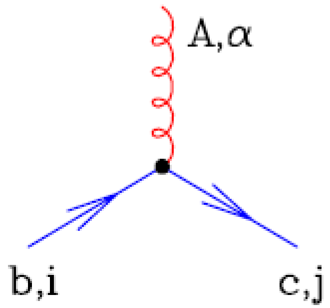
In perturbative calculations the average and sum over all possible color configuartions lead to combinatoric factors:



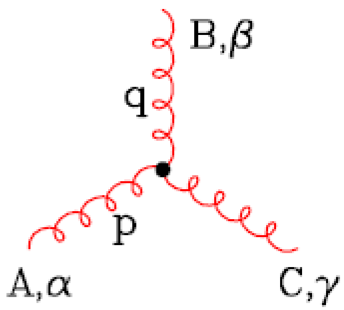
$$\left| \quad \right|^2 \sim \sum_{a=1}^{8} \sum_{k=1}^{3} (T^a)_{\alpha\eta} (T^a)_{\eta\beta}$$

$$\sum_{k,\eta} T^k_{\alpha\eta} T^k_{\eta\beta} = \delta_{\alpha\beta} C_F = \delta_{\alpha\beta} \frac{4}{3}$$

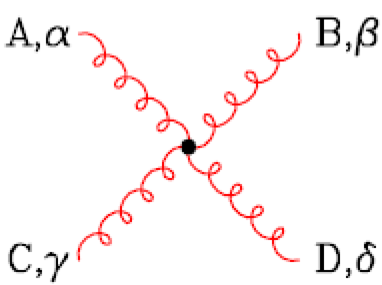
Feynman rules for QCD: vertex factors



$$-ig\ (t^A)_{cb}\ (\gamma^\alpha)_{ji}$$



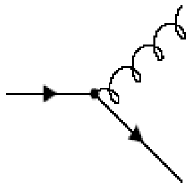
$$-g\ f^{ABC}[(p-q)^\gamma g^{\alpha\beta}+(q-r)^\alpha g^{\beta\gamma}+(r-p)^\beta g^{\gamma\alpha}]$$



$$-ig^2\ f^{XAC}f^{XBD}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$
$$-ig^2\ f^{XAD}f^{XBC}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}]$$
$$-ig^2\ f^{XAB}f^{XCD}\ [g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$

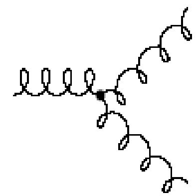
Color factors relevant for 4-jet events

Different relative angular distribution



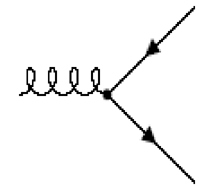
$|\;\;|^2 \sim C_F = \frac{4}{3}$

$q \to qg$



$|\;\;|^2 \sim N_C = 3$

$g \to gg$



$|\;\;|^2 \sim T_F = \frac{1}{2}$

$g \to q\bar{q}$

$$\sum_{k,\eta} T^k_{\alpha\eta} T^k_{\eta\beta} = \delta_{\alpha\beta} C_F = \frac{N_C{}^2 - 1}{2N_C} \cdot \mathbf{1} = \frac{4}{3} \cdot \mathbf{1}$$

Casimir operator of SU(N_C), here: SU(N_C=3).

$$\sum_{a,b} f^{abc} f^{abd} = \delta_{cd} C_A \qquad C_A = N_C$$

Casimir operator of adjoint representation of gluons.

$$\sum_{\alpha,\beta} T^a_{\alpha\beta} T^b_{\beta\alpha} = \delta_{ab} T_F \qquad T_F = \frac{1}{2}$$

$$\boxed{\frac{T_F}{C_F} = \frac{N_C}{N_A} = \frac{\#colors}{\#gluons}}$$

C_F, C_A describe the effective color charge of quark/gluon.

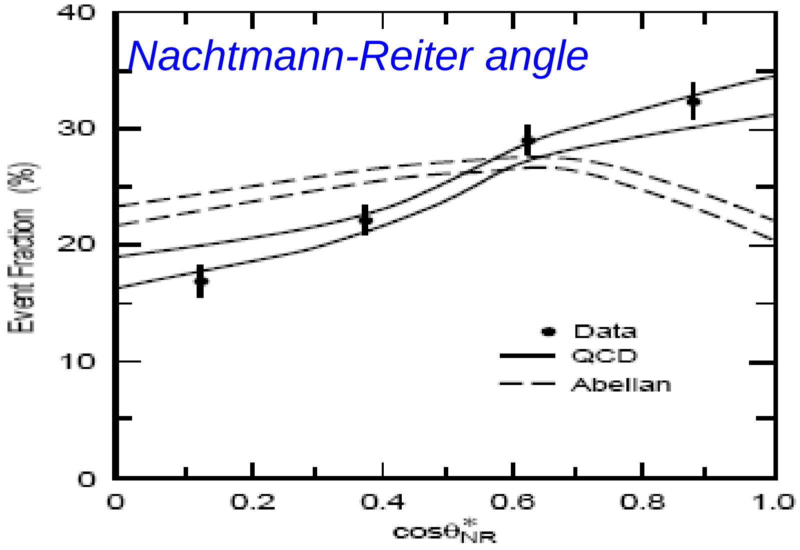
Angular correlation of jets in 4-jet events

4-jet cross section:

$$\frac{1}{\sigma_0}d\sigma^4 = \left(\frac{\alpha_s C_F}{\pi}\right)^2 \left[F_A + \left(1 - \frac{1}{2}\frac{N_C}{C_F}\right)F_B + \frac{N_C}{C_F}F_C\right]$$

$$+ \left(\frac{\alpha_s C_F}{\pi}\right)^2 \left[\frac{T_F}{C_F}N_f F_D + \left(1 - \frac{1}{2}\frac{N_C}{C_F}\right)F_E\right]$$

$F_{A,B,C,D,E}$ are kinematic functions



Exploiting the angular distribution of 4-jets:

- Bengston-Zerwas angle

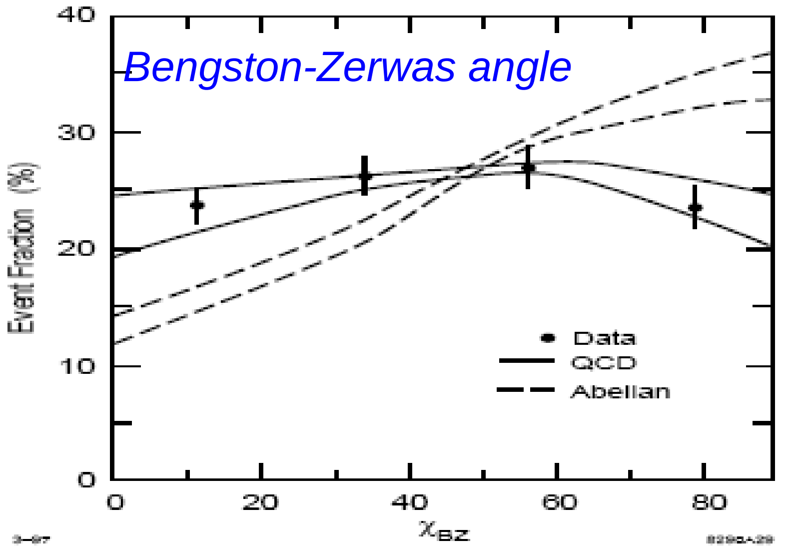
$$\cos\chi_{BZ} \propto (\vec{p}_1 \times \vec{p}_2)\cdot(\vec{p}_3 \times \vec{p}_4)$$

