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The Standard Model of Particle Physics - SoSe 2010 Assignment 12

Due July 14

1 Mixing of neutral B mesons

In case of neutral B mesons the mass eigenstates B_H and B_L are different from the flavor eigenstates B^0 and $\bar{B}^0$. The mass eigenstaes are obtained by diagonalizing the phenomenlogical Schrödinger equation[1] for the flavor states,

$$i\frac{d}{dt}\begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix} = \left(M - i\frac{\Gamma}{2}\right)\begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix}$$

where the matrices M and Γ are 2×2 hermitian matrices. For the mass eigenstates one finds:

$$|B_L\rangle = p\,|B^0\rangle + q\,\left|\bar{B}^0\right\rangle$$
$$|B_H\rangle = p\,|B^0\rangle - q\,\left|\bar{B}^0\right\rangle$$

The complex coefficients q and p obey the normalization condition $|p|^2 + |p|^2 = 1$. The time evolution of the mass states can be expressed using the eigenvalues $M_H + i\Gamma_H/2$ and $M_L + i\Gamma_L/2$,

$$|B_{H,L}(t)\rangle = e^{-i(M_{H,L} - i\Gamma_{H,L})t}\,|B_{H,L}\rangle \tag{1}$$

where $|B_{H,L}\rangle = |B_{H,L}(t=0)\rangle$ denote the mass states at t=0. The eigenvalues $M_H + i\Gamma_H/2$ and $M_L + i\Gamma_L/2$ correspond to the mass and the decay width of the heavy and light B meson states. One usually defines:

$$m = \frac{M_H + M_L}{2} = M_{11} \qquad \Gamma = \frac{\Gamma_H + \Gamma_L}{2} = \Gamma_{11}$$
$$\Delta m = M_H - M_L \qquad \Delta\Gamma = \Gamma_L - \Gamma_H$$

(The sign convention for the width difference is chosen such that the width difference is positive in the Standard Modell.)

Express the time evolution of the flavor states $|B^0(t)\rangle$ and $\left|\bar{B}^0(t)\right\rangle$) in the flavor basis. The states $|B^0(t)\rangle$ and $\left|\bar{B}^0(t)\right\rangle$ denote the time evolution of states which have been produced as pure flavor states B^0 and $\bar{B}^0$ at $t = 0$.

Give the time dependent probability to observe the B meson in its original flavor $|\langle B^0|B^0(t)\rangle|^2$ or as mixed (opposite) flavor $\left|\left\langle \bar{B}^0|B^0(t)\right\rangle\right|^2$. For simplicity, assume that the width difference $\Delta\Gamma \approx 0$.

<u>Remark:</u> For $\Delta\Gamma \neq 0$ the B meson will never reach a pure flavor state again for $t > 0$.

Show that the mixing asymmetry, defined as (unmixed - mixed)/(unmixed + mixed) is indeed equal to $\cos(\Delta m t)$.

[1] Wigner-Weisskopf approximation

2 GIM-Suppression

In the Standard Model, the mixing of neutral B mesons $B^0 \to \bar{B}^0$ is a result of box diagrams of the following kind:

The corresponding transition amplitudes are given by:

$$A(q_k, q_l) \;=\; (V_{q_k d} \;\cdot\; V^*_{q_k b})(V^*_{q_l d} \;\cdot\; V_{q_l b}) \;\cdot\; f(m_{q_k}, m_{q_k}) \;\cdot\; A_0,$$

where A_0 is a process specific constant which also considers the binding of the quarks inside the meson. The function $f(m_{q_k}, m_{q_k})$ describes the effect of the masses of the internal quarks.

Show that the mixing probability vanishes in case of equal quark masses. Hint: Exploit the unitarity of the CKM matrix.

3 Penguin decays

Give a Feynman-diagram for the flavor changing neutral current (FCNC) decay $B^0 \to K^*\gamma$. Remark: This decay is (as all FCNC decays) possible only at loop-level.