

# Flavor Mixing and CP Violation

1. CKM Matrix
2. Mixing of neutral mesons
3. CP violation

# 1. CKM Matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

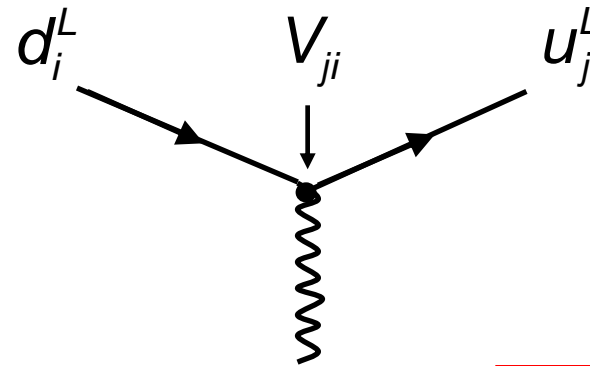
Unitarity

$$V_{CKM} V_{CKM}^+ = 1$$

weak  
eigenstates

CKM matrix

mass  
eigenstates



Charged currents:

$$J_{\mu}^{+} \propto (\bar{u}, \bar{c}, \bar{t}) \gamma_{\mu} (1 - \gamma_5) \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

weak

$$= (\bar{u}, \bar{c}, \bar{t}) \gamma_{\mu} (1 - \gamma_5) \mathbf{V}_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

mass/  
flavor

# 1.1 Parameters of CKM matrix

Number of independent parameters:



18 parameter (9 complex elements)

-5 relative quark phases (unobservable)

-9 unitarity conditions

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=4 independent parameters: 3 angles + 1 phase

## PDG parametrization

3 Euler angles

$$\theta_{23}, \theta_{13}, \theta_{12}$$

1 Phase

$$\delta$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where  $c_{ij} = \cos\theta_{ij}$ ,  $s_{ij} = \sin\theta_{ij}$

Magnitude  
of elements

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} \text{u} & \text{d} & \text{s} & \text{b} \\ \text{c} & \text{r} & \text{r} & \text{r} \\ \text{t} & \text{r} & \text{r} & \text{r} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

komplex  
in  $O(\lambda^3)$

Wolfenstein parametrization: reflects hierarchical structure of CKM matrix

$\lambda, A, \rho, \eta$  with  $\lambda = 0.22$

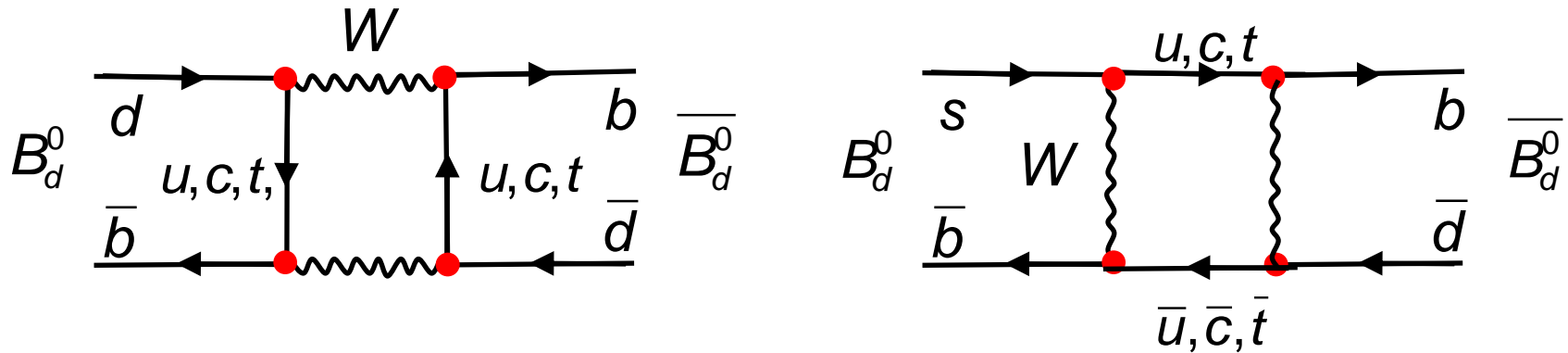
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3 e^{-i\gamma} \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3 e^{-i\beta} & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

## 2. Mixing of neutral mesons

The quark mixing results into several interesting “loop” effects:

➔ Standard Model predicts at loop-level **Flavor Changing Neutral Currents**

Mixing of neutral mesons, e.g.:  $B_d^0 \Leftrightarrow \bar{B}_d^0$



Neutral mesons:  $|P^0\rangle$ :  $K^0 = |d\bar{s}\rangle$   $D^0 = |\bar{u}c\rangle$   $B_d^0 = |d\bar{b}\rangle$   $B_s^0 = |s\bar{b}\rangle$   
 $|\bar{P}^0\rangle$ :  $\bar{K}^0 = |\bar{d}s\rangle$   $\bar{D}^0 = |\bar{u}c\rangle$   $\bar{B}_d^0 = |d\bar{b}\rangle$   $\bar{B}_s^0 = |s\bar{b}\rangle$

discovery of mixing

1960

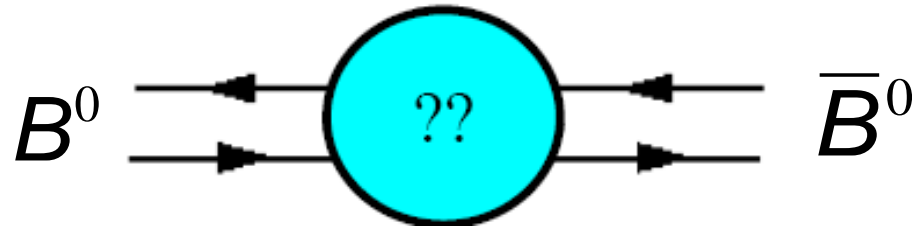
2007

1987

2006

# 2.1 Mixing Phenomenology

Applies to all neutral mesons!



$$i \frac{d}{dt} \begin{pmatrix} B^0(t) \\ \bar{B}^0(t) \end{pmatrix} = \underbrace{\left( \mathbf{M} - \frac{i}{2} \mathbf{\Gamma} \right)}_{\mathbf{H}} \begin{pmatrix} B^0(t) \\ \bar{B}^0(t) \end{pmatrix}$$

Flavor states  
= No mass  
eigenstates

Diagonalizing H:

Mass eigenstates:  $|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$  with  $m_L, \Gamma_L$  light

complex coefficients  $|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$  with  $m_H, \Gamma_H$  heavy

$$|p|^2 + |q|^2 = 1$$

$$|B_{H,L}(t)\rangle = |B_{H,L}(0)\rangle \cdot e^{-im_{H,L}t} \cdot e^{-\frac{1}{2}\Gamma_{H,L}t}$$

Flavor eigenstates:  $|B^0\rangle = \frac{1}{2p}(|B_L\rangle + |B_H\rangle)$   $|\bar{B}^0\rangle = \frac{1}{2q}(|B_L\rangle - |B_H\rangle)$

# Mixing of neutral mesons

$$\underbrace{P(B^0 \rightarrow B^0) = P(\bar{B}^0 \rightarrow \bar{B}^0)}_{\text{CPT}} = \frac{1}{4} \left[ e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos \Delta m t \right]$$

CPT

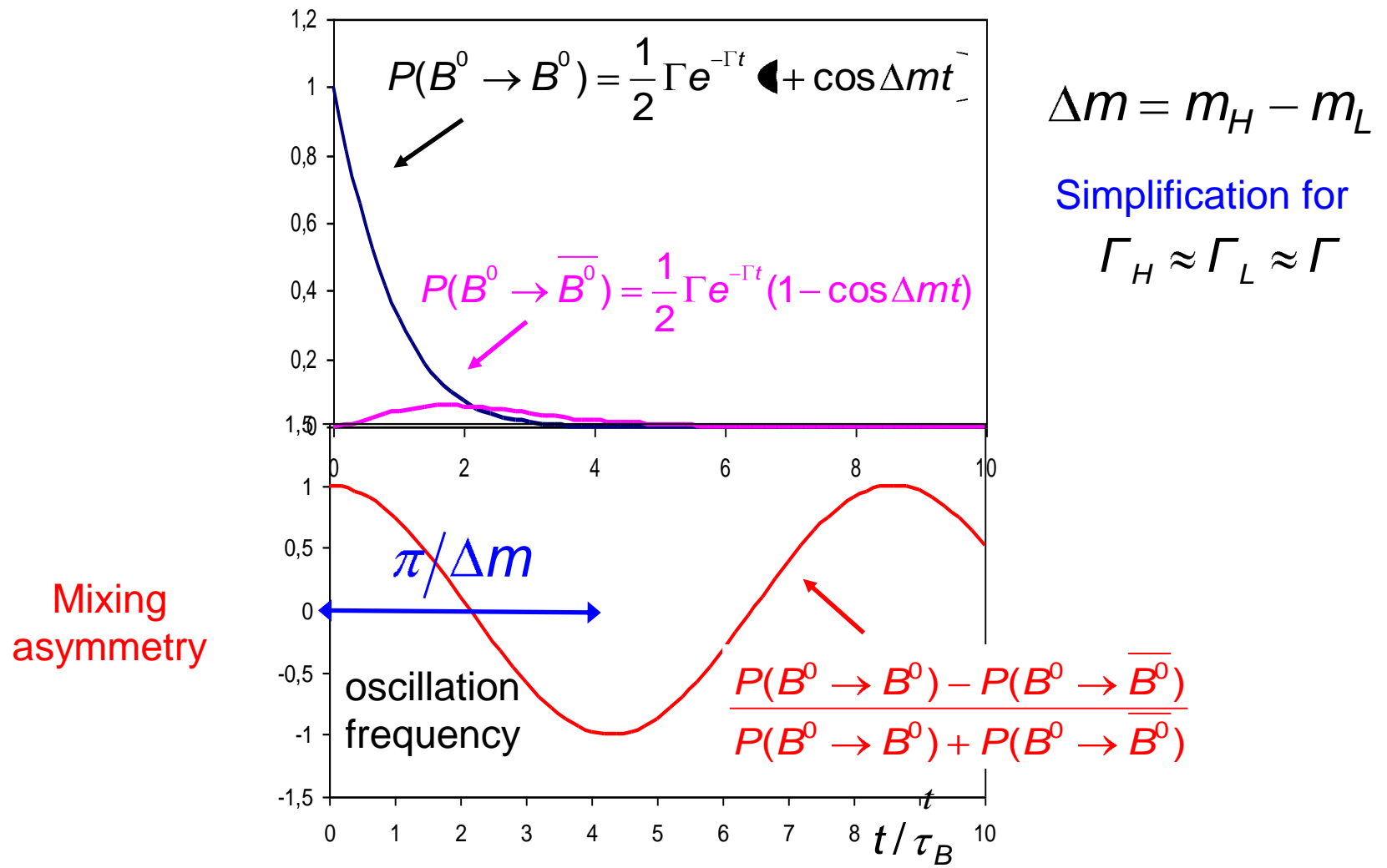
$$P(B^0 \rightarrow \bar{B}^0) = \frac{1}{4} \left| \frac{q}{p} \right|^2 \left[ e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos \Delta m t \right] \quad \Delta m = m_H - m_L$$

$$P(\bar{B}^0 \rightarrow B^0) = \frac{1}{4} \left| \frac{p}{q} \right|^2 \left[ e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos \Delta m t \right]$$

CP, T- violation in mixing:

$$P(B^0 \rightarrow \bar{B}^0) \neq P(\bar{B}^0 \rightarrow B^0) \Rightarrow \left| \frac{q}{p} \right| \neq 1$$

