$\boxed{5\text{-}1}$ e^+e^- B-factories

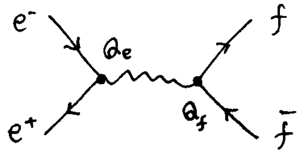
a) (Number of events) = (Cross section) × (Integrated luminosity)

$$= \sigma\,[\mathrm{cm}^2] \times L\,[\mathrm{cm}^{-2}\,\mathrm{s}^{-1}] \times T\,[\mathrm{s}]$$

$$= 1\,[\mathrm{nb}] \times 10^{34}\,[\mathrm{cm}^{-2}\mathrm{s}^{-1}] \times 10^{7}\,[\mathrm{s}]$$

$$= 10^{8}$$

※ $1\,\mathrm{nb} = 10^{-33}\,\mathrm{cm}^2$
$1\,\text{year} \sim 3\times 10^{7}\,\mathrm{s}$

b) 

$$\sigma(e^+e^-\to f\bar{f}) = \frac{1}{2s}\int d\Phi_2\, \frac{1}{4}\sum_{\text{spins}}\underbrace{\sum_{\text{colors}}}_{\times 3} \underbrace{|\mathcal{M}|^2}_{\mathcal{M}\,\propto\, Q_e Q_f}$$

$$= 3\left(\sum_{f=u,d,s,c} Q_f^2\right)\sigma(e^+e^-\to\mu^+\mu^-)$$

The cross section for $e^+e^-\to\mu^+\mu^-$:

$$\sigma(e^+e^-\to\mu^+\mu^-) = \frac{1}{2s}\int \underbrace{d\Phi_2}_{\frac{1}{8\pi}\frac{d\cos\theta}{2}\frac{d\phi}{2\pi}}\ \underbrace{\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2}_{=\,e^4(1+\cos^2\theta)}$$

$$= \frac{1}{2s}\,\frac{1}{16\pi}\,e^4\,\frac{8}{3}$$

$$= \frac{4\pi\alpha^2}{3s} \qquad \left(※\ \alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}\right)$$

$$= \frac{2.232\times 10^{-4}}{s}\,[\mathrm{GeV}^{-2}] \qquad \left(※\ 1\,\mathrm{GeV}^{-2} = 0.3893\times 10^{6}\,\mathrm{nb}\right)$$

$$= \frac{2.232\times10^{-4}\times 0.3893\times 10^{6}}{(\sqrt{s}\text{ in GeV})^2}\,[\mathrm{nb}]$$

$$= \frac{86.9}{(\sqrt{s}\text{ in GeV})^2}\,[\mathrm{nb}]$$

$$\longrightarrow\ 0.776\ \mathrm{nb}\ @\ \sqrt{s} = 10.58\,\mathrm{GeV}$$

Therefore, the cross section for ligt-quark (u,d,s,c) pairs at $\sqrt{s} = 10.58$ GeV is

$$\sigma(e^+e^-\to f\bar{f}) = 3\times\left[2\left(\tfrac{2}{3}\right)^2 + 2\left(\tfrac{1}{3}\right)^2\right]\sigma(e^+e^-\to\mu^+\mu^-)$$

$$= \frac{10}{3}\times 0.776\ \mathrm{nb}$$

$$= 2.59\ \mathrm{nb}$$

※ R-ratio :

$$R(s) = \frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)} \ \underset{\text{simple prediction}}{\longrightarrow}\ 3\times\sum_f Q_f^2$$