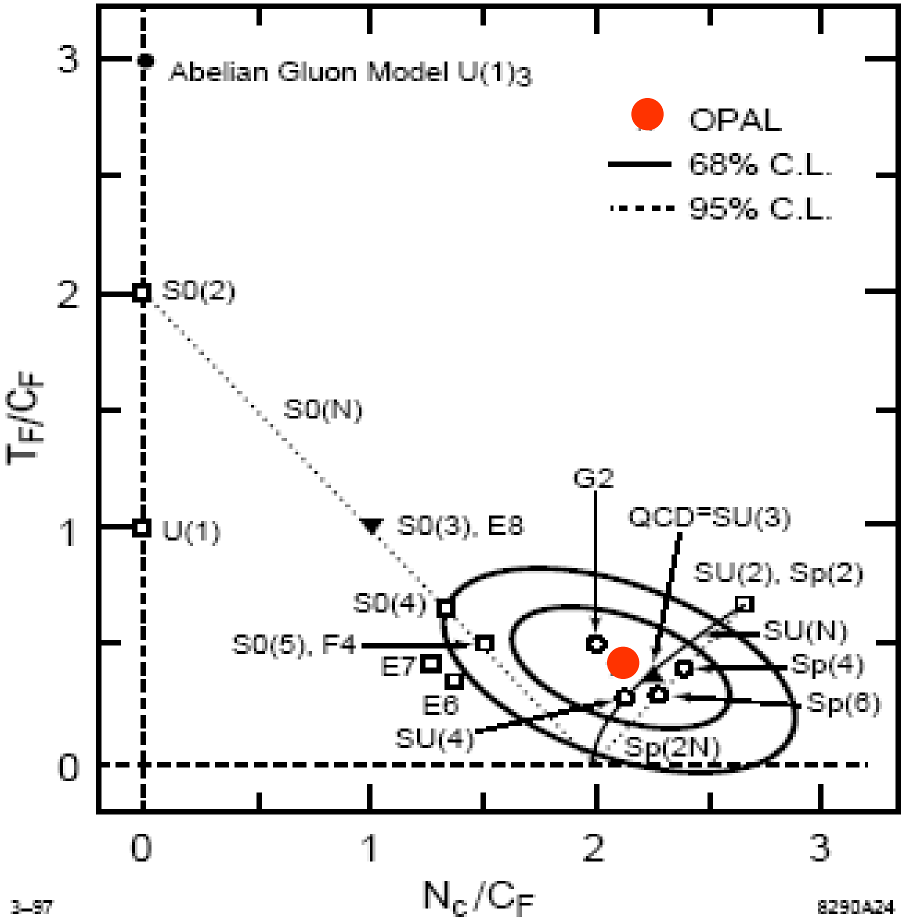
- Nachtmann-Reiter angle

$$\cos\theta_{NR} \propto (\vec{p}_1 - \vec{p}_2)\cdot(\vec{p}_3 - \vec{p}_4)$$

Allows to measure the ratios T_F/C_F and N_C/C_F

SU(3) predicts: $T_F/C_F = 0.375$ and $N_C/C_F = 2.25$



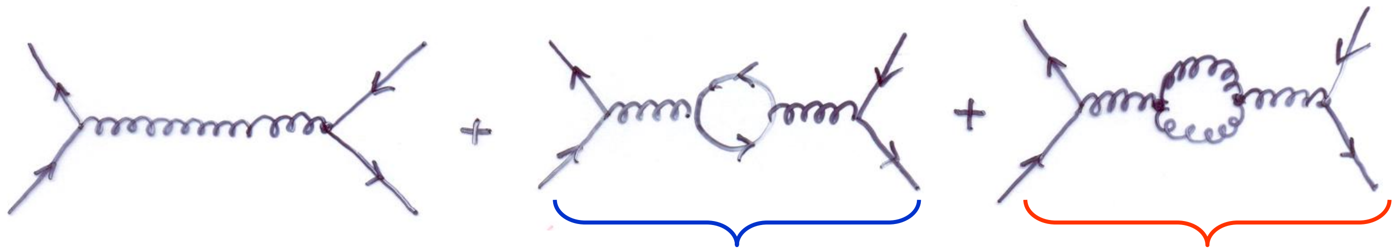


Confirms QCD prediction (SU(3)) and gluon self-coupling:

$T_F/C_F = 0.375$ and $N_C/C_F = 2.25$

2. “Running” of the strong coupling α_s

Propagator corrections:



Strong coupling $\alpha_s(Q^2)$

$$\boxed{\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1+\alpha_s(\mu^2)\underbrace{\frac{1}{12\pi}(33-2n_f)}_{\beta_0 = \frac{1}{12\pi}(33-2n_f)}\log\frac{Q^2}{\mu^2}}}$$

n_f = active quark flavors
μ^2 = renormalization scale
conventionally $\mu^2 = M_Z^2$

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda_{QCD}^2)}$$

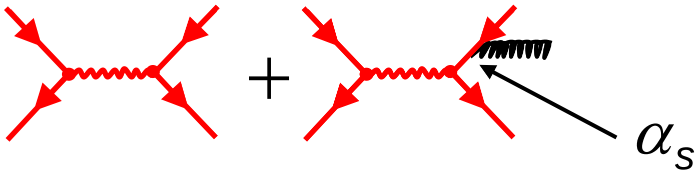
with $\Lambda_{QCD} \approx 200\text{MeV}$
scale at which perturbation theory diverges

Measurement of Q^2 dependence of α_s

➡ α_s measurements are done at given scale Q^2: $\alpha_s(Q^2)$

a) α_s from total hadronic cross section

$$e^+e^- \rightarrow q\bar{q}(g)$$



Final state gluon radiation.

$$\sigma_{had}(s) = \sigma_{had}^{QED}(s)\left[1 + \frac{\alpha_s(s)}{\pi} + 1.411 \cdot \frac{\alpha_s(s)^2}{\pi^2} + \ldots\right]$$

$$R_{had} = \frac{\sigma(ee \rightarrow hadrons)}{\sigma(ee \rightarrow \mu\mu)} = 3\sum Q_q^2 \left\{1 + \frac{\alpha_s}{\pi} + 1.411\frac{\alpha_s^2}{\pi^2} + \ldots\right\}$$

$\boxed{\Rightarrow \alpha_s(s)}$

b) α_s from hadronic event shape variables

3-jet rate: $R_3 \equiv \frac{\sigma_{3-jet}}{\sigma_{had}}$ depends on α_s

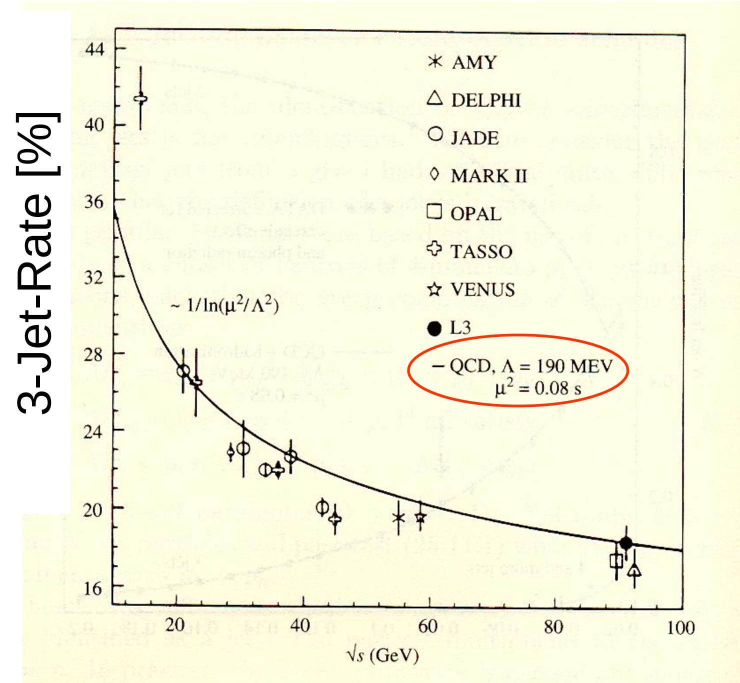
3-jet rate is measured as function of a jet resolution parameter y_{cut}

QCD calculation provides a theoretical prediction for $R_3^{theo}(\alpha_s, y_{cut})$

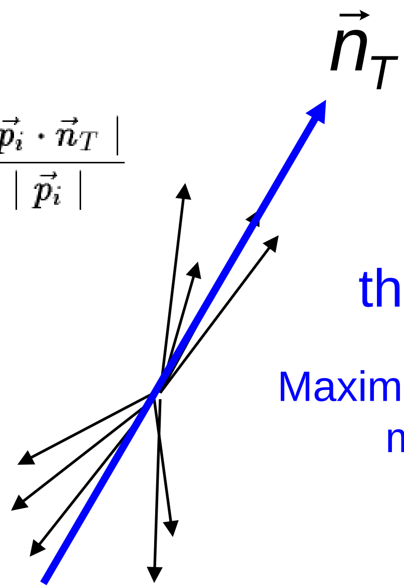
$\rightarrow$ fit $R_3^{theo}(\alpha_s, y_{cut})$ to the data to determine α_s

Similarly other event shape variables (sphericity, thrust,…) can be used to obtain a prediction for α_s