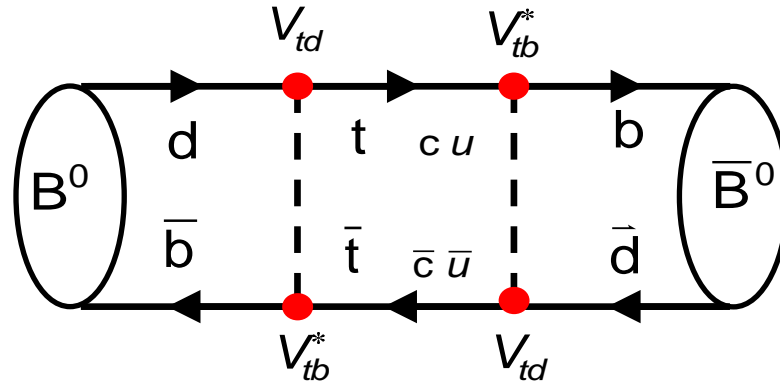
# B<sup>0</sup>-B<sup>0</sup> Mixing





## 2.2 Standard Model Prediction

$$B_d^0 - \bar{B}_d^0$$



$$\Delta m_d \sim m_t^2 \cdot O(\lambda^6)$$

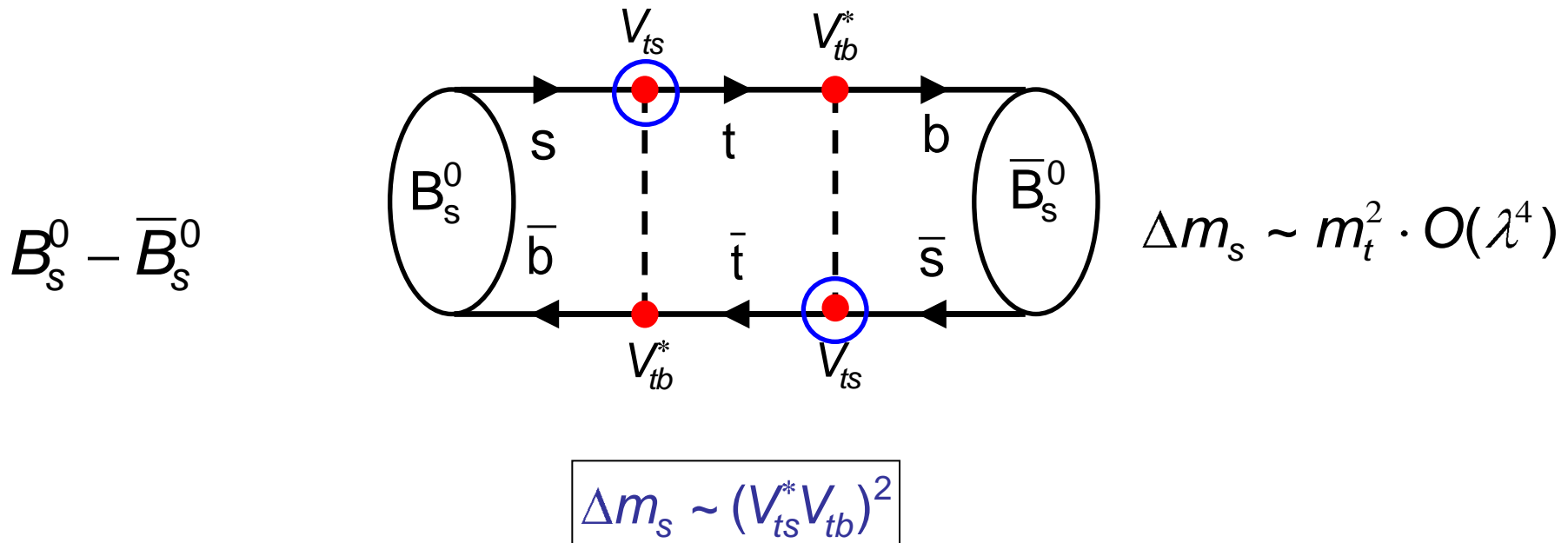
Dominant contribution from top-loop:

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_B f_B^2 B_B (V_{td}^* V_{tb})^2 m_W^2 \eta_B F\left(\frac{m_t^2}{m_W^2}\right)$$

$\eta_B = 0.55 \pm 0.01$   
 NLO QCD  
 ← e.w. correction

$$f_B^2 B_B = (235 \pm 33 \pm 12)^2 \text{ MeV}^2 \quad \text{from lattice QCD}$$

Describes the binding of the quarks to a meson



Oscillation is about 35 times stronger than in the case of  $B_d$   
 ( $V_{ts}$  much larger than  $V_{td}$ )

**B oscillation:**

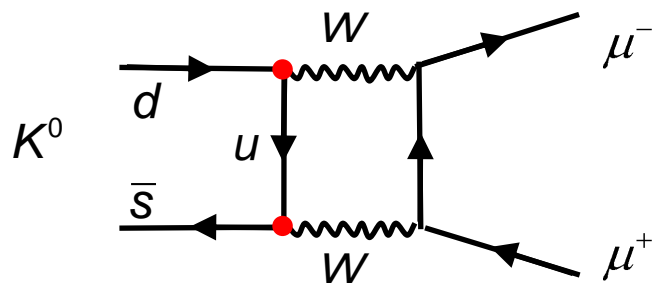
Deactivation of GIM suppression because of large mass splitting:

What would be the mixing if all quarks had the same masses?  
 (Unitarity of CKM matrix -> cancellation of FCNC!)

# Non-observed FCNC and GIM mechanism

FCNC in the 3 quark model:  $K^0 \rightarrow \mu^+ \mu^-$

Historical retrospect



$$M \sim \sin\theta_c \cos\theta_c$$

Theoretically one predicts large BR, in contradiction with experimental limits for this decay:

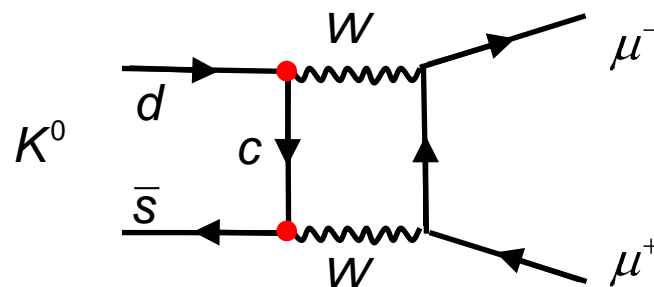
$$\frac{BR(K_L \rightarrow \mu^+ \mu^-)}{BR(K_L \rightarrow all)} = (7.2 \pm 0.5) \cdot 10^{-9}$$

Proposal by Glashow, Iliopoulos, Maiani, 1970:

There exists a fourth quark which builds together with the s quark a second doublet:

**GIM**

$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -\sin\theta_c \cdot d + \cos\theta_c \cdot s \end{pmatrix}$$



$$M \sim -\sin\theta_c \cos\theta_c$$

➔ Additional Feynman-Graph for  $K^0 \rightarrow \mu\mu$  which compensates the first one:

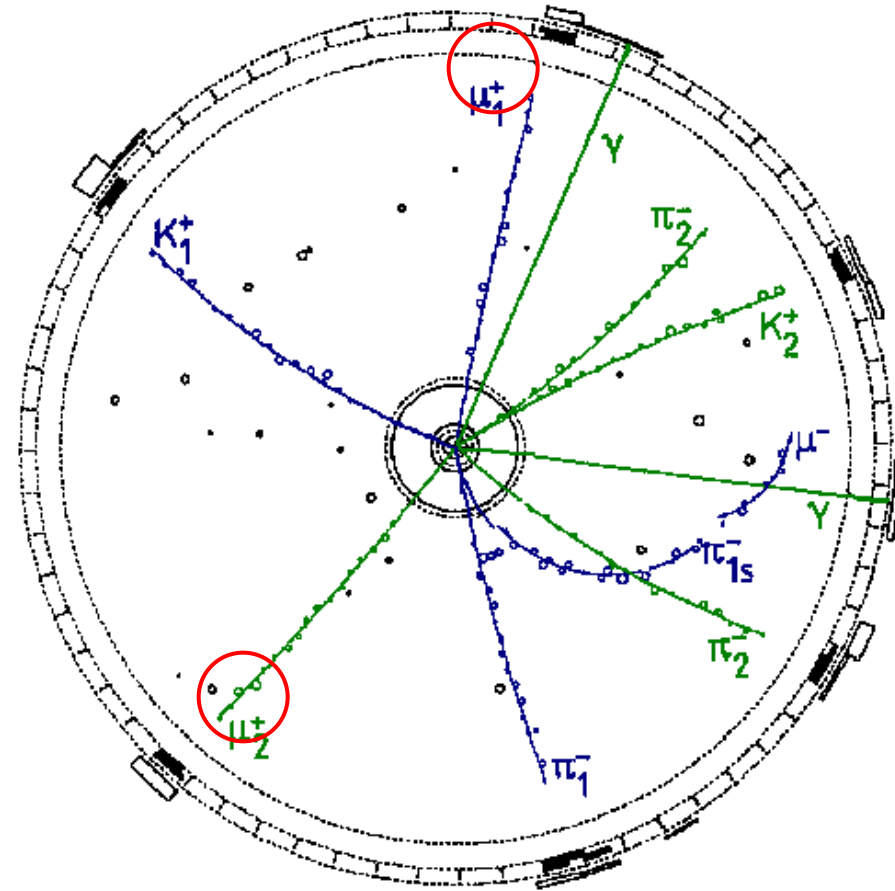
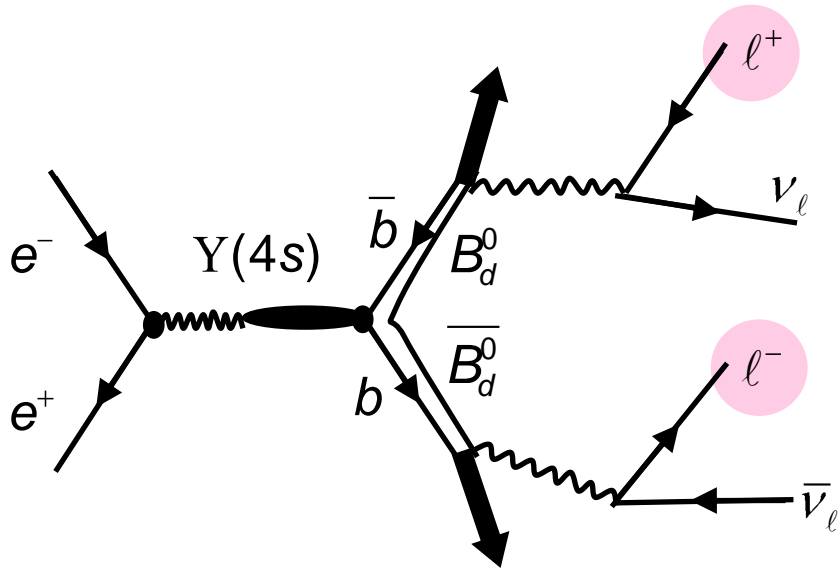
Prediction of a fourth quark:  
Mass prediction  $BR=f(m_c, \dots)$

# 2.3 Discovery of $B^0$ mixing

**ARGUS 1987**

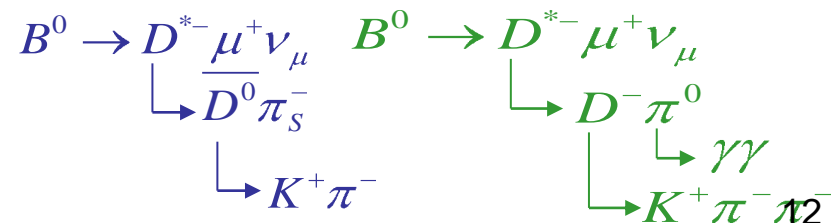
First  $e^+e^-$  B factory at DESY:

$$\left. \begin{array}{l} \text{at } \sqrt{s} = 10.58 \text{ GeV :} \\ e^+e^- \rightarrow Y(4S) \rightarrow B^0\bar{B}^0 \end{array} \right\} \sigma(B\bar{B}) \approx 1\text{nb}$$



Unmixed:  $B^0\bar{B}^0 \rightarrow l^+l^-$

Mixed:  $B^0B^0 \rightarrow l^+l^+$   
 $\bar{B}^0\bar{B}^0 \rightarrow l^-l^-$  } Same charge



