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The Standard Model of Particle Physics - SoSe 2010 Assignment 10

(Due: June 30, 2010)

1 Quark interactions

Starting from the gauged kinetic terms for the left- and righthanded quarks, derive the precise form of the charged and neutral electroweak current and the electromagnetic current in the quark sector. What is the underlying reason for absence of flavour-changing neutral currents?

2 No gauge anomalies in the Standard Model

In a general QFT, the symmetries exhibited by the tree-level lagrangian may not be respected by the full quantum theory. In this case one calls the symmetry anomalous. While the anomalies of global symmetries pose no harm, anomalies of gauge symmetries render the theory inconsistent - after all gauge symmetry is needed for a consistent quantisation of the theory.

According to the Adler-Bardeen theorem, perturbative gauge anomalies can occur in chiral gauge theories and at most at one-loop level: Gauge anomalies are related to amplitudes with 3 external gauge bosons and with chiral fermions running in the loop. These potentially anomalous diagrams are called triangle diagrams.

Consider a chiral gauge theory theory with gauge group G generated by T_a and fermions in various representations R thereof. Adler-Bell-Jackiw (ABJ) showed that the triangle diagram with external gauge bosons of type i, j, k is proportional to the totally symmetric quantity

$$\mathcal{A}_{ijk} = \sum_R Tr\Big(\{T_i(R), T_j(R)\} T_k(R)\Big), \tag{1}$$

where $\{,\}$ denotes the anticommutator of matrices. Here we sum over all fermion fields, each transforming in representation R, and evaluate the trace of the matrices acting on the representations R. Furthermore, left-handed fermions in representation R are weighed with $+1$ and right-handed fermions with -1, illustrating the chiral character of the potential anomaly.

Consider now the electro-weak $SU(2) \times U(1)_Y$ theory. There are potential triangle diagrams with only SU(2) bosons - the $SU(2)^3$ anomaly - diagrams with one $U(1)_Y$ boson - the mixed $U(1)_Y - SU(2)^2$ anomalies - and analogously the $U(1)_Y^2 - SU(2)$ and the $U(1)_Y^3$ anomaly.

Show by explicit evaluation of the expression (1) for the full quark and lepton spectrum of the Standard Model that all these anomalies vanish provided each quark is weighed by an extra factor of 3. Interpret this extra factor!

3 Global symmetries of the electro-weak lagrangian

Consider the lagrangian of quarks and leptons in the electro-weak theory (with 3 families) without Higgs-sector and without the Yukawa couplings. This symmetry exhibits a large *global, continuous* family symmetry group beyond the gauged $SU(2) \times U(1)_Y$ symmetry. Determine this symmetry group. Now take into account also the Yukawa couplings (no right-handed neutrinos.) Which remnant of the original global symmetry group which is respected also by the Yukawa couplings?

4 Charge conjugation and CP violation

To determine the action of charge conjugation on Dirac spinors we make the ansatz

$$\psi(x) \to \eta_C \psi^C(x), \qquad \psi^C = C\bar{\psi}(x)^T \tag{2}$$

for some matrix C. This matrix must be such that if $\psi(x)$ satisfies the Dirac equation, so does $\psi^C(x)$.

a.) Show that this is the case as long as

$$C(\gamma^\mu)^T C^{-1} = -\gamma^\mu. \tag{3}$$

b.) Use this to show that

$$C[\gamma^\mu, \gamma^\nu]^T C^{-1} = -[\gamma^\mu, \gamma^\nu], \qquad C\gamma_5^T C^{-1} = \gamma_5, \qquad C(\gamma^\mu\gamma_5)^T C^{-1} = \gamma^\mu\gamma_5. \tag{4}$$

Show also that

$$(C^{-1})^T \gamma^\mu C^T = C^{-1}\gamma^\mu C \qquad \gamma^\mu C^T C^{-1} = C^T C^{-1}\gamma^\mu, \tag{5}$$

and similarly for the remaining γ-combinations appearing in (4). Deduce that $C^T C^{-1} \propto 1$.

Hint: Use that the 16 matrices $\Gamma^A \in \{\gamma^\mu, [\gamma^\mu, \gamma^\nu], \gamma, \gamma^\mu\gamma^5, 1\}$ form a basis of 4×4 matrices.

Finally argue that

$$C^T = -C \tag{6}$$

by requiring that $\Gamma^A C$ yields 10 symmetric and 6 anti-symmetric matrices.

c.) By similar manipulations one can show that

$$C^\dagger = C^{-1}. \tag{7}$$

Convince yourself that the representation

$$C = i\gamma^0\gamma^2 \tag{8}$$

satisfies the two defining conditions (6) and (7).

d.) Convince yourself that the transformation (2) indeed exchanges particle and anti-particle excitations (up to phases) by analysing its action on the mode expansion of $\psi(x)$. You may use (or show) that the elementary spinors in the expansion are related via $v(p, \lambda) = C\bar{u}(p, \lambda)^T$ for the matric C above.

e.) Now consider the charged electro-weak current in the Standard Model. Derive its transformation under the successive application of parity and charge conjugation and deduce that the presence of the CKM matrix leads to CP violation.