$\Rightarrow \alpha_s(s)$

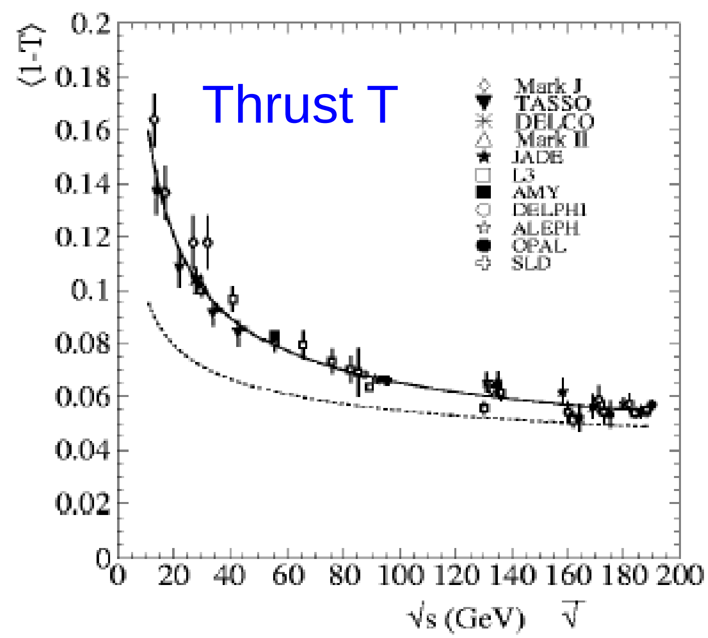


$$T = \max \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}$$



thrust axis

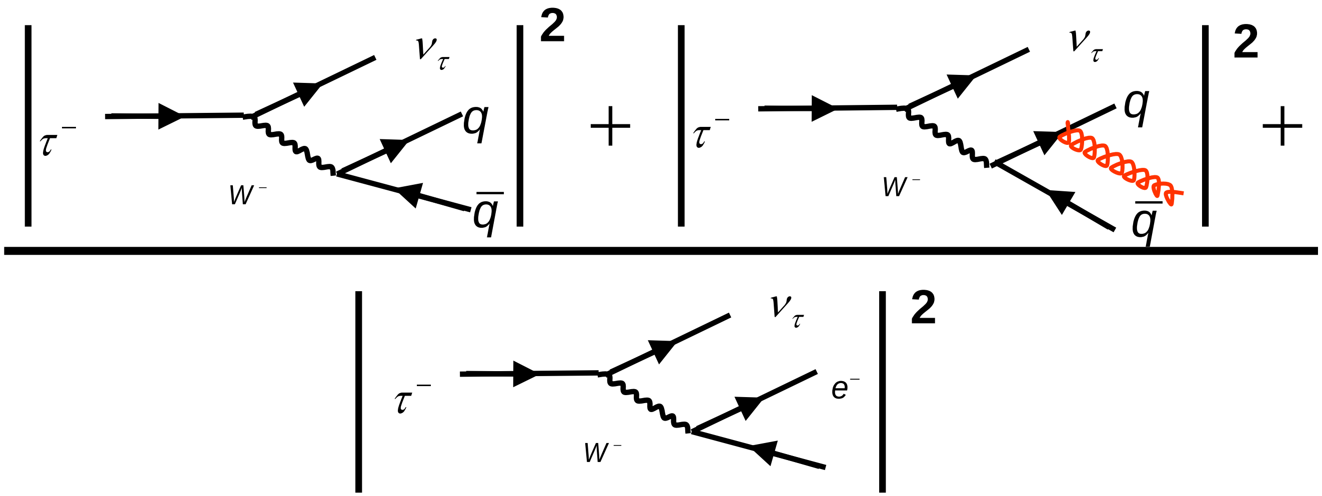
Maximizes longitudinal momentum



c) α_s from hadronic τ decays

$$R^{\tau}_{had} = \frac{\Gamma(\tau \to \nu_\tau + Hadrons)}{\Gamma(\tau \to \nu_\tau + e\overline{\upsilon}_e)} \sim f(\alpha_s)$$

$R^{\tau}_{had} =$

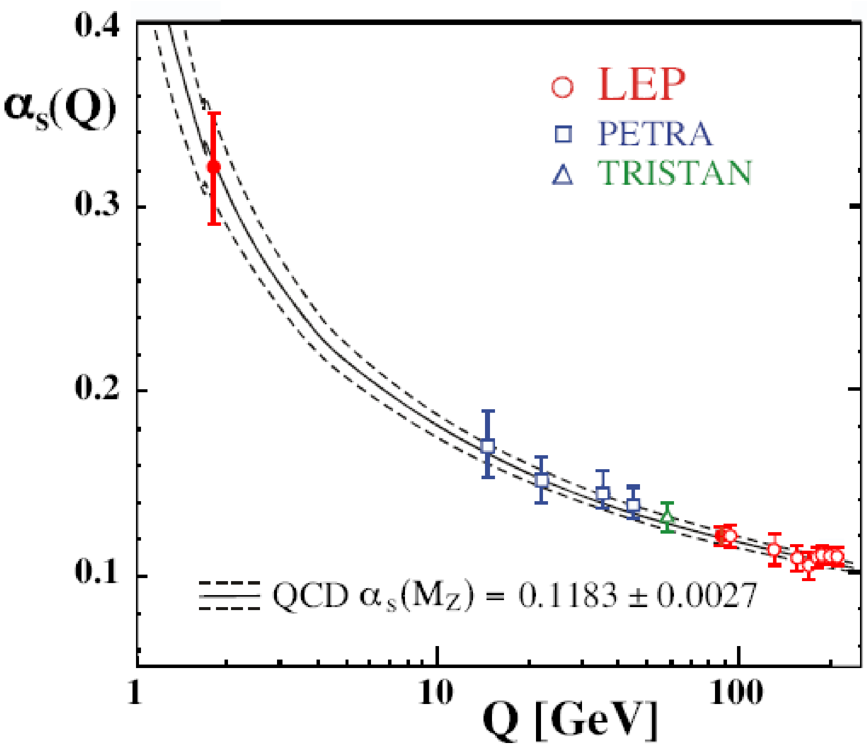


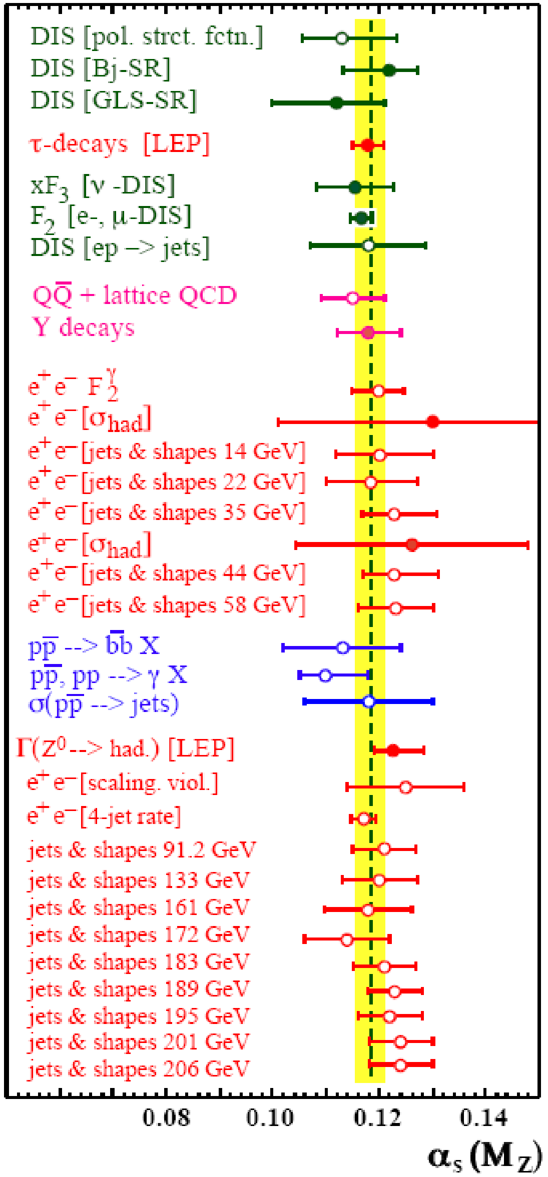
$$R^{\tau}_{had} = R^{\tau,0}_{had}\left(1 + \frac{\alpha_s(m_\tau^2)}{\pi} + \ldots\right)$$

d) α_s from DIS (deep inelastic scattering)