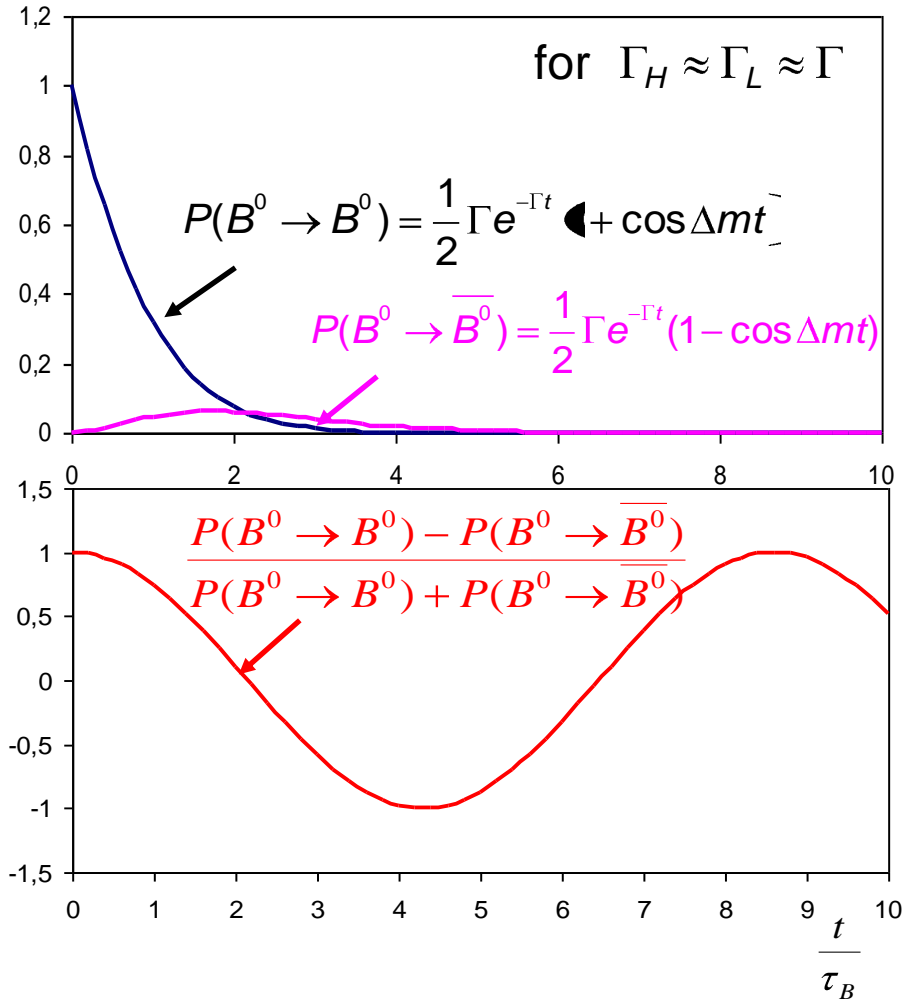
## Historical remark:

The observation of the  $B_d$  meson mixing put the first lower limit on the top mass:  $m_{\text{top}} > 50 \text{ GeV}$ .

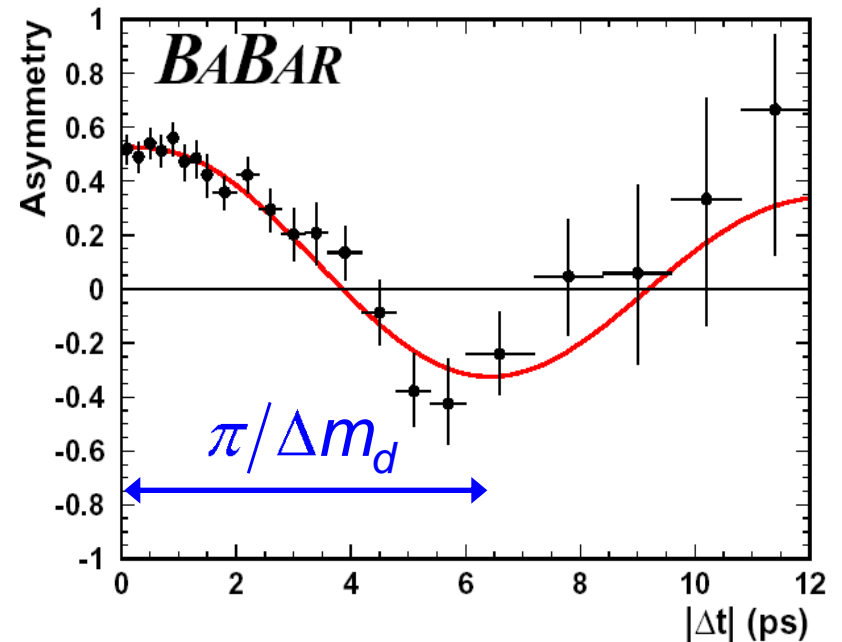
If the top mass was lower the GIM mechanism would in a small  $\Delta m$ , i.e. the B would oscillate very slowly and would decay before mixing.

The GIM mechanism is a result of the unitarity of the CKM matrix. Only different quark masses lead to a non-perfect cancellation and are the sources of observable FCNCs at loop level.

# Experimental Status of B meson mixing

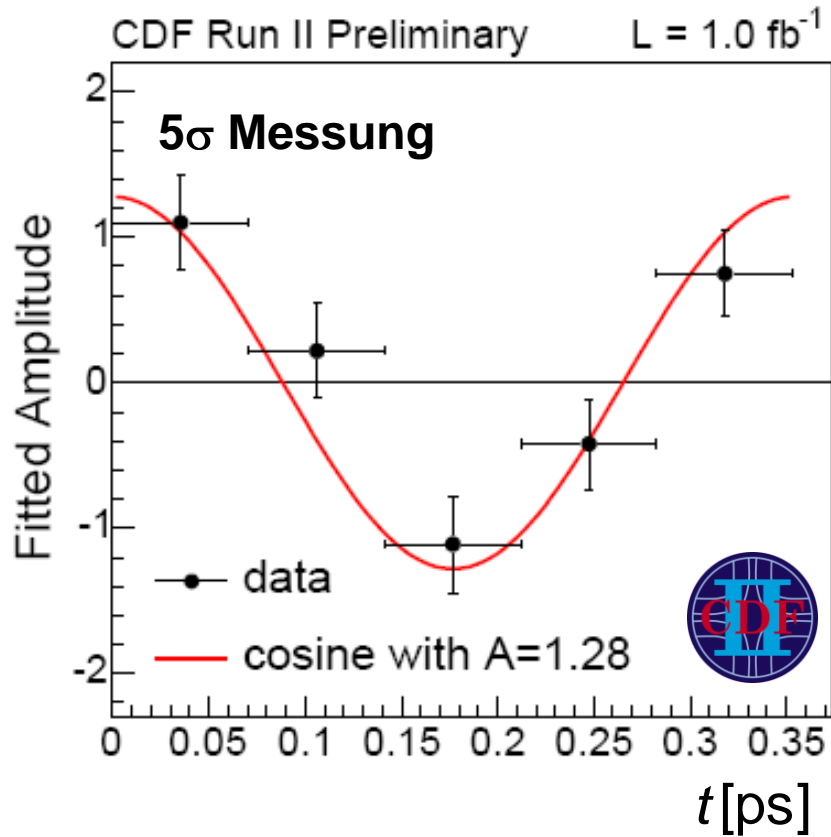


$$A = \frac{\text{unmixed} - \text{mixed}}{\text{unmixed} + \text{mixed}}$$

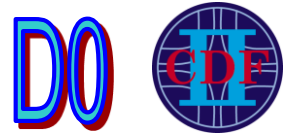


$$\Delta m_d = 0.506 \pm 0.006 \pm 0.004 \text{ ps}^{-1}$$

$$\approx \frac{0.774}{\tau_B}$$



**Observation:  
Spring 2006**



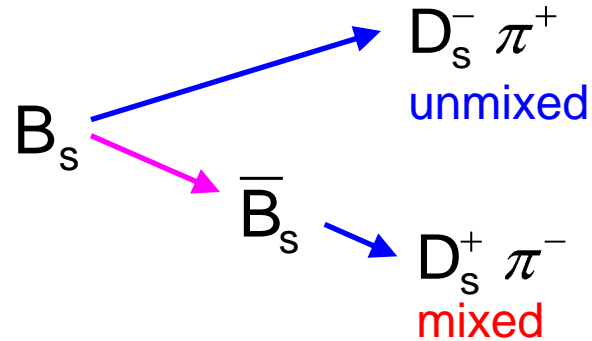
$$\Delta m_s = 17.77 \pm 0.10(\text{stat.}) \pm 0.07(\text{syst.}) \text{ ps}^{-1} = \frac{26}{\tau}$$

(CDF Collaboration, September 2006)

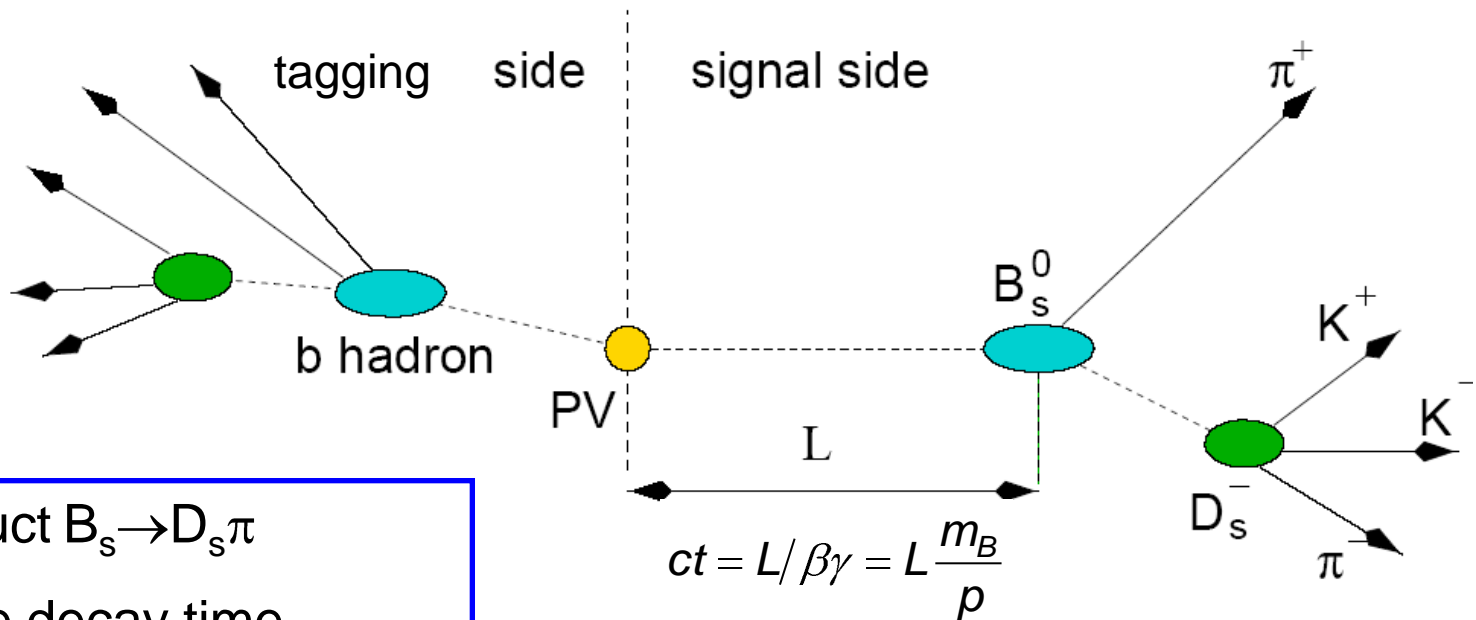
35 times faster  
than  $B^0$

# Measuring $B_s$ Mixing

Principle:



$$A(t) = \frac{\text{unmixed}(t) - \text{mixed}(t)}{\text{unmixed}(t) + \text{mixed}(t)} = \cos(\Delta m t)$$



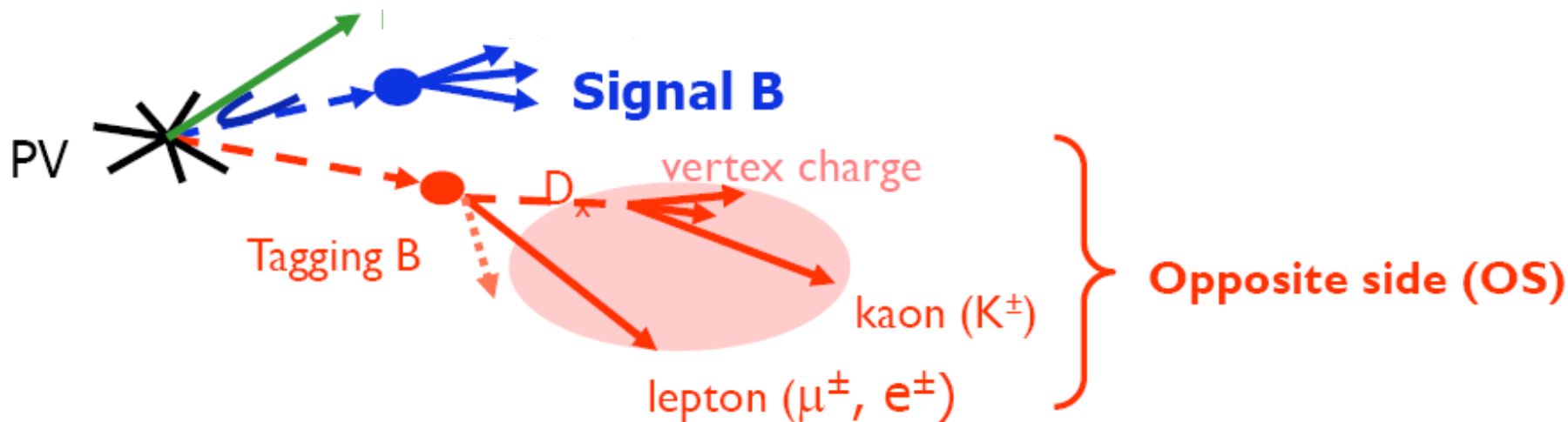
Reconstruct  $B_s \rightarrow D_s \pi$

Determine decay time

Determine production flavor



# Determination of Production Flavor = Tagging



## Flavour tagging algorithms are not perfect!

- Backgrounds in tagger selections
- The *tagging B* can oscillate incoherently (unlike in B-factories):

40%  $B^\pm$ , 10% baryons : no oscillation ☺

40%  $B_d$ :  $\Delta m_d \sim \Gamma_d \Rightarrow$  oscillated 17.5% ☺

- 10%  $B_s$ :  $\Delta m_s \gg \Gamma_s \Rightarrow$  oscillated 50% ☹

at LHC

## Characterization:

$\epsilon_{\text{tag}}$  = tagging effi.

