Prof Dr. Ulrich Uwer/ Dr. T. Weigand 2. Juni 2010

The Standard Model of Particle Physics - SoSe 2010 Assignment 7

Note: This examples sheet will be discussed together with sheet 6 in the tutorial on June 10, 2010.

Abschlussklausur (Final Exam): The date for the final exam is Monday, July 26, 2010, 9:15 in Großer Hörsaal, Philosophenweg 12.

1 Massive abelian gauge theories

Consider massive QED,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M^2 A_\mu A^\mu. \tag{1}$$

How many physical degrees of freedom does A^μ have? Explain this result and compare it to the number of physical degrees of freedom for massless $U(1)$ theory.

2 The non-abelian Higgs mechanism

Consider the Lagrangian of a complex scalar field Φ

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_\Phi, \tag{2}$$

$$\mathcal{L}_\Phi = \frac{1}{2}(D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi^\dagger\Phi), \qquad D_\mu\Phi = \partial_\mu\Phi + igA_{\mu\,a}T_a\Phi, \tag{3}$$

which is gauge invariant under gauge group G generated by T_a. $\mathcal{L}_{gauge}$ is the Yang-Mills Lagrangian for gauge group G.

Suppose the vacuum Φ_0 of the potential V breaks the symmetry spontaneously from G to $H \subset G$, where H is generated by t_i, and $T_{\hat{a}}$ generates the coset G/H.

a) Take $G = U(4)$ and suppose that Φ transforms in the fundamental representation of G, i.e. Φ is a four-component vector with complex entries. Assume that the symmetry is broken spontaneously to $H = U(3)$. Give the number of gauge fields associated with the original gauge group G and with the residual gauge group H. Give the number of real Goldstone modes that you would expect from Goldstone's theorem if it were not for the Higgs effect. Give also the number of massive real degrees of freedom that you expect.

b) Now take G and H general again. Make the ansatz

$$\Phi = U^{-1}(\Phi_0 + f) \qquad \text{subject to} \qquad f^\dagger T_{\hat{a}}\Phi_0 - \Phi_0^\dagger T_{\hat{a}} f = 0 \tag{4}$$

$$A^\mu_{\hat{a}} T_{\hat{a}} = U^{-1} B^\mu_{\hat{a}} T_{\hat{a}} U + \frac{i}{g}(\partial_\mu U^{-1})U, \tag{5}$$

where U is a gauge transformation $U = \exp(i\eta_{\hat{a}} T_{\hat{a}})$ associated with G/H.

Show that there exists a gauge transformation such that

$$\Phi = \Phi_0 + f, \qquad A^\mu_{\hat{a}} = B^\mu_{\hat{a}}. \tag{6}$$

c) Now go to the unitary gauge as above in (6) and extract all quadratic terms from $\mathcal{L}_{gauge} + \mathcal{L}_\Phi$. In particular you should reproduce:

- There are no cross-terms $\partial_\mu f B^\mu_{\hat{a}}$. Why is this important to extract the physical degrees of freedom?
- The gauge fields $A_{\mu\, i}$ remain massless.
- The mass matrix for the fields $B_{\mu\, \hat{a}}$ is

$$\mathcal{M}_{\hat{a}\hat{b}} = g^2 (\Phi_0^\dagger T_{\hat{a}})\,(T_{\hat{b}}\Phi_0). \tag{7}$$

How many massless scalars are there left in the theory? What can you say about the number of massive scalars?

d) The mass matrix $\mathcal{M}_{\hat{a}\hat{b}}$ is hermitian and thus has real eigenvalues and orthogonal eigenvectors of the form

$$\mathcal{M}_{\hat{a}\hat{b}}\omega_{\hat{b}} = \lambda\omega_{\hat{b}}. \tag{8}$$

Use that

$$(\sum_{\hat{a}} \omega^\dagger_{\hat{a}} \Phi_0^\dagger T_{\hat{a}})\,(\sum_{\hat{b}} T_{\hat{b}}\Phi_0\omega_{\hat{b}}) \geq 0 \tag{9}$$

in order to show that all eigenvalues of $\mathcal{M}_{\hat{a}\hat{b}}$ are strictly positive: $\lambda > 0$.