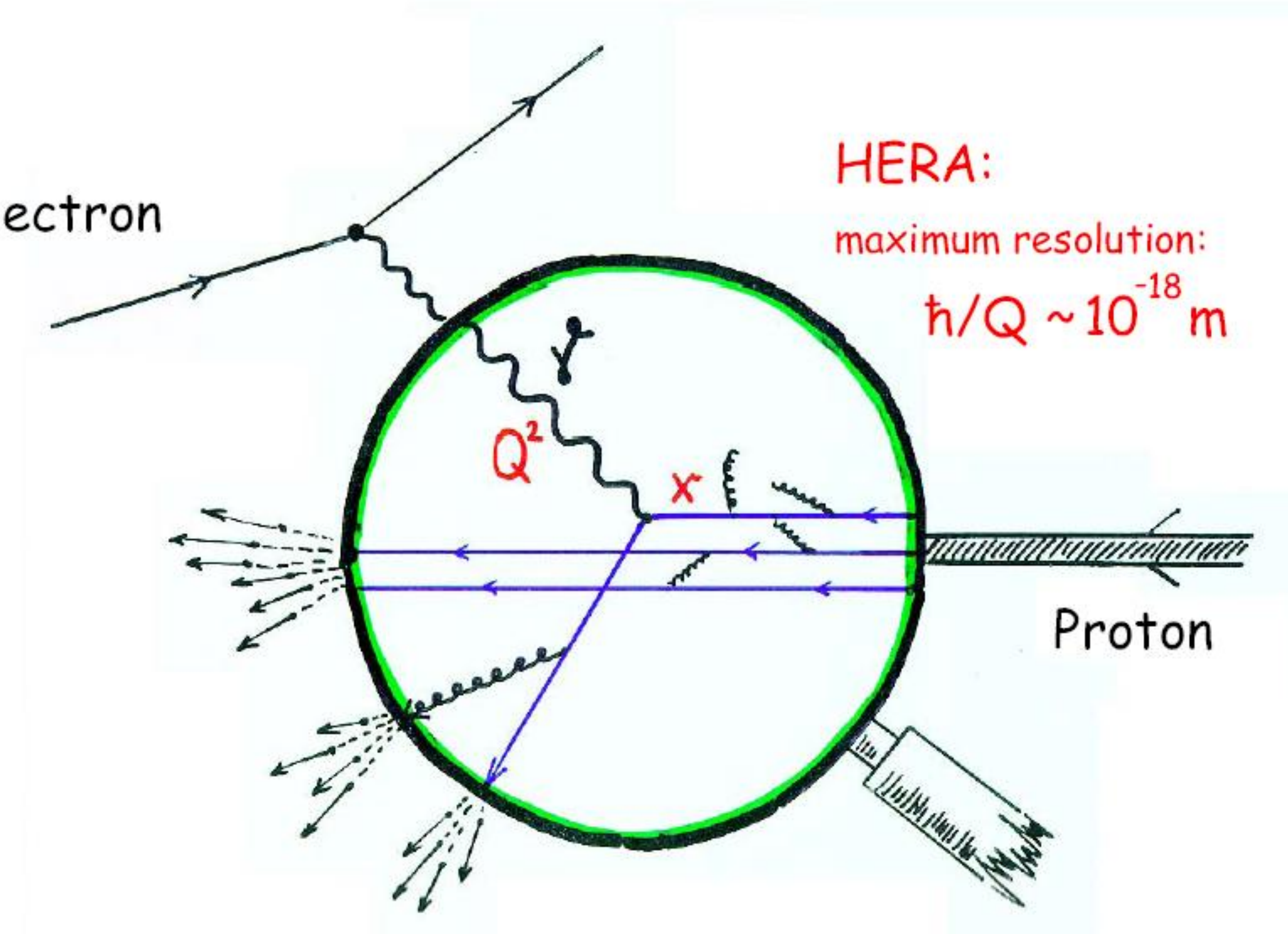
Running of α_s and asymptotic freedom

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda^2_{QCD})}$$



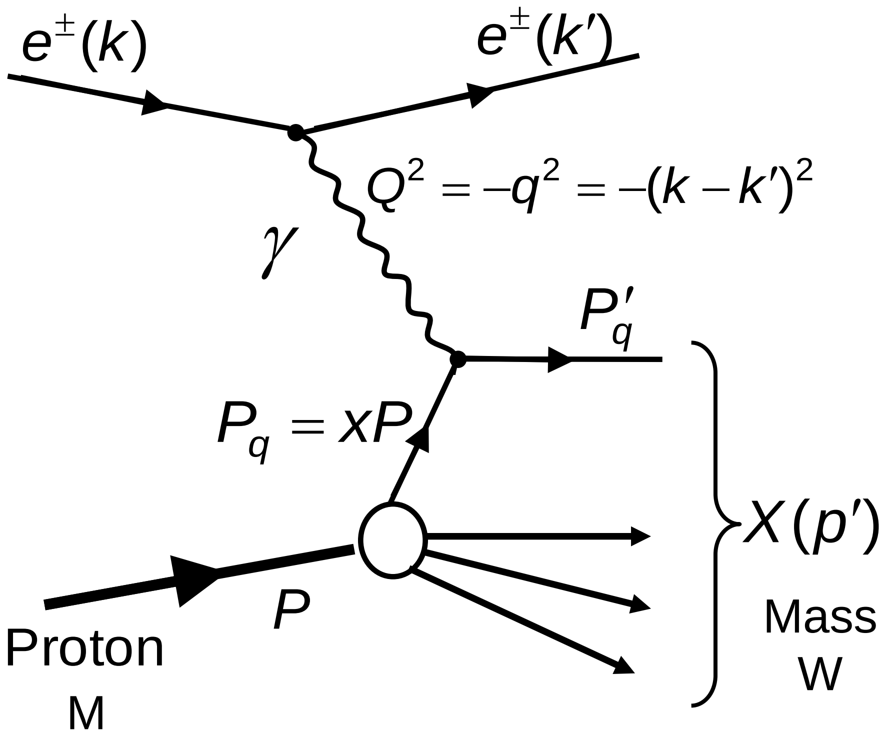


3. Study of QCD in deep inelastic scattering (DIS)



Courtesy: H.C. Schultz-Coulon

3.1 DIS in the quark parton model (QPM)



- Elastic scattering: W = M

  $\Rightarrow$ only one free variable

$$\frac{Q^2}{2M\nu} = 1$$

- Inelastic scattering: W $\neq$ M

  $\Rightarrow$scattering described by 2 independent variables

$$(E,\nu),\ (Q^2,x),\ (x,y),\ \ldots$$

x = fractional momentum of struck quark

y = Pq/Pk = elasticity, fractional energy transfer in proton rest frame

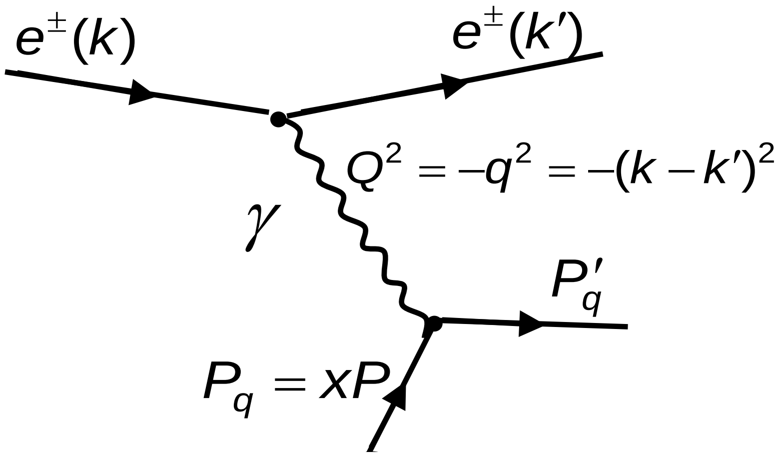
ν = E - E′ = energy transfer in lab

$$Q^2 = sxy$$ s = CMS energy

$$x = \frac{Q^2}{2M\nu}$$ (Bjorken x)

Cross section in quark parton model (QPM)

Elastic scattering on single quark



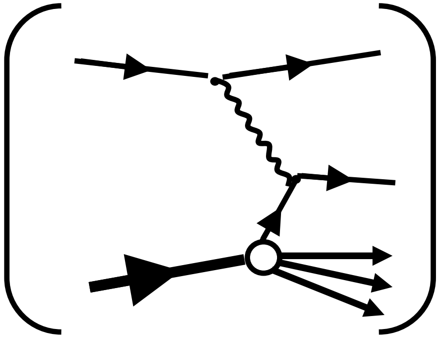
Starting point:
electron muon scattering

$$\boxed{\left\{\ldots\ldots\right\}^{elastic}_{e\mu\to e\mu} = \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}\right)}$$

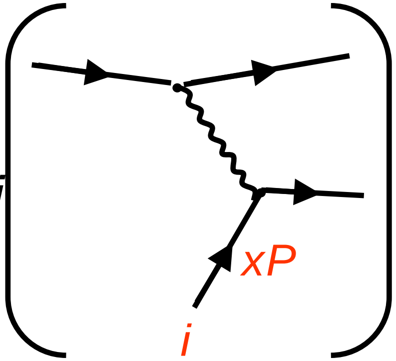
Electron-quark scattering (quark momentum fraction x):

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot \underset{\uparrow \text{charge}}{e_i^2}\left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

$$\sigma\left(\right.$$



$$\left.\right) = \sum_i q_i(x)\,\sigma_i\left(\right.$$



$$\left.\right)$$

Parton density $q_i(x)dx$: Probability to find parton i in momentum interval [x, x+dx]

$$\boxed{\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot\underbrace{\sum_i\int_0^1 e_i^2\cdot q_i(\xi)\cdot\delta(x-\xi)d\xi\left(\cos^2\frac{\theta}{2}+\frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)}}$$

Structure functions

$$F_2(x) = x\sum_i\int_0^1 e_i^2 q_i(\xi)\cdot\delta(x-\xi)d\xi = x\sum_i e_i^2 q_i(x)$$

$$F_1(x) = \frac{1}{2}\sum_i e_i^2 q_i(x)$$

ignore (include) anti-quarks!