$\omega$  = wrong tag fraction

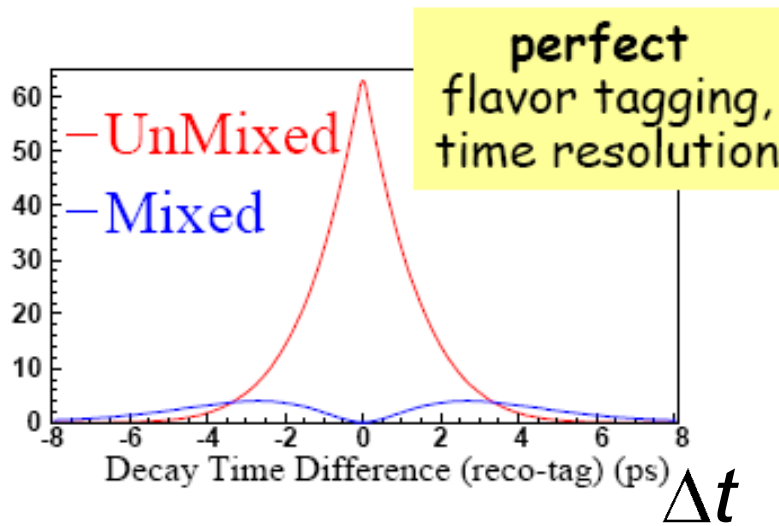
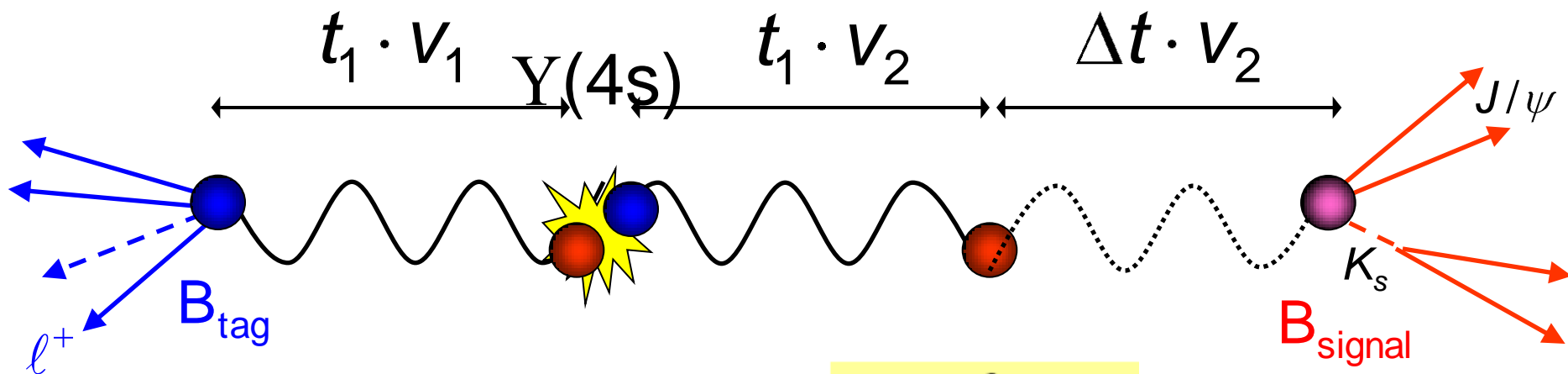
➔ Advantage of e+e-  
B-factories

# Interlude: $Y(4S) \rightarrow B^0 B^0$

Coherent state!

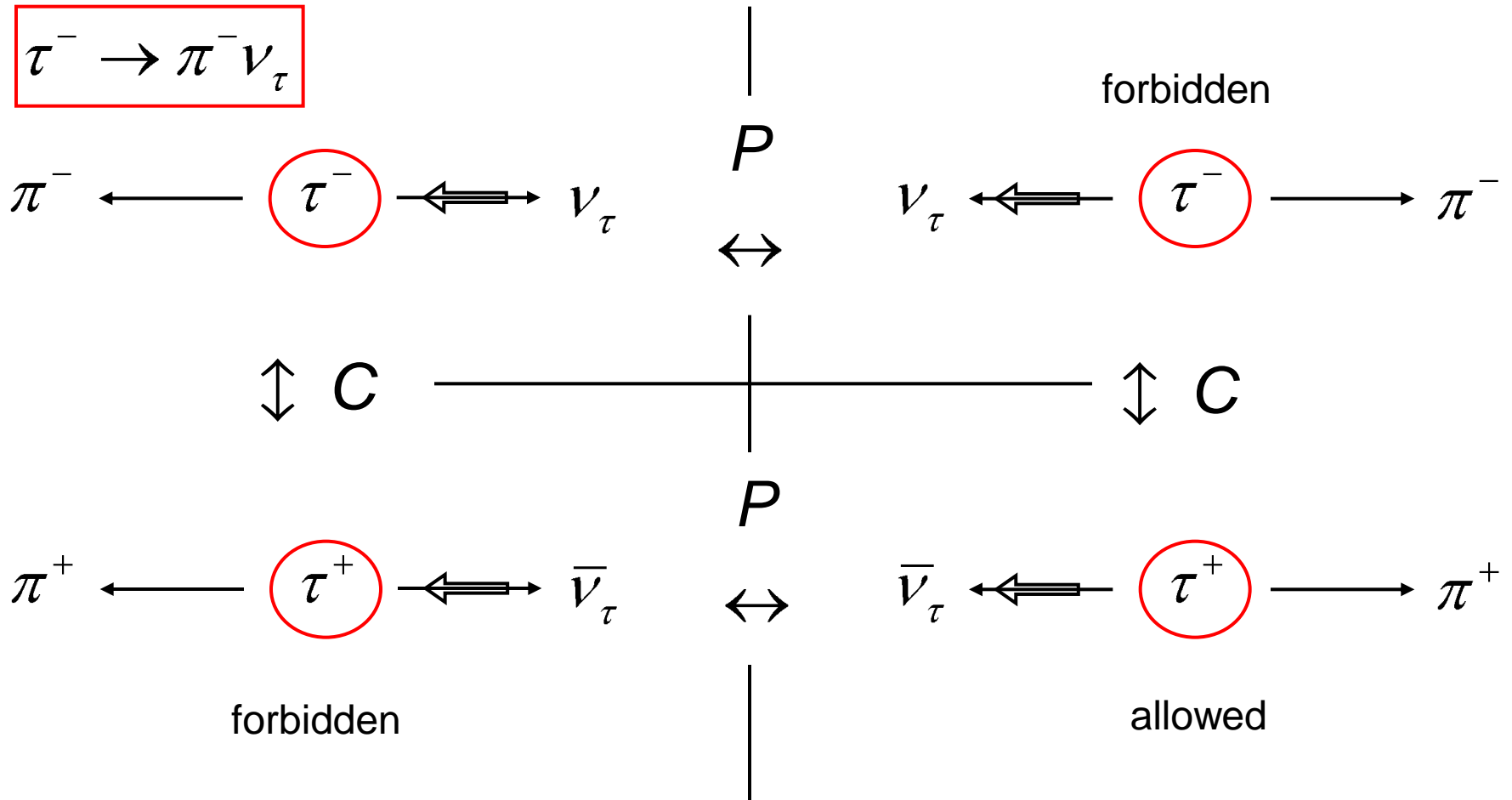


$$Y(4S) \rightarrow \frac{1}{\sqrt{2}} \left( |B^0 \bar{B}^0\rangle - |\bar{B}^0 B^0\rangle \right)$$



### 3. CP Violation

Reminder: Maximum C and P violation in weak decays:



- C and P violated in weak decays
- CP conserved in weak interaction ? → No !

# 3.1 Discovery of CP Violation in Kaon Decays

Observation of two neutral kaons  $K_L$  (long) and  $K_S$  (short) with different lifetimes:

$$\tau(K_L^0) = 1.7 \pm 0.4 \text{ ns} \gg \tau(K_S^0) = 0.089 \pm 0.001 \text{ ns}$$

$$K_L^0 \rightarrow 3\pi$$

$$CP = -1$$

$$K_S^0 \rightarrow 2\pi$$

$$CP = +1$$

$$K^0 = |d\bar{s}\rangle$$

$$\bar{K}^0 = |\bar{d}s\rangle$$

**Interpretation:** (neglecting possible CP violation)

$$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}} \left( |K^0\rangle - |\bar{K}^0\rangle \right)$$

$$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}} \left( |K^0\rangle + |\bar{K}^0\rangle \right)$$

$$CP|K_2\rangle = -|K_2\rangle$$

$$CP|K_1\rangle = +|K_1\rangle$$

Phase convention:

$$CP|K^0\rangle = |\bar{K}^0\rangle$$

$$CP|\bar{K}^0\rangle = |K^0\rangle$$

Large differences between lifetimes

$$\begin{aligned} \Delta m &= 0.5303 \pm 0.0009 \cdot 10^{10} \text{ } \hbar\text{s}^{-1} \\ &= 0.49 \pm 0.006 \cdot 10^{-12} \text{ MeV} \end{aligned}$$

$$\Delta\Gamma = -11.182 \cdot 10^9 \text{ } \hbar\text{s}^{-1}$$

If no CPV:

$$|K_L\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle) \quad CP = -1$$

should always decay into  $3\pi$ :

$$CP(|3\pi\rangle) = -1$$

and never into  $2\pi$   $CP(|2\pi\rangle) = +1$

Explanation:

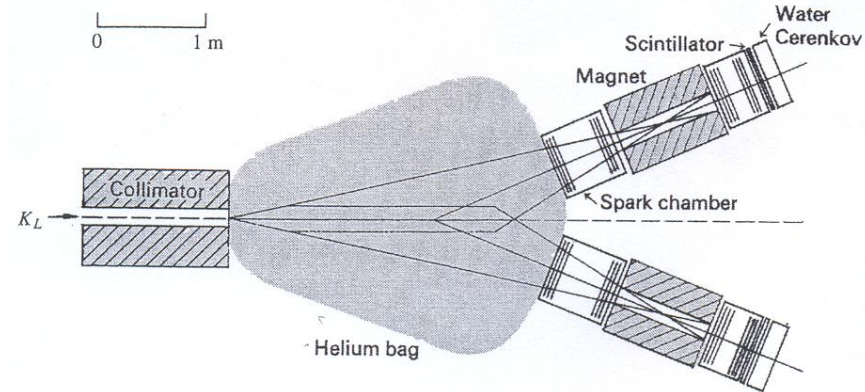
$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} (|K_2\rangle - \varepsilon|K_1\rangle)$$

