Problem sheet 9 - Physics V - WS 2006/2007

Due: December 21/22, 2006

Problem 9.1 Form factors (50P)

The mean-square radius for a nucleus with A nucleons is given by $R_{\mathrm{ms}} = \sqrt{\langle r^2 \rangle} \approx 0.94\ \mathrm{fm} \cdot \sqrt[3]{A}$. For a homogeneous spherical charge density (sharp edges) one obtains $R_{\mathrm{K}} = 1.21\ \mathrm{fm} \cdot \sqrt[3]{A}$.

a) Electrons with an energy of 750 MeV are scattered off (spinless) ^{40}Ca nuclei. Give the differential cross-section $\mathrm{d}\sigma/\mathrm{s}\Omega$ of this reaction as a function of the scattering angle θ for the following assumptions:

   (i) scattering of spinless electrons off pointlike nuclei,

   (ii) scattering of spinless electrons off spherical nuclei with a constant charge density (for the radius of the sphere use R_{K}).

   (iii) scattering of spin $^1/_2$ electrons off pointlike nuclei,

   (iv) scattering of spin $^1/_2$ electrons off nuclei with a Gaussian charge density $f(r) = (a^2/(2\pi))^{3/2} \cdot \exp(-a^2r^2/2)$ with $a = 1/R_{\mathrm{ms}}$,

   (v) scattering of spin $^1/_2$ electrons off spherical nuclei with a constant charge density (for the radius of the sphere use R_{K}).

b) Plot the cross-sections obtained in a) separately for (i) to (v), as a function of θ. Use an algebra program like Mathematica or Maple. For the abscissa use $5° < \theta < 60°$, for the ordinate use units of cm^2 and a logarithmic scale. In a 6th plot show all cross-sections in a single diagram ($5° < \theta < 60°$, $10^{-34} < (\mathrm{d}\sigma/\mathrm{d}\Omega) < 10^{-23}$ cm^2, use a logarithmic scale for the ordinate). Which of the predicted cross-sections describes the data best (see figure 1)? Explain why?

c) In addition to the elastic electron-^{40}Ca scattering cross-section, the cross-section for elastic scattering of electrons off ^{48}Ca is shown in figure 1. Estimate the radius of the ^{48}Ca nucleus from the angle of the first minimum. Compare with the expected mean-square radius and R_{K}! The energy of the electrons is again 750 MeV.