$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot\left(\frac{F_2(x)}{x}\cos^2\frac{\theta}{2} + 2F_1(x)\frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

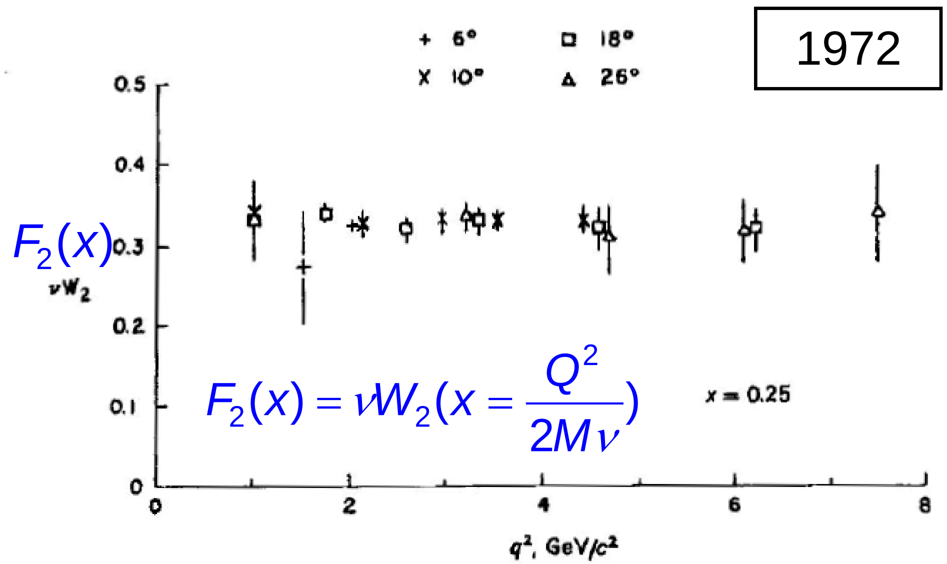
Kinematical relations

$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{Q^4x}\right)\cdot\left((1-y)F_2(x) + xy^2F_1(x)\right)$$

Deep inelastic electron-proton scattering:

- Free partons: $F_2=F_2(x)$ $\Leftrightarrow$ "scaling"
- Spin ½ partons: $2xF_1(x) = F_2(x)$
  (Callan-Gross relation)

Scaling



Structure function F_2 (=νW_2) depends only on the dimensionless variable x:

$$x = \frac{Q^2}{2M\nu}$$

$\rightarrow$ **Scale invariance: "scaling"**

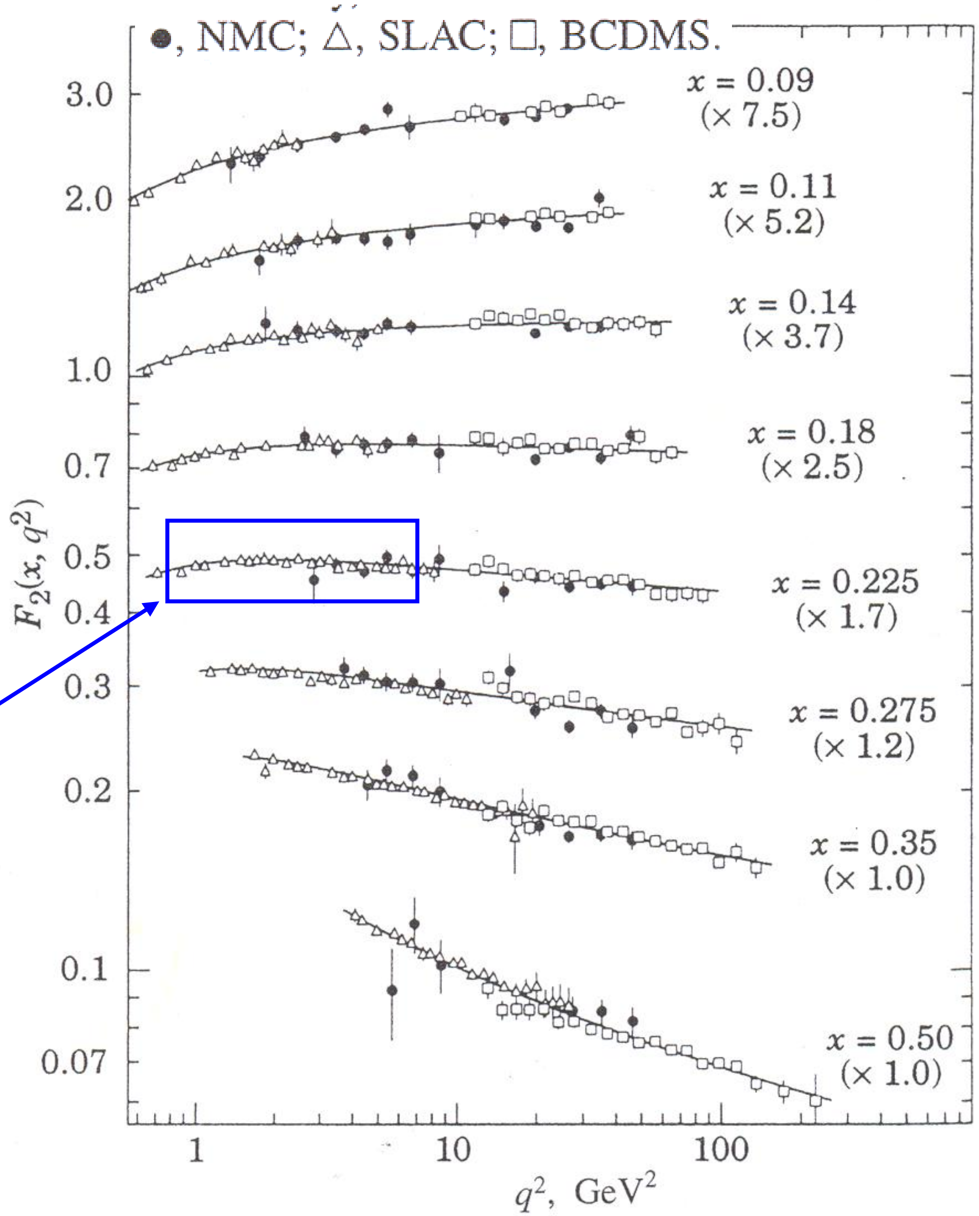
Indicates elastic scattering at point-like free constituents of the proton: partons

3.2 Scaling violation

Structure function:

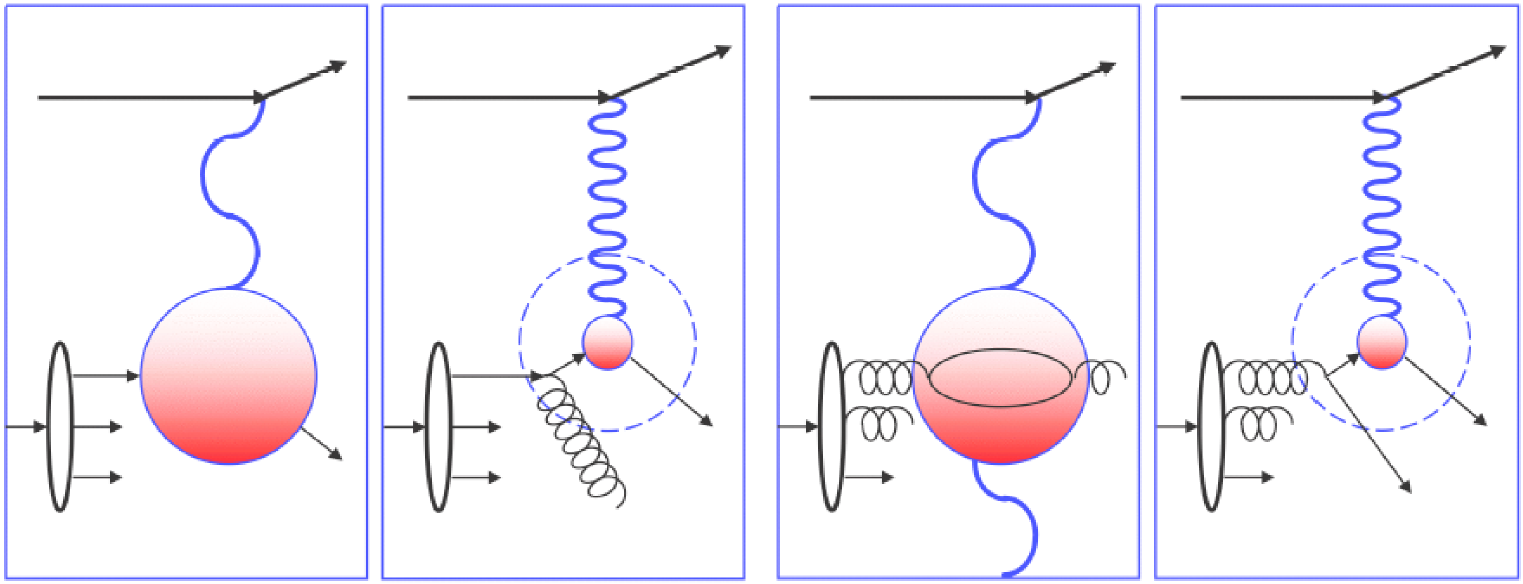
$$F_2 = F_2(x,Q^2) = x\sum_i e_i^2 q_i(x)$$

Region of 1st SLAC measurement (1972)



QCD explains observed scaling violation

<u>Large x:</u> valence quark scattering   <u>Small x:</u> Gluon+sea quark scattering

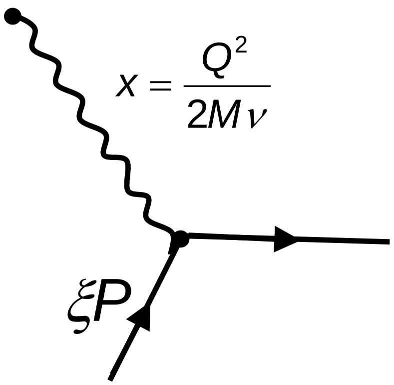


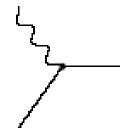
$Q^2 \uparrow \Rightarrow F_2 \downarrow$ for fixed x $Q^2 \uparrow \Rightarrow F_2 \uparrow$ for fixed (small) x

Scaling violation is one of the clearest manifestation of radiative effect predicted by QCD.