$CP = -1$       $CP = +1$

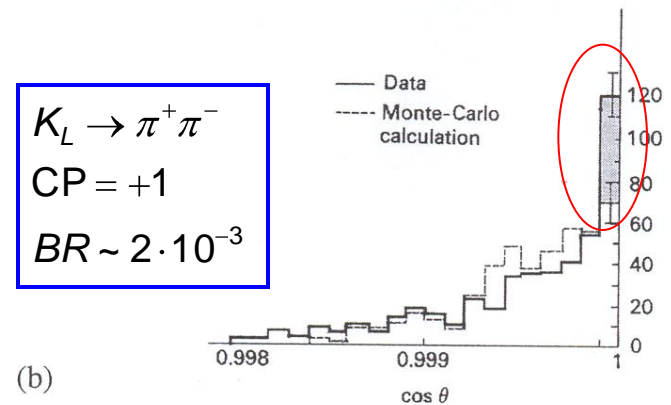
↑

**Not a CP eigenstate: CP violation !**

**Christenson, Cronin, Fitch, Turlay, 1964**



(a)



(b)

$$\theta = \angle(\vec{p}_K, (\vec{p}_{\pi^+} + \vec{p}_{\pi^-}))$$

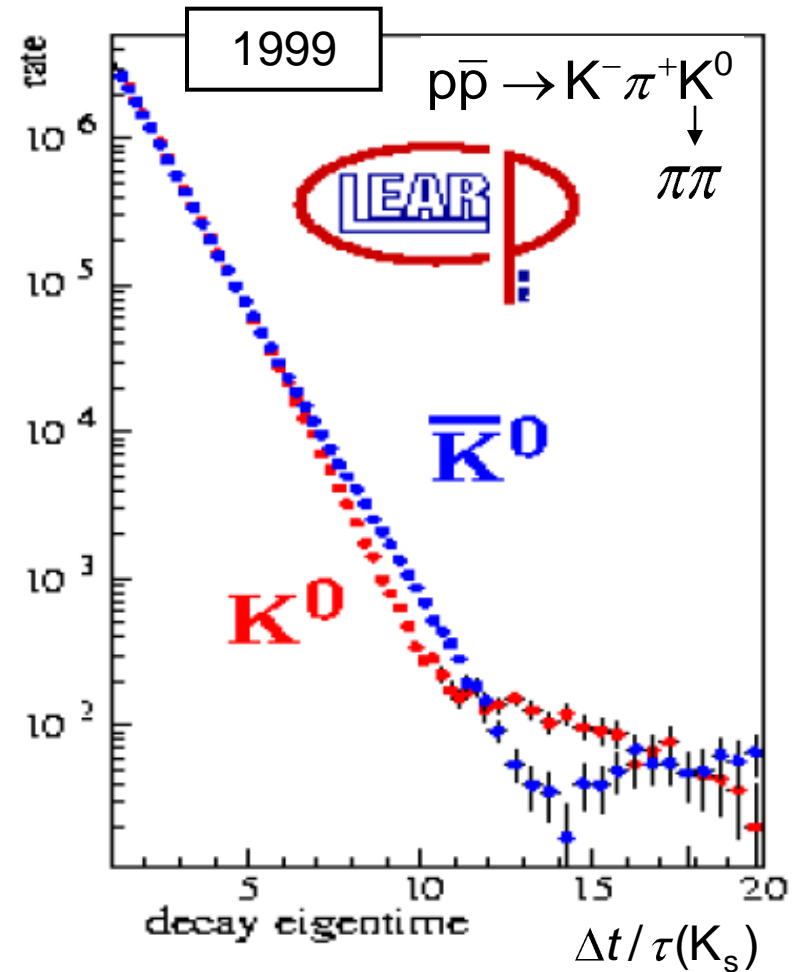
After 35 years of kaon physics:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}} \left[ |K_2\rangle + \varepsilon |K_1\rangle \right]$$

$\downarrow \varepsilon'$                        $\downarrow$   
 $\pi\pi$                                $\pi\pi$   
 (Direct CPV)                      (mixing)

$$|\varepsilon| = (2.284 \pm 0.014) \cdot 10^{-3}$$

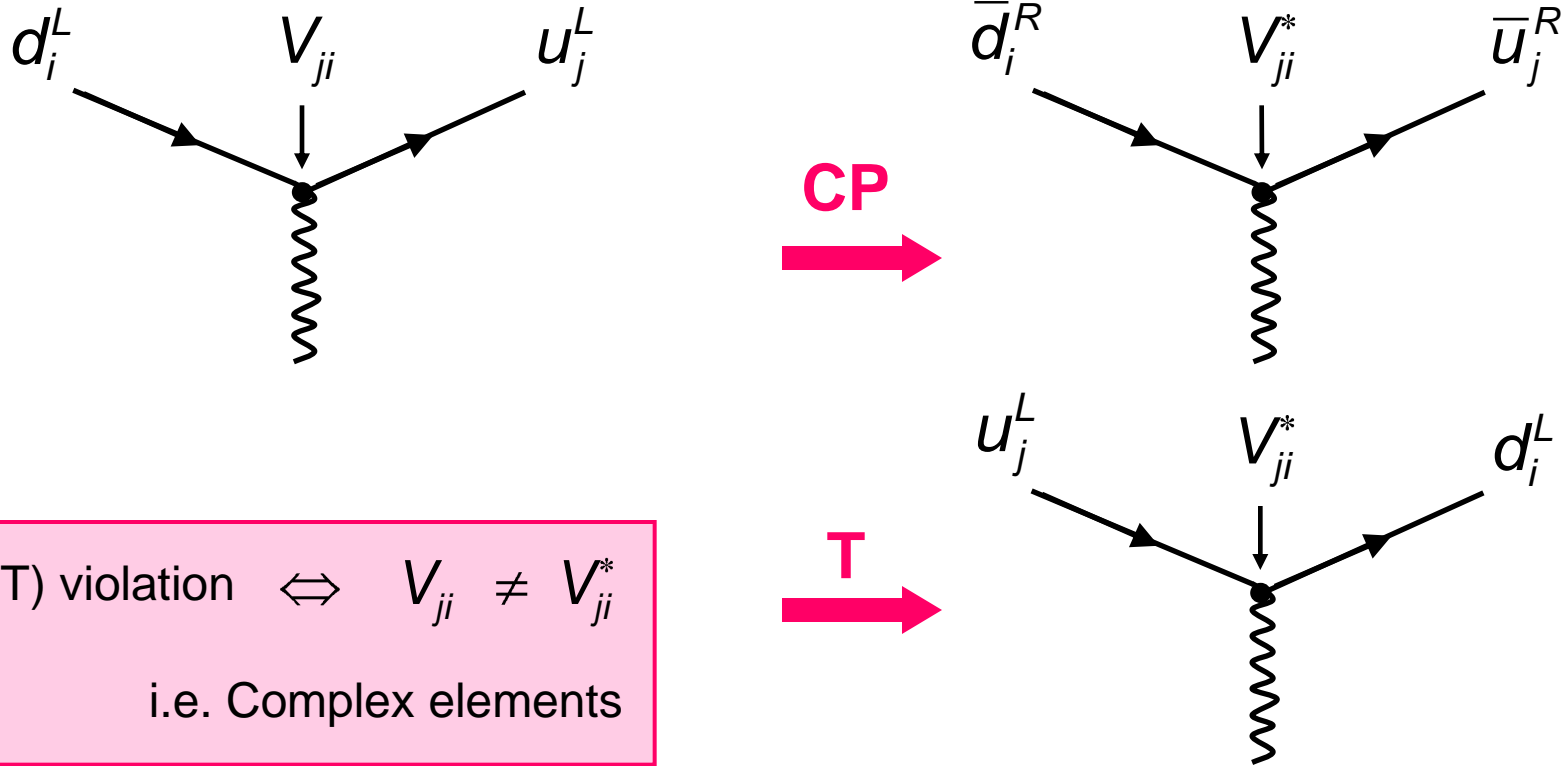
$$\text{Re}(\varepsilon'/\varepsilon) = (1.67 \pm 0.26) \cdot 10^{-3}$$



The measured CP violation in the kaon system is small – theoretical interpretation is quiet difficult !

In the B meson system effects are much larger, easier to understand and they can be calculated in the Standard Model. CPV in the B<sup>0</sup> system was observed in 2000.

## 3.2 CP Violation in Standard Model: complex CKM elements



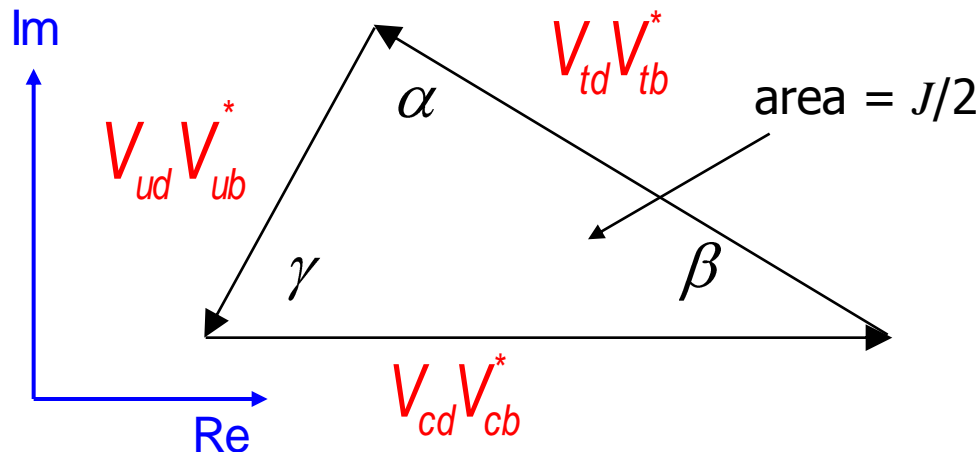
Remark: For 2 quark generations the mixing is described by the **real 2x2** Cabbibo matrix  $\rightarrow$  **no CP violation** !!. To explain **CPV** in the SM Kobayashi and Maskawa have predicted a **third quark generation**.

Moreover, as can be shown, CPV requires that all **u-type and all d-type quarks have different masses**.

# CKM Matrix and Unitarity Triangle

Unitary CKM matrix:  $\mathbf{V}\mathbf{V}^\dagger = \mathbf{1}$  → 6 “triangle” relations in complex plane:

$$\mathbf{V} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

$$V_{td} V_{ud}^* + V_{ts} V_{us}^* + V_{tb} V_{ub}^* = 0$$

Important for  $\mathbf{B}_d$  and  $\mathbf{B}_s$  decays

Non degenerated “triangles” only in case of CP violation: Tip / triangle area defines the amount/strength of CPV!

Strength of CPV characterized by Jarlskog invariant (area)  $J = \text{Im} \left( V_{ij} V_{kl} V_{il}^* V_{kj}^* \right)$