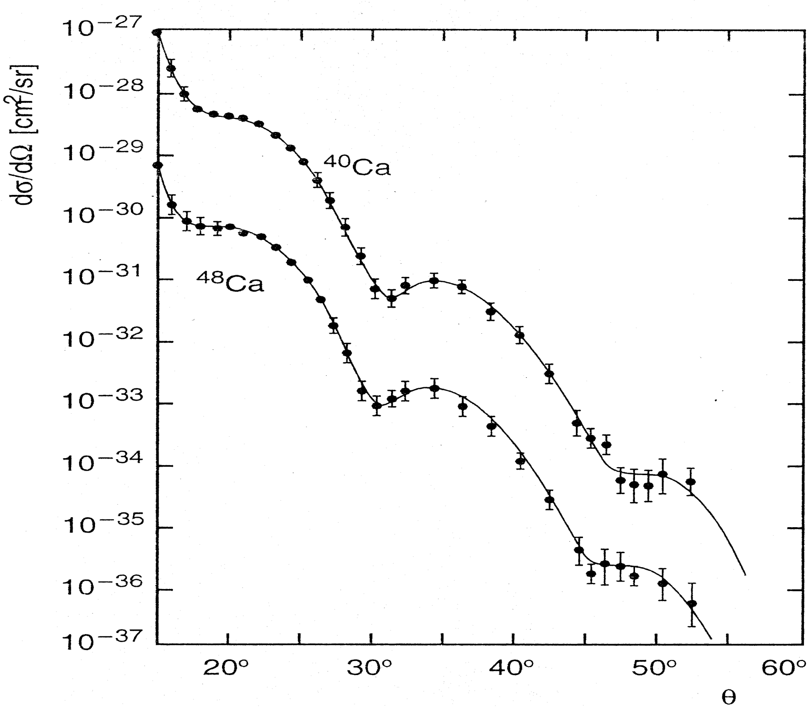

Figure 1: Differential cross-section $\mathrm{d}\sigma/\mathrm{d}\Omega$ of the elastic scattering of electrons off ^{40}Ca and ^{48}Ca nuclei as a function of the scattering angle θ. The cross-section for ^{40}Ca has been scaled up by a factor of 10; the one for ^{48}Ca has been multiplied by a factor of 0.1.

Problem 9.2 Deep-inelastic scattering – structure functions (50P)

The proton structure as measured in deep inelastic electron scattering off protons is described by the structure function $F_2(x)$. $F_2(x)$ can be expressed in terms of the quark density distributions as follows:

$$F_2^{ep}(x) = x\left(\frac{4}{9}\left(u(x)+\overline{u}(x)\right)+\frac{1}{9}\left(d(x)+\overline{d}(x)\right)+\frac{1}{9}\left(s(x)+\overline{s}(x)\right)\right) ,$$

where $u(x)$ is the u-quark density and x is the fraction of the proton momentum carried by the u-quarks. $\overline{u}(x)$ is the density of the anti-u-($\overline{u}$-)quarks. $d(x)$ ($\overline{d}(x)$) and $s(x)$ ($\overline{s}(x)$) denote the corresponding densities for the (anti-)u- and (anti-)s-quarks. Contributions of heavier quarks are neglected.

a) Rewrite the above equation in terms of valence (u_v and d_v) and sea quark distributions assuming that the sea quark distributions are the same for all quark and antiquark flavors ($u_\mathrm{s}(x) = \overline{u}_\mathrm{s}(x) = d_\mathrm{s}(x) = \overline{d}_\mathrm{s}(x) = s_\mathrm{s}(x) = \overline{s}_\mathrm{s}(x) =: q_\mathrm{s}(x)$). In other words, write $F_2^{ep}(x)$ as a combination of $u_\mathrm{v}(x)$, $d_\mathrm{v}(x)$ and $q_\mathrm{s}(x)$ only.

b) Express the structure function $F_2^{en}(x)$ for the elastic electron scattering off neutrons in terms of the quark density functions for the proton ($u(x)$, $\overline{u}(x)$, $d(x)$, $\overline{d}(x)$, $s(x)$ and $\overline{s}(x)$) and, in a second step, in terms of the valence and sea quark distributions of the proton ($u_\mathrm{v}(x)$, $d_\mathrm{v}(x)$ and $q_\mathrm{s}(x)$). Hint: Think of the isospin symmetry of proton and neutron.

c) Express the structure function $F_2^{eN}(x)$ for deep inelastic electron scattering off a isoscalar target (nucleus with same number of protons and neutrons) in terms of the valence and sea quark distributions of the proton ($u_\mathrm{v}(x)$, $d_\mathrm{v}(x)$ and $q_\mathrm{s}(x)$).

d) From deep inelastic electron scattering off protons and neutrons one can obtain $F_2^{ep}(x)$ and $F_2^{en}(x)$. Use the experimental results

$$\int_0^1 F_2^{ep}(x)\,\mathrm{d}x = 0{,}18 , \qquad \int_0^1 F_2^{en}(x)\,\mathrm{d}x = 0{,}12$$

to determine the integrals

$$\int_0^1 x u_\mathrm{v}(x)\,\mathrm{d}x , \qquad \int_0^1 x d_\mathrm{v}(x)\,\mathrm{d}x .$$

For this neglect the sea quark content in $F_2^{ep}(x)$ and $F_2^{en}(x)$. What does the result imply concerning the number of u and d valence quarks in the proton assuming that the mean momentum fraction carried by an u valence quark is the same as that of a d valence quark?

e) Using the results from part d) determine the fraction of the proton momentum carried by the valence quarks!