Quantitative description of scaling violation

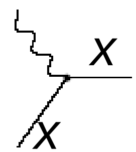
Quark Parton Model

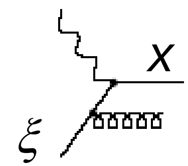




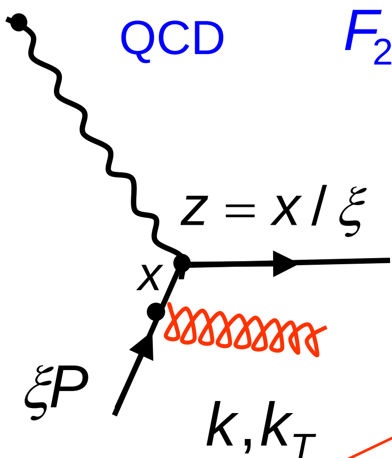
$$F_2(x) = x\sum_i e_i^2 \int_0^1 q_i(\xi)\cdot\delta(x-\xi)d\xi = x\sum_i e_i^2 q_i(x)$$

$$\boxed{\delta(ax) = \frac{1}{|a|}\delta(x)}$$





QCD



$$F_2(x,Q^2) = x\sum_i e_i^2 \int_0^1 \frac{d\xi}{\xi} q_i(\xi)\cdot\left[\delta(1-\frac{x}{\xi}) + \frac{\alpha_s}{2\pi}\mathrm{P}_{qq}(\frac{x}{\xi})\log\frac{Q^2}{\mu_0^2}\right]$$

$$\boxed{\begin{aligned} &\sim \frac{\alpha_s}{2\pi}\mathrm{P}_{qq}(z)\int_{\mu_0^2}^{Q^2}\frac{dk_T^2}{k_T^2} \\ &\sim \frac{\alpha_s}{2\pi}\mathrm{P}_{qq}(z)\log(\frac{Q^2}{\mu_0^2}) \end{aligned}}$$

In the limit of small irradiation angle

P_{qq} probability of a quark to emit gluon and becoming a quark with momentum reduced by fraction z.

μ^0 cutoff parameter

Changing to the quark (parton) densities:

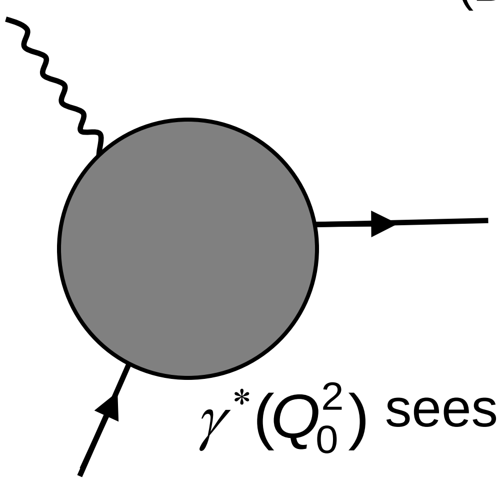
$$q_i(x,Q^2) = q_i(x) + \underbrace{\frac{\alpha_s}{2\pi}\log\frac{Q^2}{\mu_0^2}\int_0^1 \frac{d\xi}{\xi} q_i(\xi) P_{qq}\left(\frac{x}{\xi}\right)}_{\Delta q(x,Q^2)}$$

Integro-differential equation for q(x,Q^2):

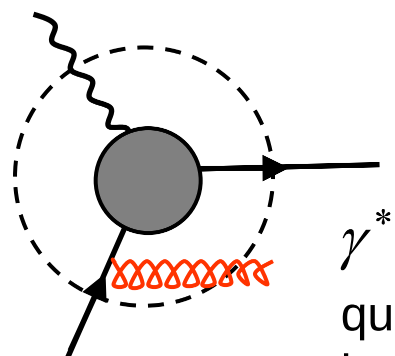
$$\frac{d}{d\log Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_0^1 \frac{d\xi}{\xi} q(\xi,Q^2) P_{qq}\left(\frac{x}{\xi}\right)$$

We have ignored gluon splitting

DGLAP evolution equation
(**D**okshitzer, **G**ribov, **L**ipatov, **A**ltarelli, **P**arisi, 1972 – 1977)



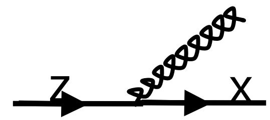
$\gamma^*(Q_0^2)$ sees q(x)

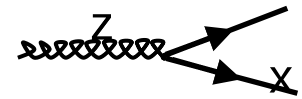


$\gamma^*(Q^2 > Q_0^2)$ sees that quarks q(x) are surrounded by softer quarks

Evolution of parton densities (quarks and gluons)

evolution of quark
density with $\ln Q^2$

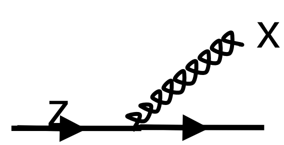


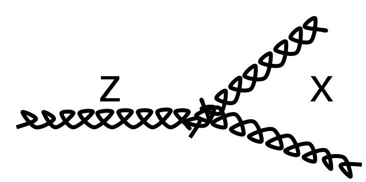


$$\frac{\partial q(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[q(z,Q^2)P_{qq}\left(\frac{x}{z}\right) + g(z,Q^2)P_{qg}\left(\frac{x}{z}\right)\right]$$

$$\frac{\partial g(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[q(z,Q^2)P_{gq}\left(\frac{x}{z}\right) + g(z,Q^2)P_{gg}\left(\frac{x}{z}\right)\right]$$

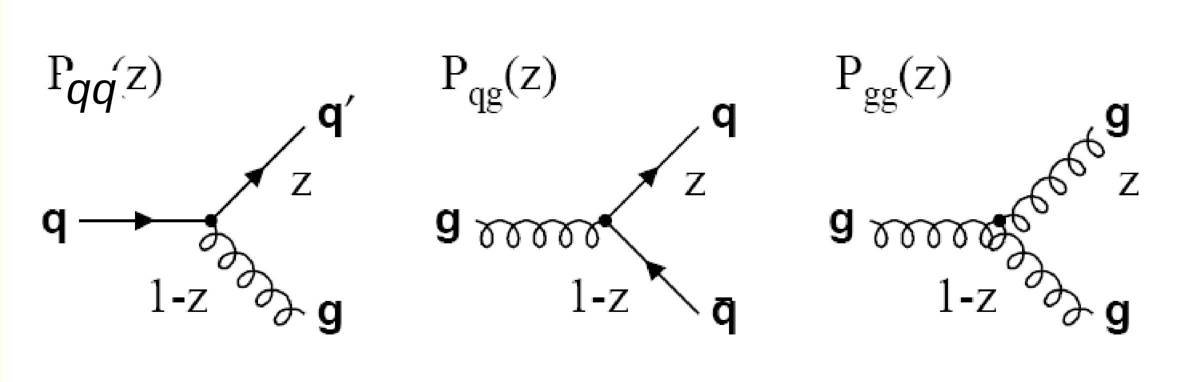
evolution of gluon
density with $\ln Q^2$





Splitting functions: Probability that a parton (quark or gluon) emits a parton **(q, g)** with momentum fraction $\varepsilon = x/z$ of the parent parton.

Splitting functions are calculated as power series in α_s up to a given order:



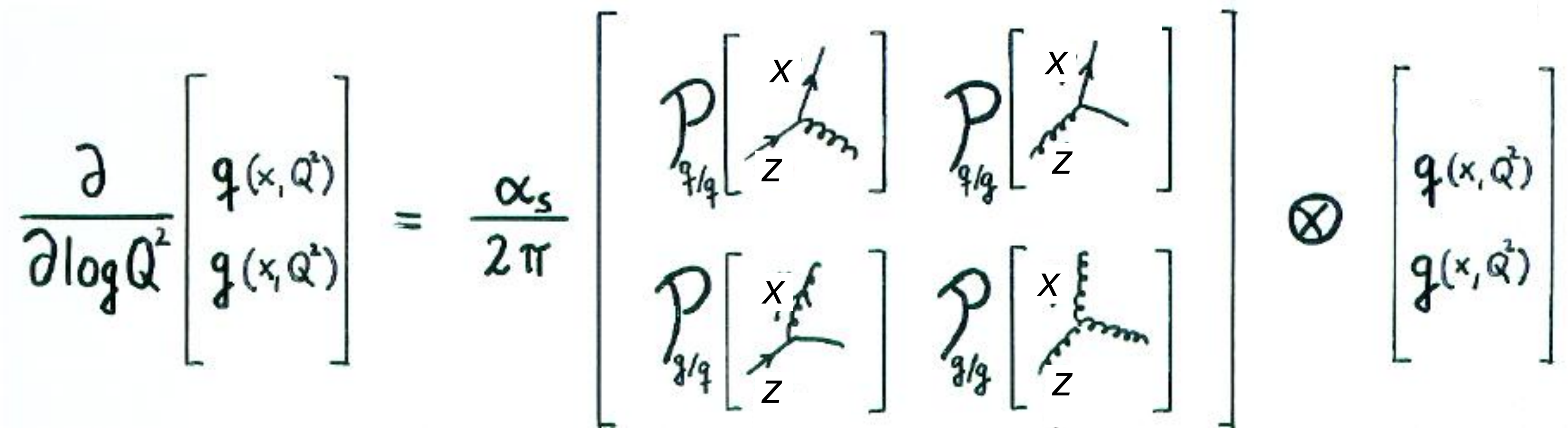
$$P_{ij}(z,\alpha_s) = P_{ij}^0(z) + \frac{\alpha_s}{2\pi} P_{ij}^1(z) + \ldots$$

