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The Standard Model of Particle Physics - SoSe 2010 Assignment 4

Note: There is no tutorial on May 13 due to Christi Himmelfahrt.

(Due: May 20, 2010)

1 Compton Scattering

Consider the Compton scattering process $e^- + \gamma \to e^- + \gamma$, or more precisely

$$|e_s^-(p)\rangle + |k, \epsilon\rangle \to |e_{s'}^-(p')\rangle + |k', \epsilon'\rangle. \tag{1}$$

Our aim is the computation of

$$\langle k', \epsilon'; e_{s'}^-(p')|S^{(2)}|e_s^-(p); k, \epsilon\rangle = i\,(2\pi)^4\delta^{(4)}(p' + k' - p - k)\,T, \tag{2}$$

where $S^{(2)}$ is the QED S-matrix expanded to order e^2.

a) First we need to annihilate the electron states with the help of Wick's theorem. Follow similar steps as the ones in the computation of $e^+e^- \to \mu^+\mu^-$ in the lecture to bring the above matrix element into the form

$$(ie)^2 \int d^4x\, d^4y\ \bar{u}_{s'}(p')\gamma^\mu\, iS_F(x - y)\,\gamma^\nu u_s(p)\, e^{-ipy+ip'x}\langle k', \epsilon'|: A_\mu(x)A_\nu(y) : |k, \epsilon\rangle, \tag{3}$$

where $iS_F(x - y)$ is the Feynman propagator of the Dirac field. Mind factors of 2.

b) Show that $\langle k', \epsilon'| : A_\mu(x)A_\nu(y) : |k, \epsilon\rangle$ splits into 2 contributions

$$(i)\ \epsilon'_\mu\epsilon_\nu e^{-iky}e^{ik'x} \qquad (ii)\ \epsilon_\mu\epsilon'_\nu e^{ik'y}e^{-ikx}. \tag{4}$$

Correspondingly we have $T = T_1 + T_2$. Compute the final expression for T_1 and for T_2. Give a graphical interpretation of the two processes in terms of Feynman diagrams.

c) Now use the Feynman rules given in the lecture and compare.

2 $e^+ + e^- \rightarrow \gamma + \gamma$

Consider the process $e^-_s(p) + e^+_{s'}(p') \rightarrow |k, \epsilon\rangle + |k', \epsilon'\rangle$ at tree-level, i.e. to order e^2.

a.) Draw the two different Feynman diagrams contributing to this process.

b.) Use the Feynman rules to write down the scattering amplitude for both processes.

3 Gamma matrix identities

Using the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ and $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ with $\gamma_5^2 = 1$, prove the following identities:

a) $\mathrm{Tr}(\gamma \cdot a_1 \, \gamma \cdot a_2) = 4 a_1 \cdot a_2$;

b) $\mathrm{Tr}(\gamma \cdot a_1 \, \gamma \cdot a_2 \ldots \gamma \cdot a_r) = 0$ if r is odd;

c) $\mathrm{Tr}(\gamma \cdot a_1 \, \gamma \cdot a_2 \, \gamma \cdot a_3 \, \gamma \cdot a_4) = \Big((a_1 \cdot a_2)(a_3 \cdot a_4) + (a_1 \cdot a_4)(a_2 \cdot a_3) - (a_1 \cdot a_3)(a_2 \cdot a_4)\Big)$;

d) $\mathrm{Tr}(\gamma_5 \, \gamma \cdot a_1 \, \gamma \cdot a_2) = 0$.

Hint: The trace is cyclic: $\mathrm{Tr}(ABC) = \mathrm{Tr}(BCA)$; sometimes insertion of $\gamma_5^2 = 1$ into a trace also helps.