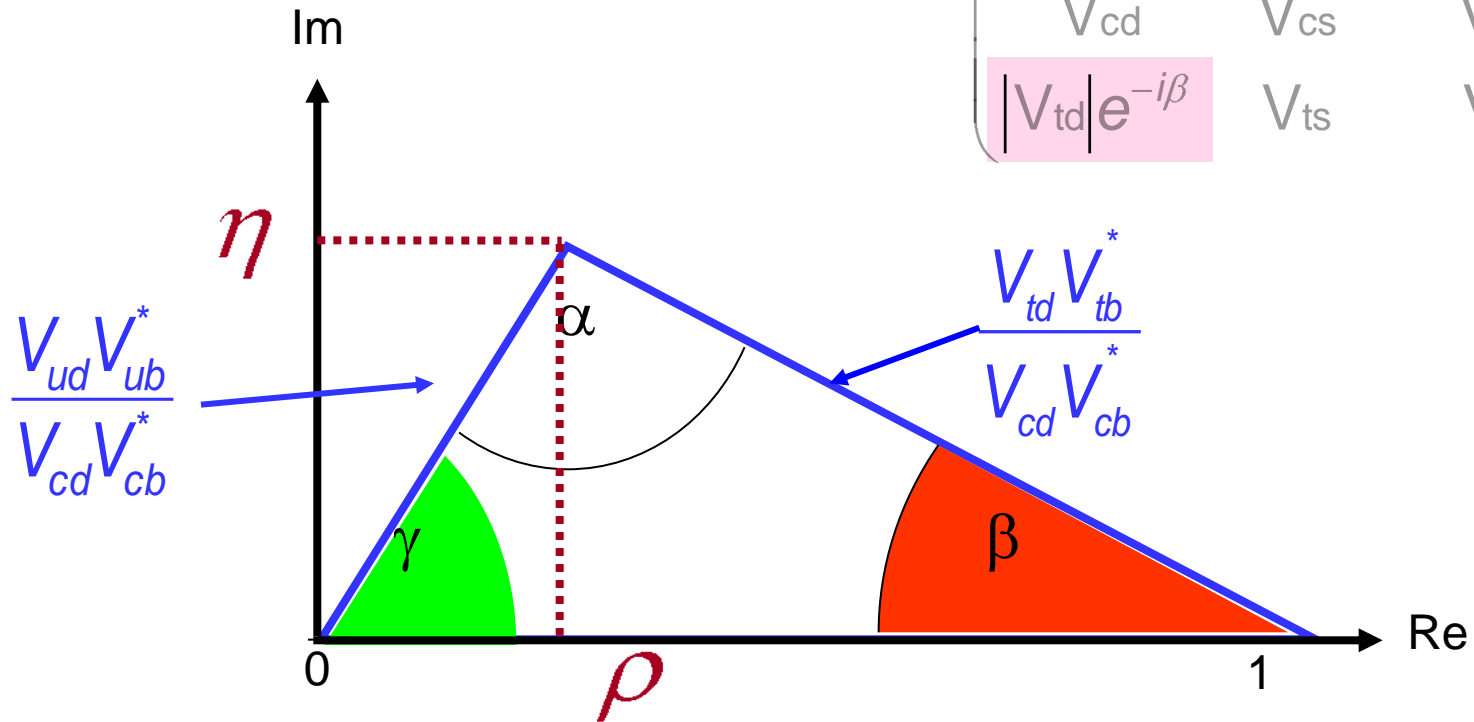
In SM:  $J = \text{Im}[V_{us} V_{cb} V_{ub}^* V_{cs}^*] = A^2 \lambda^6 \eta \left( -\lambda^2/2 \right) + O(\lambda^{10}) \sim 10^{-5}$



Rescaled unitarity condition  $V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$

“The Unitarity Triangle”

$$\begin{pmatrix} V_{ud} & V_{us} & |V_{ub}|e^{-i\gamma} \\ V_{cd} & V_{cs} & V_{cb} \\ |V_{td}|e^{-i\beta} & V_{ts} & V_{tb} \end{pmatrix}$$



$$\alpha \equiv \arg \left[ - \frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right] \quad \beta \equiv \arg \left[ - \frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right] \quad \gamma \equiv \arg \left[ - \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right]$$

## Experimental confirmation of UT:

The sides of the UT can be measured via

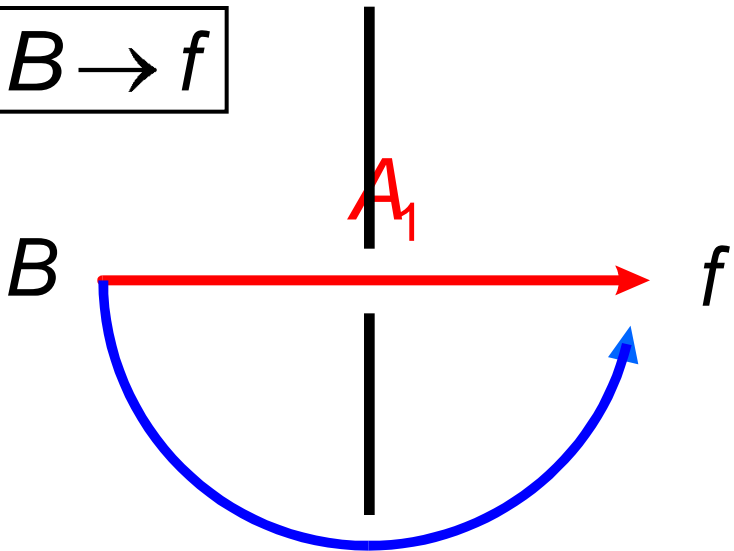
- $B_d$  and  $B_s$  oscillation
- Semileptonic B decays with a  $b \rightarrow c$  or  $b \rightarrow u$  quark transition.

The angles (phases) can be determined from CP asymmetries in B decays! Observation of CP violating phases require presence of interfering amplitudes!

# 3.3 Observation of CP Violation

→ Phase measurement  
→ Interference experiment

$$B \rightarrow f$$



$$A_2 e^{i\phi_{CP}} e^{i\delta}$$

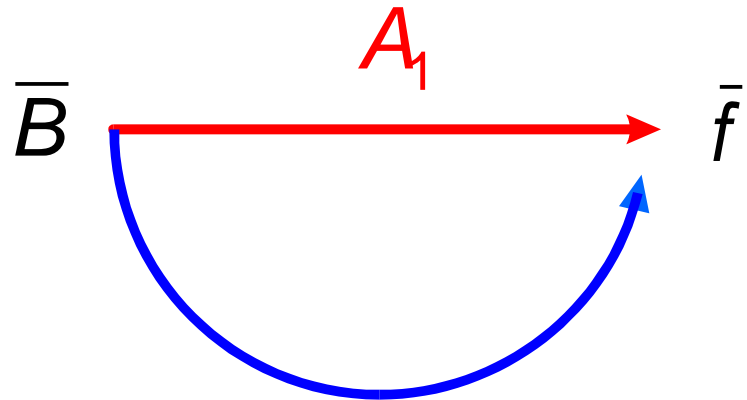
Weak and CP invariant phase difference

$$|A|^2 = A_1^2 + A_2^2$$

$$+ 2A_1A_2 \cos(\phi_{CP} + \delta)$$

CP

$$\bar{B} \rightarrow \bar{f}$$



$$A_2 e^{-i\phi_{CP}} e^{i\delta}$$

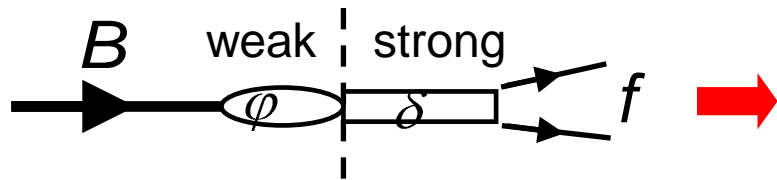
$$|\bar{A}|^2 = A_1^2 + A_2^2$$

$$+ 2A_1A_2 \cos(\phi_{CP} - \delta)$$

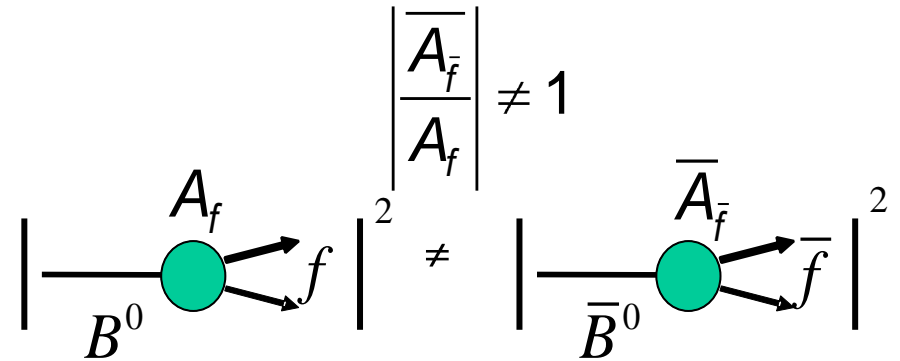
Need two phase differences between  $A_1$  and  $A_2$ : Weak difference which changes sign under CP and another phase difference (strong) which is unchanged.

# “3 Ways” of CP violation in meson decays

## a) Direct CP violation



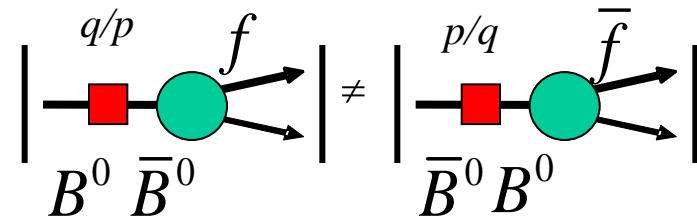
$$A(B \rightarrow f) = |A| e^{i\varphi} e^{i\delta}$$



$$P(\bar{B} \rightarrow \bar{f}) \neq P(B \rightarrow f)$$

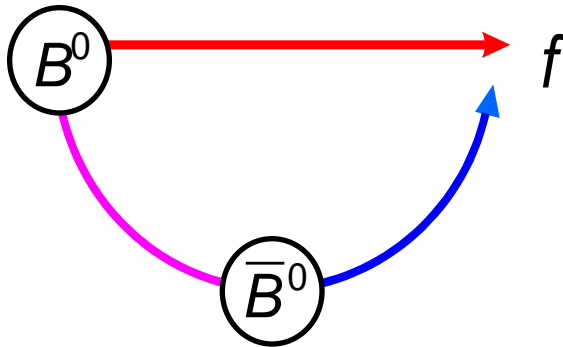
## b) CP violation in mixing

$$\left| \frac{q}{p} \right| \neq 1$$



$$P(B^0 \rightarrow \bar{B}^0) \neq P(\bar{B}^0 \rightarrow B^0)$$

## c) CP violation through interference of mixed and unmixed amplitudes

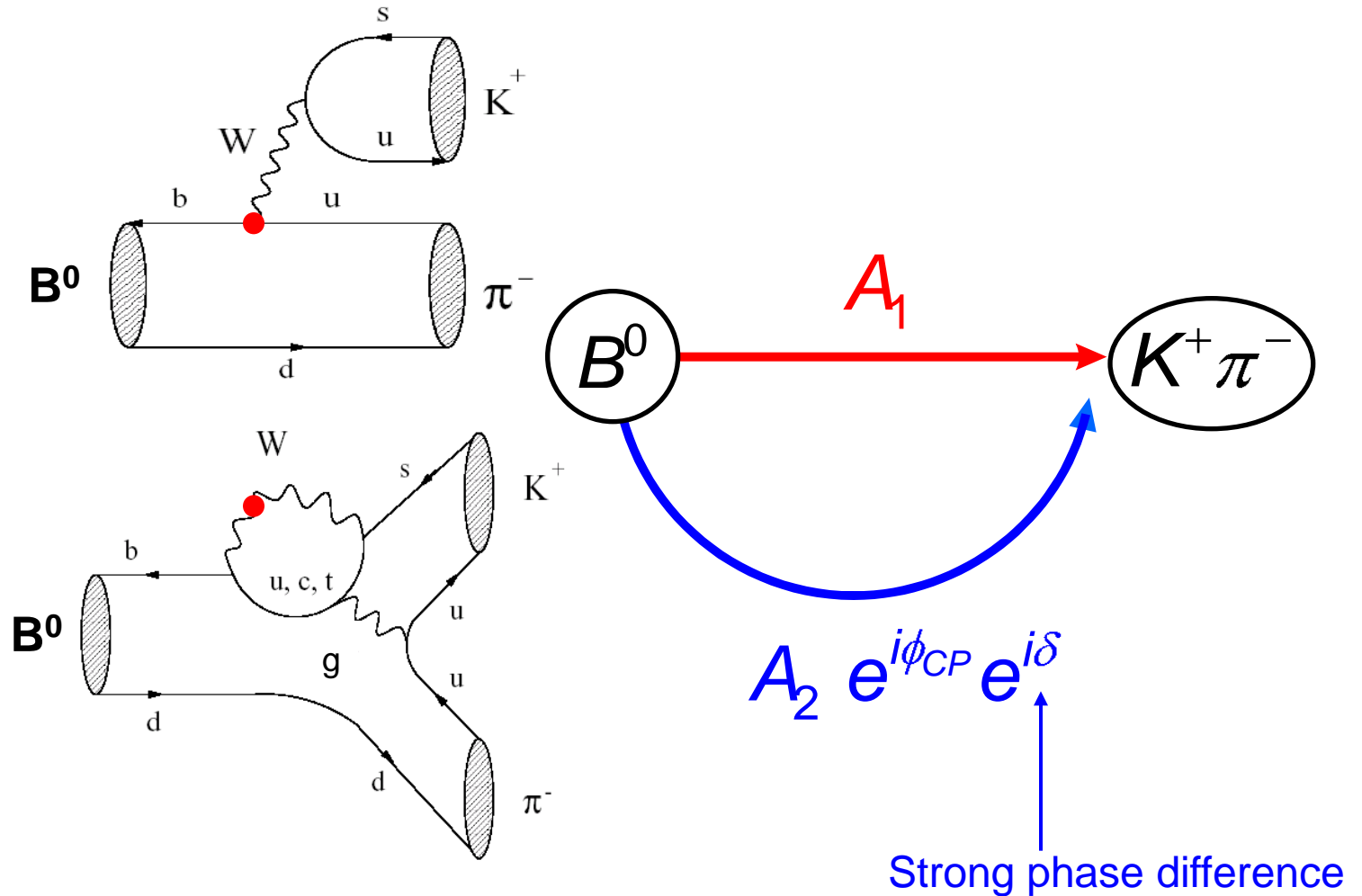


$$\Gamma(B_{t=0}^0 \rightarrow f)(t) \neq \Gamma(\bar{B}_{t=0}^0 \rightarrow f)(t)$$