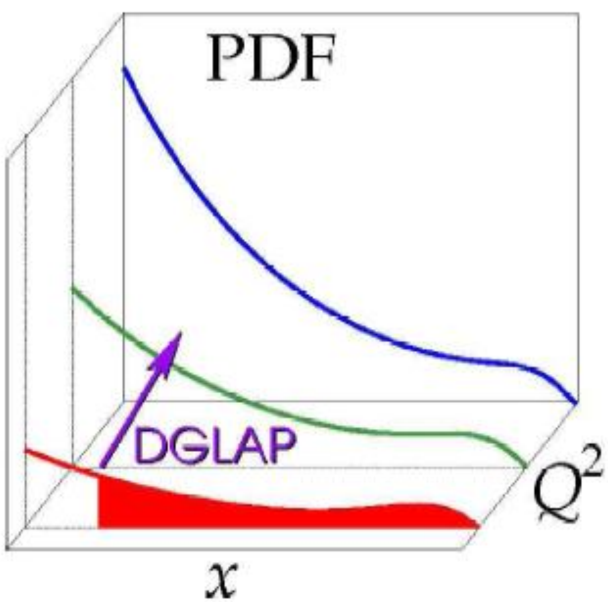
In leading order: $P_{ij}(z,\alpha_s) \equiv P_{ij}^0(z)$

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z} \qquad P_{gq}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}$$

$$P_{qg}(z) = \frac{z^2+(1-z)^2}{2} \qquad P_{gg}(z) = 6\left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right)$$

DGLAP Evolution ("symbolic"):





$$P \otimes f(x,Q^2) = \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}\right) f(z,Q^2)$$

QCD evolution:

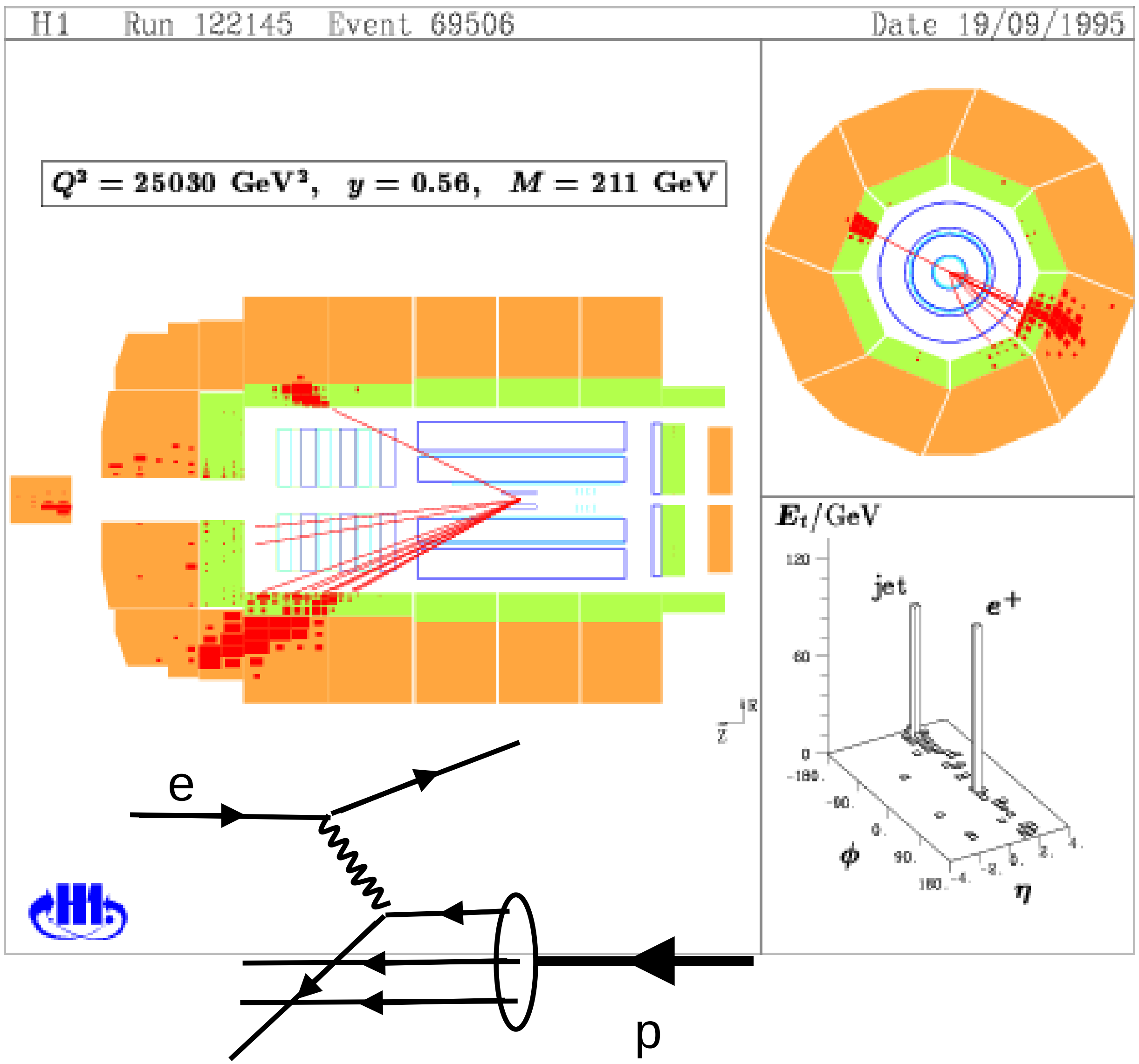
QCD predicts the PDF behavior for a scale Q^2 once the PDF was measured at another scale.

HERA

$e^{\pm}$ → 30 GeV

← p 900 GeV

$$s = 4E_eE_p \approx 10^5\,\mathrm{GeV}^2$$


H1 Run 122145 Event 69506 Date 19/09/1995
$Q^2 = 25030\ \mathrm{GeV}^2,\quad y = 0.56,\quad M = 211\ \mathrm{GeV}$
E_t/GeV
jet
e^+
ϕ
η
e
p

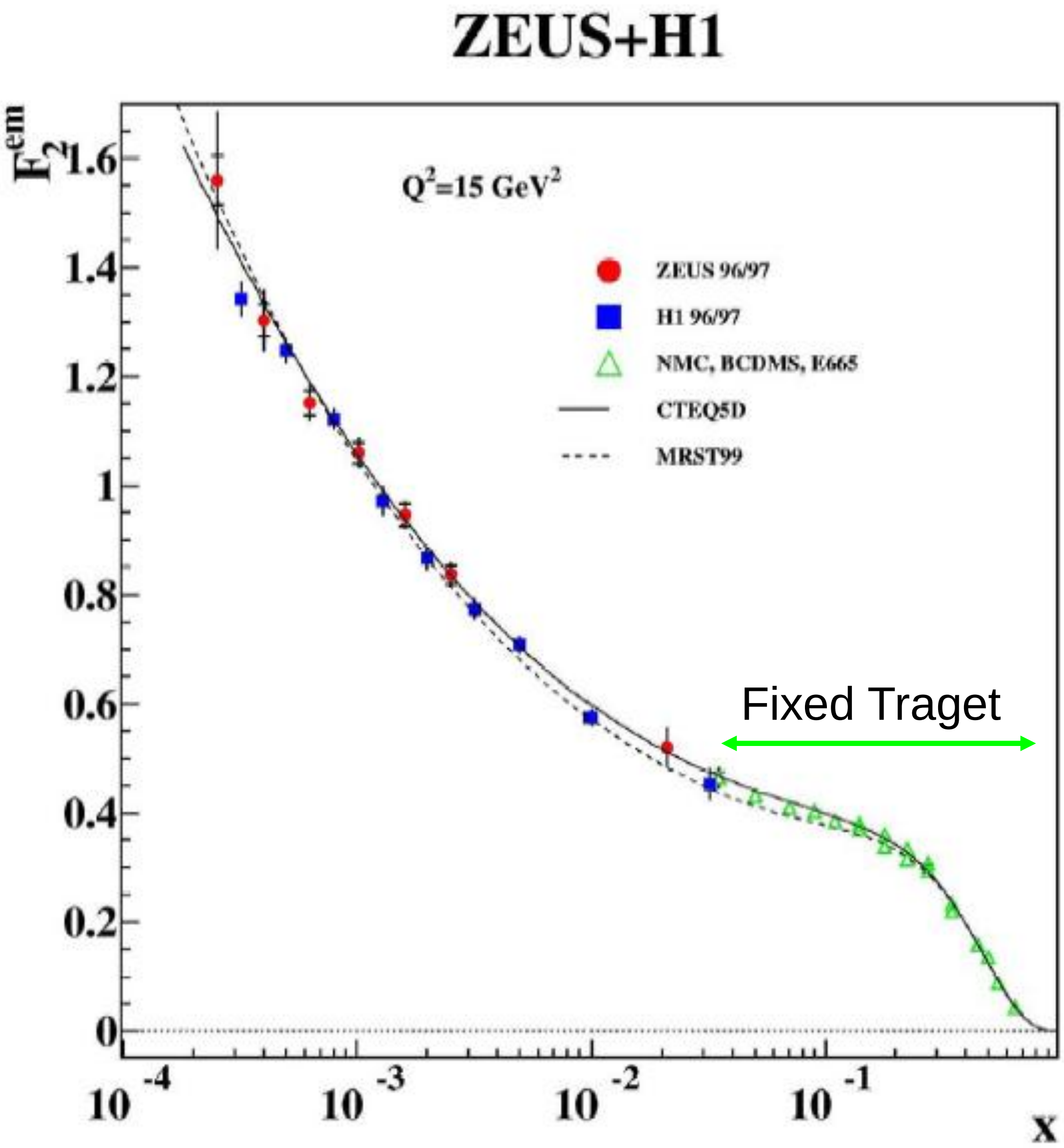
Measurement of the parton densities / F_2

