Name: **Group:**

Problem Set 6, Physics V, WS 06/07

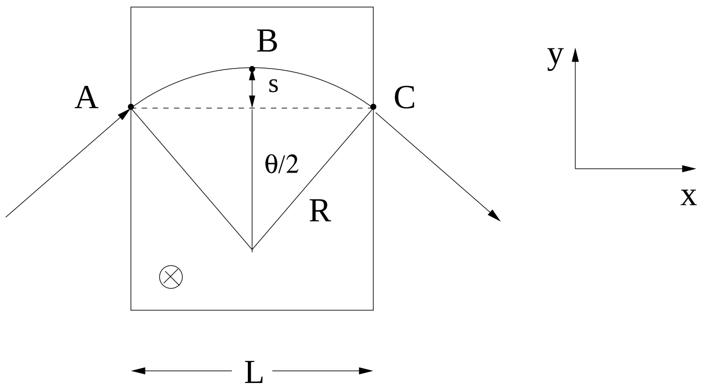
Due November 30 / December 1, 2005

Problem 6.1 (25 Points) Momentum Measurement

The momentum of a charged particle can be determined in a homogeneous magnetic field B of length L (see sketch).

a) Derive an expression for the radius of curvature R as a function of the particle's momentum p_t transverse to the magnetic field. Express the sagitta s of the curved track segment as a function of R, assuming that the angle of deflection θ is small, i.e. $L/R \ll 1$.

b) The trajectory of the particle will be measured at three equidistant points A, B, and C with equal resolution in the y-direction σ_y, from which the sagitta can be determined. Assume that the three measurements are uncorrelated. Determine the relative momentum resolution σ_{p_t}/p_t obtained from this measurement.
Hint: Note that one endpoint of the sagitta is determined by the measurement at point B, the other one by two measurements at points A and C.
(numerical example: $p_t = 1$ GeV, $L = 1$m, $B = 1$T, and $\sigma_y = 200\mu$m).



Problem 6.2 (25 Points) Sampling Calorimeter

The relative energy resolution $\sigma(E)$ of a calorimeter depends on statistical fluctuations of the number of detected photons ($\propto 1/\sqrt{E}$), calibration (energy independent) as well as on readout noise ($\propto 1/E$). Let us ignore readout noise and consider a specific calorimeter for which the relative energy resolution can be parametrized as (these values can be reached with a $Bi_4Ge_3O_{12}$ (BGO) calorimeter)

$$\sigma(E)/E = \sqrt{(0.02/\sqrt{E})^2 + (0.0075)^2}, \qquad \text{with } E \text{ in GeV.}$$

a) Consider the decay $\pi^0 \to \gamma\gamma$. If the energies E_1, E_2 and the opening angle θ between the decay photons can be measured in the calorimeter, the invariant mass M of the mother particle can be reconstructed.
Calculate the relative resolution of the invariant mass σ_M/M of the two photons. Neglect at first the error on the opening angle.
Compute the relative mass resolution numerically for the above calorimeter, first for the symmetric case with both photons having an energy of 1.2 GeV, and then for the asymmetric case with $E_1 = 1.2$GeV and $E_2 = 0.4$GeV.

b) Now do not neglect the error on the opening angle. Discuss which terms in the energy resolution dominate at high photon energies.

Problem 6.3 (25 Points) Addition of Angular Momentum

Baryons are three-quark-states (qqq) with quarks having half integer spin $s_{\mathrm{q}} = 1/2$. Write down all possible spin states (total spin S and its third component S_z) for the baryons for the case that all relative orbital angular momenta are zero $l = 0$.
Hint: Add at first the spins of two quarks and note the quantum mechanical rules for the addition of angular momenta.

Problem 6.4 (25 Points) Spin-Statistics

The neutral $\rho(770)$ meson (spin S_ρ= 1) decays into two charged pions (spin $S_\pi = 0$) $\rho^0 \rightarrow \pi^+\pi^-$ with a branching ratio BR $\approx$ 100%. The decay $\rho^0 \rightarrow \pi^0\pi^0$ is not observed. Show that the decay $\rho^0 \rightarrow \pi^0\pi^0$ is indeed forbidden.
Hint: The π^0s in the final state are identical particles and obey Bose statistics, i.e. the final state wave funtion has to be symmetric under the exchange of the particles.