Asymmetrie modulated by  $\sim \sin \Delta m t$

**Combinations of the 3 ways are possible!**

# ad a) Direct CP violation (B system)



CP Asymmetrie 
$$|\overline{A}|^2 - |A|^2 = 4|A_1||A_2| \sin\phi \sin\delta$$

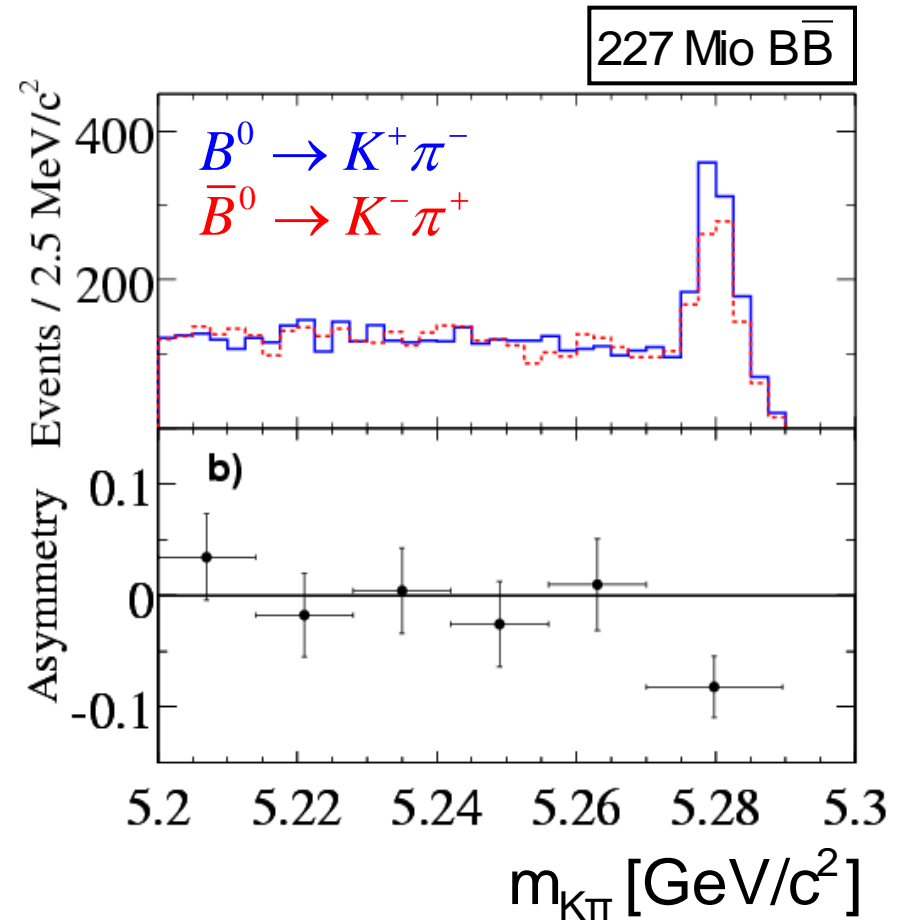


$$N(B^0 / \bar{B}^0 \rightarrow K^\pm \pi^\mp) = 1606 \pm 51$$

$$A_{CP} = \frac{N(\bar{B}^0 \rightarrow K^+ \pi^-) - N(B^0 \rightarrow K^- \pi^+)}{N(\bar{B}^0 \rightarrow K^+ \pi^-) + N(B^0 \rightarrow K^- \pi^+)}$$

$$A_{CP} = -0.133 \pm 0.030 \pm 0.009$$

**4.2 $\sigma$**



*PRL93(2004) 131801.*

## b) CP (T) violation in mixing

$$\left| \frac{q}{p} \right| \neq 1 \quad \overset{\text{T violation}}{\longleftrightarrow} \quad P(B^0 \rightarrow \overline{B^0}) \neq P(\overline{B^0} \rightarrow B^0)$$

Evidence of anomalous CP-violation in the mixing of neutral B mesons:

### Evidence for an anomalous like-sign dimuon charge asymmetry

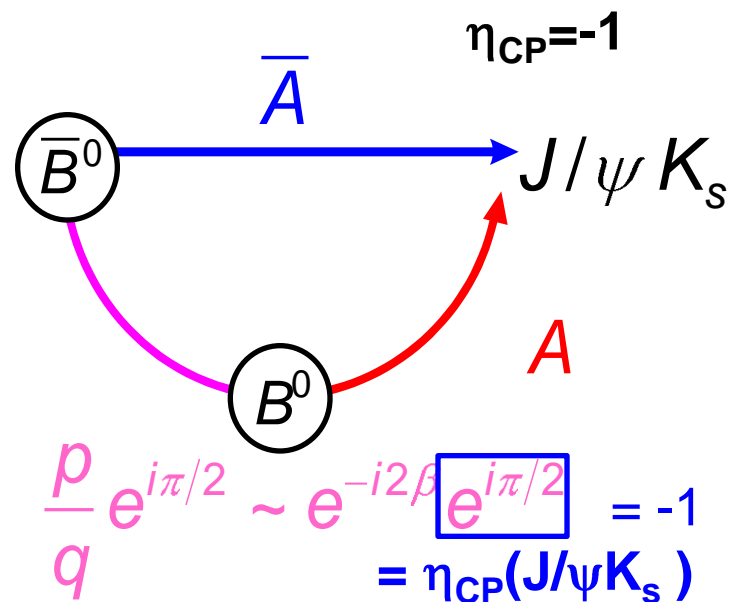
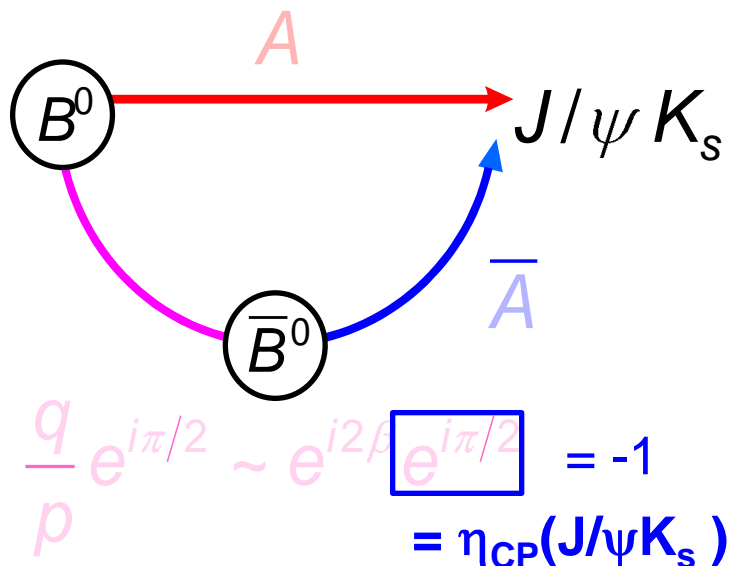
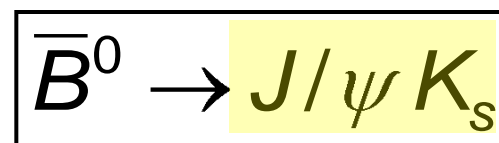
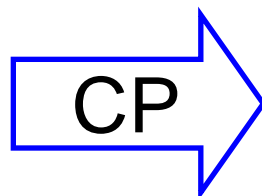
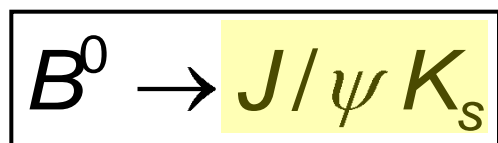
We measure the charge asymmetry  $A$  of like-sign dimuon events in  $6.1 \text{ fb}^{-1}$  of  $p\bar{p}$  collisions recorded with the D0 detector at a center-of-mass energy  $\sqrt{s} = 1.96 \text{ TeV}$  at the Fermilab Tevatron collider. From  $A$ , we extract the like-sign dimuon charge asymmetry in semileptonic  $b$ -hadron decays:  $A_{\text{sl}}^b = -0.00957 \pm 0.00251 \text{ (stat)} \pm 0.00146 \text{ (syst)}$ . This result differs by 3.2 standard deviations from the standard model prediction  $A_{\text{sl}}^b(\text{SM}) = (-2.3_{-0.6}^{+0.5}) \times 10^{-4}$  and provides first evidence of anomalous CP-violation in the mixing of neutral  $B$  mesons.

arXiv:1005.2757v1 [hep-ex] 16 May 2010

Skipped.



## c) CP violation in interference between mixing and decay



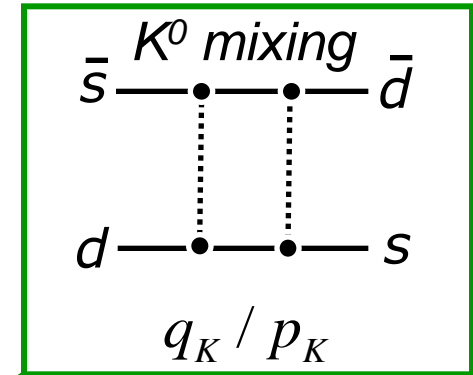
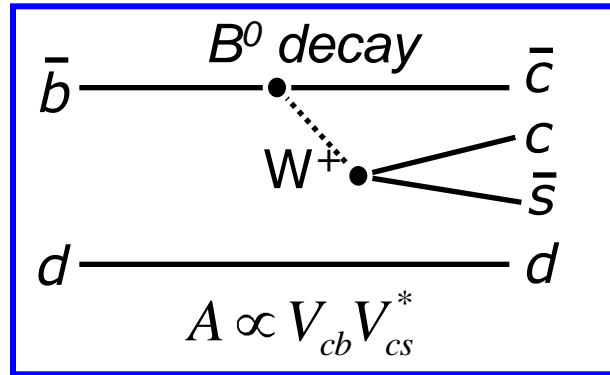
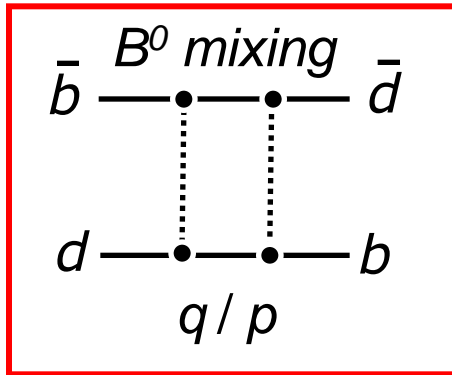
$$\Gamma(t) \sim e^{-\Gamma t} \left[ -\sin 2\beta \sin(\Delta m t) \right]$$

$$\Gamma(t) \sim e^{-\Gamma t} \left[ +\sin 2\beta \sin(\Delta m t) \right]$$

$$A_{CP}(t) = \frac{\Gamma(\bar{B}^0 \rightarrow f)(t) - \Gamma(B^0 \rightarrow f)(t)}{\Gamma(\bar{B}^0 \rightarrow f)(t) + \Gamma(B^0 \rightarrow f)(t)} = \sin 2\beta \sin(\Delta m t)$$

# SM prediction for $\lambda = A_1 / A_2$ (amplitude ratio)

Same for all  $c\bar{c}K^0$  channels



$$\frac{q}{p} \sim e^{2i\beta}$$

$$\text{Ratio } A_1 / A_2 = \lambda_{\text{CP}} = \frac{q \bar{A}}{p A} = - \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} = - \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}} = - e^{-2i\beta}$$