σ and R in e^+e^- Collisions

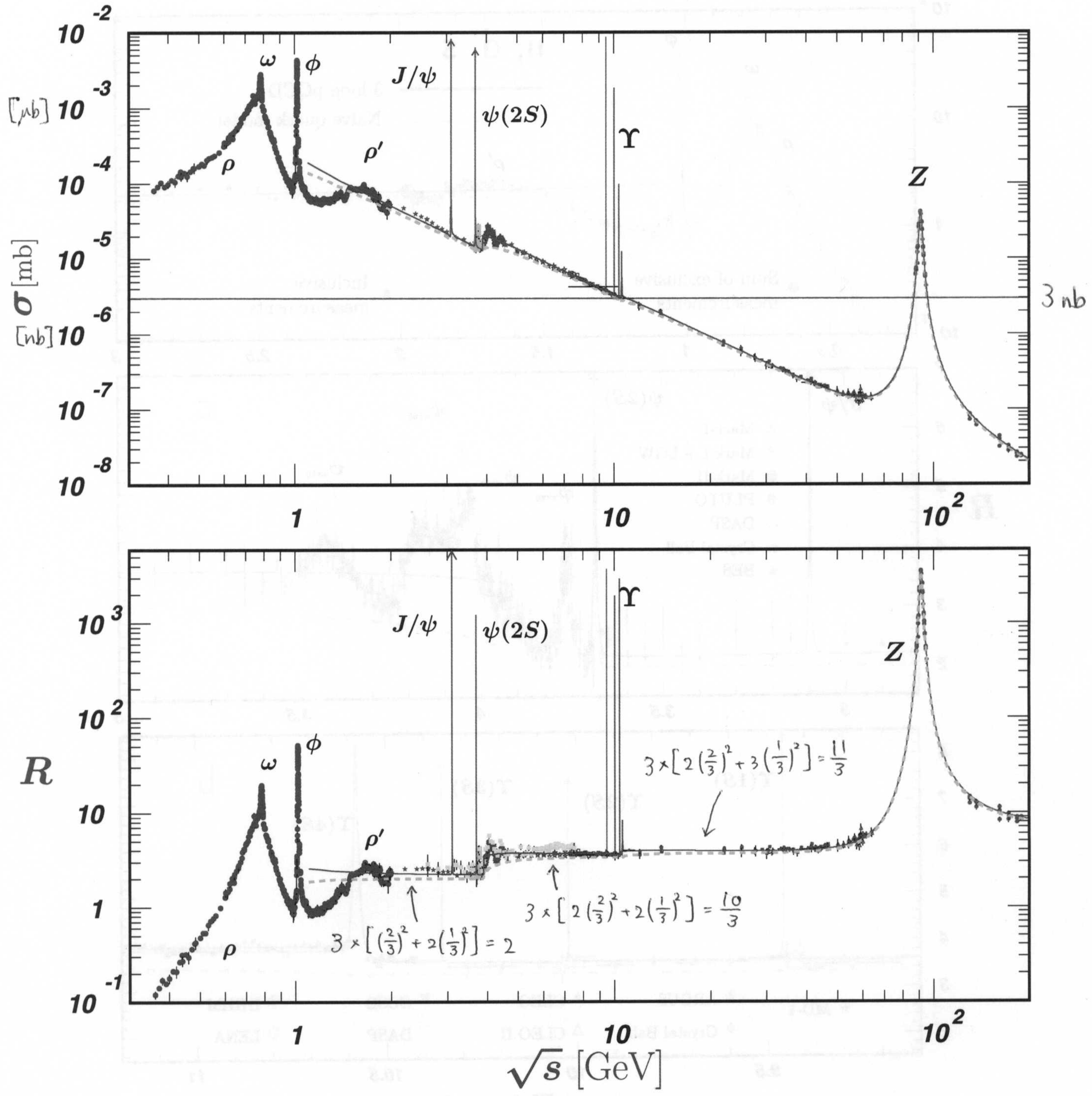
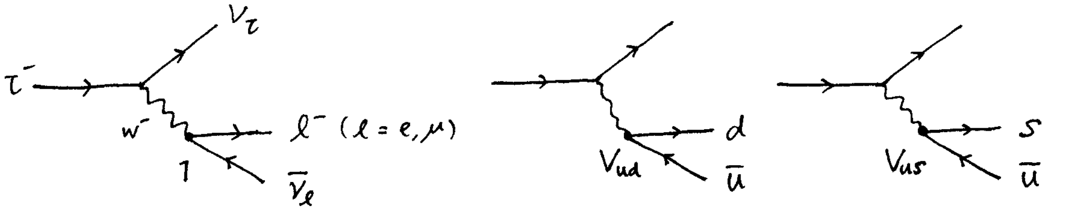


Figure 40.6: World data on the total cross section of $e^+e^- \to hadrons$ and the ratio $R(s) = \sigma(e^+e^- \to hadrons, s)/\sigma(e^+e^- \to \mu^+\mu^-, s)$. $\sigma(e^+e^- \to hadrons, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \to \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one (green) is a naive quark-parton model prediction, and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this *Review*, Eq. (9.12) or, for more details, K. G. Chetyrkin *et al.*, Nucl. Phys. **B586**, 56 (2000) (Erratum *ibid.* **B634**, 413 (2002)). Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS), n = 1,2,3,4$ are also shown. The full list of references to the original data and the details of the R ratio extraction from them can be found in `[arXiv:hep-ph/0312114]`. Corresponding computer-readable data files are available at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, August 2007. Corrections by P. Janot (CERN) and M. Schmitt (Northwestern U.))

5-2 Tau - decays

a)



$V_{ud} \sim \cos\theta_c \sim 0.975 \qquad V_{us} \sim \sin\theta_c \sim 0.22$

b) $B(\tau^- \to e^-\bar{\nu}_e \nu_\tau) : B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) : B(\tau^- \to \nu_\tau \text{ hadrons})$

$= 1^2 : 1^2 : 3 \times \{|V_{ud}|^2 + |V_{us}|^2\}$

$= 1 : 1 : 3$

color sum

c) $$\frac{1}{\tau_\mu} = \Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \left(\frac{M_\mu}{M_\tau}\right)^5 \Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$$

$$\left(※ \; \Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{G_F^2 M_\mu^5}{192\pi^3} \qquad G_F = 1.166 \times 10^{-5}\ \text{GeV}^{-2} \right)$$

Fermi constant

d) $$\tau_\tau = \frac{1}{\Gamma_\tau} = \frac{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} = B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)\, \tau_\mu \left(\frac{M_\mu}{M_\tau}\right)^5$$

$$\left(※ \; \begin{aligned} \tau_\mu &= 2.197 \times 10^{-6}\ \text{s} \\ M_\mu &= 0.1057\ \text{GeV} \\ M_\tau &= 1.777\ \text{GeV} \end{aligned} \right)$$

Therefore, the τ-lifetime is

$$\tau_\tau = B(\tau^- \to e^- \bar{\nu}_e \nu_\tau) \times 1.636 \times 10^{-12}\ \text{s}$$

$$\approx 1/5 \times 1.636 \times 10^{-12} = 3.27 \times 10^{-13}\ \text{s} \qquad \approx 2.90 \times 10^{-13}\ \text{s (exp.)}$$

$$\left(※ \; B(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = 17.8\ \% \text{ (exp.)} \right)$$