$$\frac{d^2\sigma}{dx\,dQ^2} = \left(\frac{2\pi\alpha^2}{xQ^4}\right)\cdot\left(2\cdot(1-y)F_2(x,Q^2) + y^2F_2(x,Q^2)\right)$$

e.g. for y=1 $\qquad Q^2 = sxy$

$$\frac{d^2\sigma}{dx\,dQ^2} = \left(\frac{2\pi\alpha^2}{xQ^4}\right)\cdot F_2(x,Q^2)$$

$$F_2(x,Q^2) = x\sum_q e_q^2\left[q(x,Q^2) + \bar{q}(x,Q^2)\right]$$



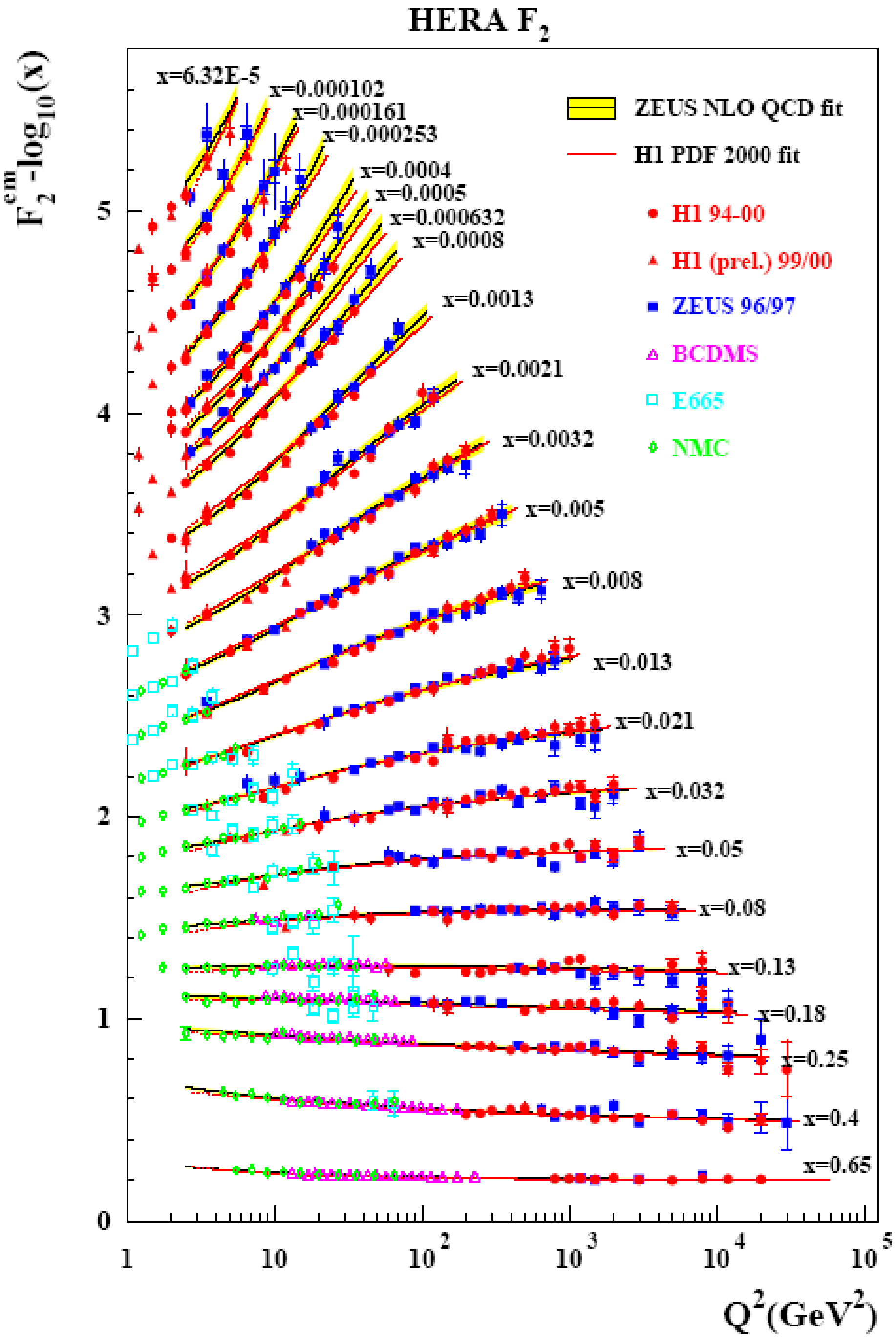
$F_2(x)$

Large increase of $F_2(x)$ for very small x - unexpected

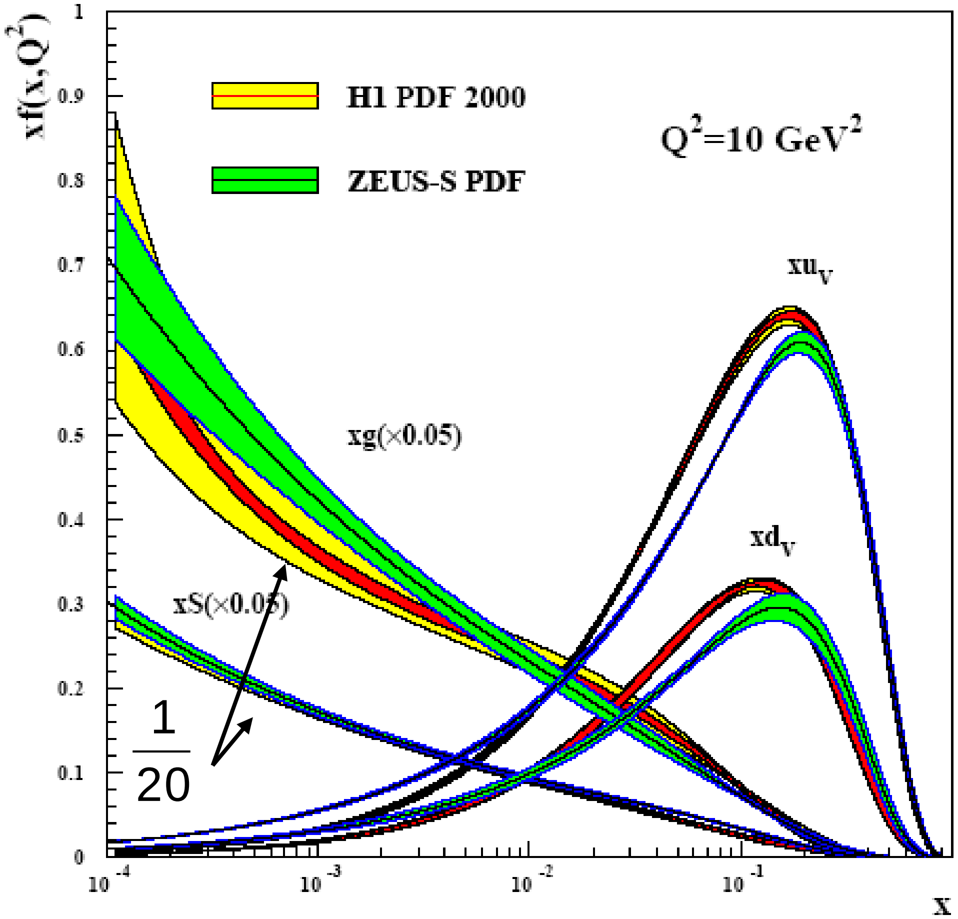
When does the rise stop ??

$F_2(x,Q^2)$

Q² dependence is correctly described by QCD evolution



Structure of the proton as seen by HERA



Valenzquarks = $\int u_v(x) + d_v(x)\,dx = 3$

Gluonen = $\int g(x)\,dx > 30$