Beside  $V_{td}$  all other CKM elements are real

$$V_{td} \approx |V_{td}| e^{-i\beta}$$

$\Rightarrow$

$$|\lambda_{\text{CP}}| = 1$$

$$\text{Im}(\lambda_{\text{CP}}) = \sin(2\beta)$$

no direct CPV, no CPV in mixing

# Calculation of the time-dependent CP asymmetry

$$\Gamma(B^0 \rightarrow f_{CP})(t) \propto \frac{e^{-|\Delta t|/\tau_{B^0}}}{1 + |\lambda_{CP}|^2} \left[ \frac{1 + |\lambda_{CP}|^2}{2} - \text{Im} \left[ \lambda_{CP} \sin m_d t \right] + \frac{1 - |\lambda_{CP}|^2}{2} \cos m_d t \right]$$

$$\neq$$

$$\Gamma(\bar{B}^0 \rightarrow f_{CP})(t) \propto \frac{e^{-|\Delta t|/\tau_{B^0}}}{1 + |\lambda_{CP}|^2} \left[ \frac{1 + |\lambda_{CP}|^2}{2} + \text{Im} \left[ \lambda_{CP} \sin m_d t \right] - \frac{1 - |\lambda_{CP}|^2}{2} \cos m_d t \right]$$

$$A_{CP}(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow f_{CP}) - \Gamma(B^0(t) \rightarrow f_{CP})}{\Gamma(B^0(t) \rightarrow f_{CP}) + \Gamma(\bar{B}^0(t) \rightarrow f_{CP})} = S_f \sin m_d t - \underbrace{C_f \cos m_d t}_{\text{negligible}}$$

**Time resolved**

$$S_f = \frac{2 \text{Im} \lambda_{CP}}{1 + |\lambda_{CP}|^2} \quad C_f = \frac{1 - |\lambda_{CP}|^2}{1 + |\lambda_{CP}|^2}$$

Interference  
=  $\sin 2\beta$  for  $B^0 \rightarrow J/\psi K_S$

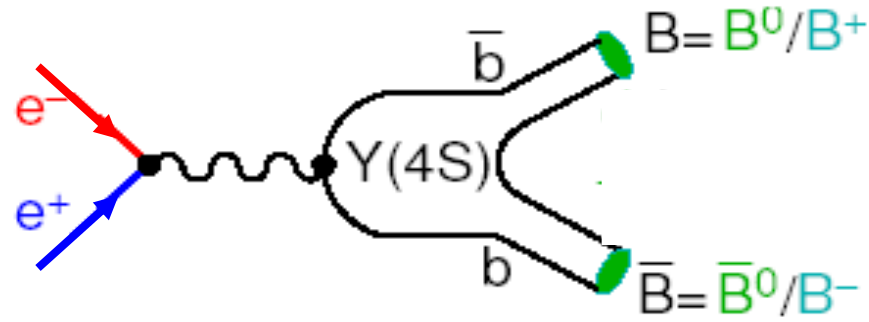
indicates direct CP violation  
if  $|q/p| \neq 1$

## To measure CP violation in $B_d$ system:

- Need many B (several  $100 \times 10^9$ )
- Need to know the flavor of the B at  $t=0$
- Need to reconstruct the decay length to measure  $t$

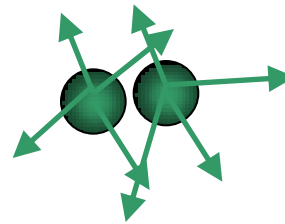
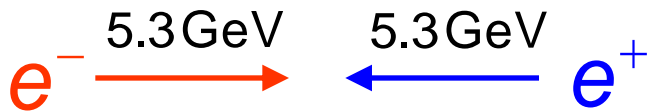
# 3.4 Measurement of $\sin 2\beta$ : Asymmetric $e^+ e^-$ B factory

$E_{\text{CMS}} = 10.58 \text{ GeV}$



Y(4S) resonance decays entirely to B pairs

**Symmetric:**

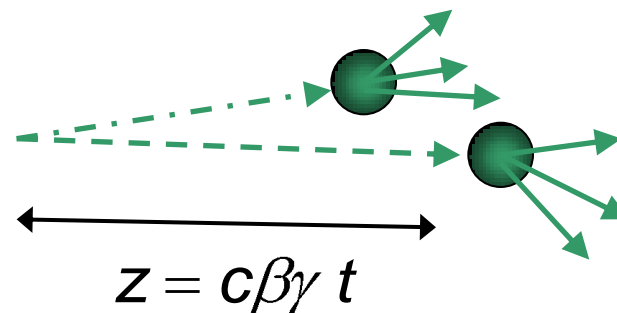


B mesons decay at rest  
→ decay length  $z \approx 0$

**Asymmetric:**



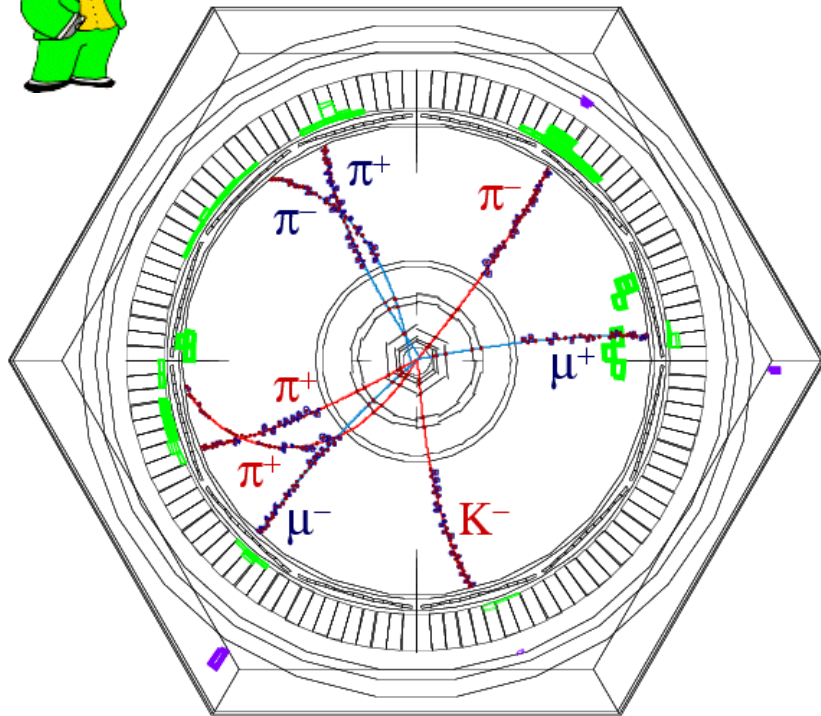
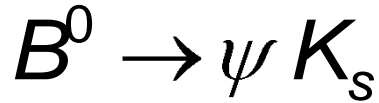
**Boost  $\beta = 0.56$**



decay length  
 $z \approx 250 \mu\text{m}$



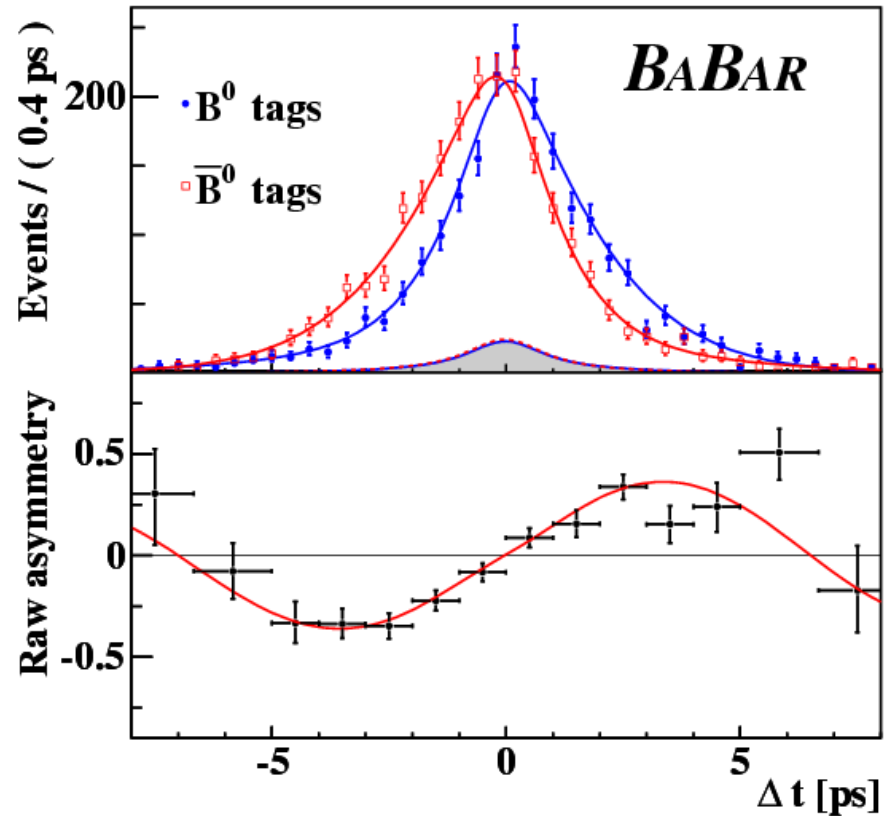
# Measurement of $\sin 2\beta$ : Golden decay channel $B^0 \rightarrow \psi K_s$



$$A_{CP}(t) = \sin 2\beta \sin(\Delta m t)$$

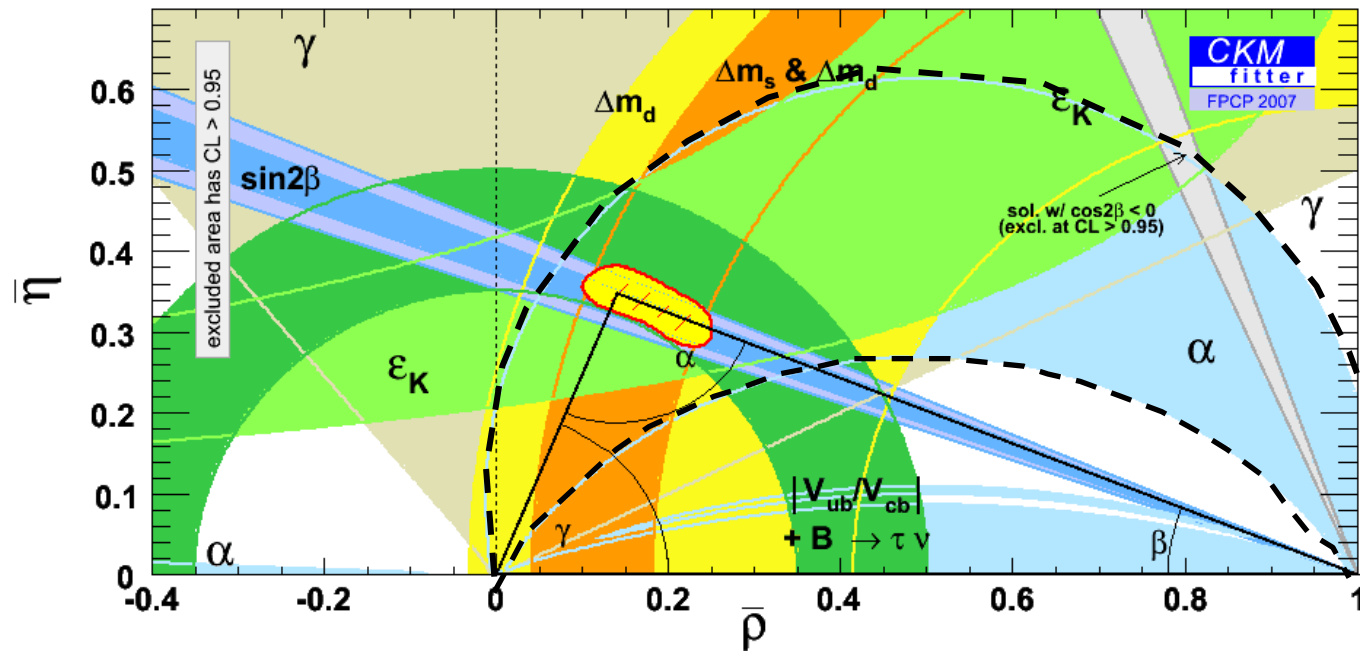
PRL 94, 161803.

227 Mio  $B\bar{B}$



$$\sin 2\beta = 0.722 \pm 0.040 \pm 0.023$$

## 3.5 Experimental status of the Unitarity Triangle



### Standard Model CKM mechanism confirmed

1. Large CP Violation in B decays
2. Large direct CP violation observed
3. CPV parameter related to magnitude of non-CP observables