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The Standard Model of Particle Physics - SoSe 2010 Assignment 11

(Due: July 8, 2010)

1 Lepton Mixing in Models with Majorana Masses

a.) Consider a single family of isodoublet (active) neutrino ν_L supplemented by a sterile right-handed neutrino ν_R^c. Show that the full mass terms

$$\mathcal{L}_{mass} = \mathcal{L}_M + \mathcal{L}_D, \tag{1}$$

$$\mathcal{L}_D = -M_D(\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R), \qquad L_M = -M_M(\bar{\nu}_R^c \nu_R + \bar{\nu}_R \nu_R^c) \tag{2}$$

can be rewritten in terms of the Majorana spinors

$$\chi = \nu_L + (\nu_L)^c, \qquad \omega = \nu_R + (\nu_R)^c \tag{3}$$

as

$$\mathcal{L}_{mass} = -(\bar{\chi}, \bar{\omega})\, \mathcal{M} \begin{pmatrix} \chi \\ \omega \end{pmatrix}, \qquad \mathcal{M} = \begin{pmatrix} 0 & \frac{1}{2} M_D \\ \frac{1}{2} M_D & M_M \end{pmatrix}. \tag{4}$$

Deduce that for $M_D \ll M_M$ there exists one light and one heavy mass eigenstate. Discuss the necessary order of magnitude of M_M in order to generate eV-scale masses for the light neutrinos.

b.) Generalise the above Majorana masses to 3 families of ν_L and n families of ν_R^c and discuss the mixing in the charged current interaction. I.e. promote χ to the 3 vector $\vec{\chi}$ and ω to the n- vector $\vec{\omega}$ and $\mathcal{M}$ to a complex, symmetric matrix

$$\mathcal{M} = \begin{pmatrix} 0 & \frac{1}{2} M_D \\ \frac{1}{2} M_D^T & M_M \end{pmatrix}, \tag{5}$$

where M_D and M_M are suitable complex matrices and $M_M = M_M^T$. Why must the Majorana mass matrix be symmetric?

Then diagonalise

$$\mathcal{M} = \mathcal{V}^T \tilde{\mathcal{M}} \mathcal{V}, \qquad \tilde{\mathcal{M}} = \begin{pmatrix} \epsilon^2 M_1 & 0 \\ 0 & M_2 \end{pmatrix}, \qquad \mathcal{V} = \begin{pmatrix} \epsilon^2 U_{11} & \epsilon U_{12} \\ \epsilon U_{21} & U_{22} \end{pmatrix}, \tag{6}$$

where we scaled out a small parameter $\epsilon \ll 1$ for illustration of the see-saw philosophy.

Express the electroweak eigenstates in terms of $\vec{\chi}$ and $\vec{\omega}$ in terms of the above matrices. Derive the charged current and discuss the various mixings that occur. Compare to the case where only Dirac masses are present for the right-handed neutrinos. Compare the numbers of degrees of freedom of the mixing matrices in both cases that are responsible for mixing between charged leptons and the light neutrino mass eigenstates.

c.) Discuss also the uncharged neutrino current and show that there exists mixing between ν_L and ν_R^c.

2 Symmetries of Majorana masses

Exercise 3 on Examples Sheet 11 is devoted to the *continuous, global* symmetries of the electro-weak Lagrangian in absence of neutrino masses. Such non-gauged global symmetries are sometimes called **accidental** because they are not put into the theory as an organising principle. Rather they happen to be a consequence of the structure of the tree-level Lagrangian defined in terms of the

- gauge invariant
- renormalisable (mass dimension 4 such as Yukawa couplings or less)

couplings.

Redo this analysis in the theory that includes right-handed neutrinos ν_R

a) for the case of pure Dirac masses

b) for the case of Majorana masses (as in exercise 1).

3 Type II See-Saw

Consider the group $SU(N)$ and a rank n Tensor $T_{i_1,\ldots,i_n}$ transforming by multiplication with n SU(N) matrices.

a.) Recall from your favourite group theory book that this tensor representation splits into irreducible representations

$$T_{i_1,\ldots,i_n} = A_{[i_1,\ldots,i_n]} + S_{(i_1,\ldots,i_n)} \tag{7}$$

defined as the antisymmetric and symmetric representations, respectively.

b.) Now consider the Standard Model supplemented by an extra scalar field $S_{(ij)}$ transforming in the symmetric representation of $SU(2)$. This representation is called iso-triplet representation. Argue that the renormalisable coupling

$$L^T_{(i} C^{-1} L_{j)}\, S^{(ij)}, \tag{8}$$

where $SU(2)$ indices are raised and lowered with the SU(2) invariant ϵ^{ij} tensor, is $SU(2)$ gauge invariant. Give the Y charge of S for hypercharge invariance. Deduce that a suitable vacuum expectation value of S gives a Majorana mass to the neutrino. This is an example of a Type II see-saw model that does not introduce any new fermionic degrees of freedom.