Name: **Gruppe:**

Problem sheet 12 – Physics V – WS 2006/2007

Due: January 25/26, 2007 (in your group)

12.1 Phenomenology of flavor oscillation (40 Points)

It is known that the properties of some weakly interacting neutral particles vary with time. For instance, a kaon, originally produced as a $\overline{K^0}$, oscillates into a K^0 and back. This mixing appears because K^0 and $\overline{K^0}$ are no eigenstates of the weak interaction. They are combinations of two weak eigenstates K_1^0 and K_2^0 (the small CP violation in the kaon system is ignored):

$$|K^0(t=0)>=1/\sqrt{2}\cdot(|K_1^0(t=0)>+|K_2^0(t=0)>) \qquad \overline{|K^0(t=0)}>=1/\sqrt{2}\cdot(|K_1^0(t=0)>-|K_2^0(t=0)>)$$

. The time dependence of K_1^0, K_2^0 is given by

$$|K_1^0(t)>=|K_1^0(t=0)>\cdot e^{-im_1t-\frac{1}{2}\Gamma_1 t} \qquad |K_2^0(t)>=|K_2^0(t=0)>\cdot e^{-im_2t-\frac{1}{2}\Gamma_2 t}$$

where the m_i are the masses and Γ_i are the decay widths.

a) Write down the time dependence of the wave function of an initial $\overline{K^0}$ beam.

b) Because the strong interaction properties of K_0 and $\overline{K_0}$ differ ($K^0 p \to \Lambda\pi^+$ is allowed, $\overline{K^0} p \to \Lambda\pi^+$ is forbidden) it is possible to measure the probability $|<K_0(t=0)|\overline{K_0(t)}>|^2$ that an original $\overline{K_0}$ converted into a K_0. Compute this probability and show that it oscillates with a frequency proportional to $m_1 - m_2$.

c) The mean lifetimes are $0.895\cdot 10^{-10}$s for K_1^0 and $5.1\cdot 10^{-8}$s for $\overline{K_2^0}$. The time period of the oscillation, divided by 2π, is $1.9\cdot 10^{-10}$s. Compute the ratios of $\Gamma_1-\Gamma_2$ and m_1-m_2 to the mean mass $1/2\cdot(m_1+m_2) =$ 498MeV. The differences beween K_1^0 and K_2^0 are due to the breaking of a discrete symmetry. Which one ?

12.2 Z decays (35 Points)

The results for Z boson decays are corner stones of the present standard model. The partial decay width for the Z decay into a fermion antifermion pair is given by

$$\Gamma_f = \frac{G_F m_Z^3}{3\pi\sqrt{2}}\cdot\left(\left(g_L^f\right)^2+\left(g_R^f\right)^2\right)$$

for leptons and

$$\Gamma_f = \frac{G_F m_Z^3}{3\pi\sqrt{2}}\cdot N_C\cdot\left(\left(g_L^f\right)^2+\left(g_R^f\right)^2\right)\cdot\left(1+\frac{\alpha_s(m_Z^2)}{\pi}\right).$$

for quarks, where $N_C=3$ and $\alpha_S(m_Z^2)=0.121$. The couplings g_L^f, g_R^f were given in the lectures. They depend on the weak mixing angle and are different for different fermion types (neutrinos, charged leptons, up-type or down-type quarks).

a) Compute, for a weak mixing angle $\sin^2\Theta_W = 0.231$, the relative sizes of the decay widths for the allowed final fermion antifermion states.

b) Compute the branching ratios for the following decays and compare them with the experimental results (in brackets): Z$\to$ e^-e^+ (3.36%), Z$\to$ $\mu^-\mu^+$ (3.37%), Z$\to$ $\tau^-\tau^+$ (3.37%), Z$\to$ $b\overline{b}$ pair (15.1%), Z$\to$ $c\overline{c}$ pair (12.0%), Z$\to$ arbitrary quark antiquark pair (69.9%). (Note: If electroweak radiative corrections and higher order QCD corrections are taken into account, the agreement becomes even better).

c) Assume that, in addition to the known decay products, a fourth massless neutrino antitineutrino generation is produced. Compute for a fixed weak mixing angle the relative changes of the total decay width and the branching ratios due to this additional particle production.

12.3 Neutrinos from the sun (25 Points)

The Sudbury Neutrino Observatory (SNO) uses a heavy water (D_2O) tank equipped with photomultipliers to detect neutrinos that have been produced in the sun. It allows to measure the electron neutrino flux and the total neutrino flux.

a) Draw the Feynman graphs (on quark level) of the possible neutrino interactions between the 3 neutrino species and the SNO heavy water. Assume that there are no antineutrinos produced in the sun.

b) Consider that the energy of the neturinos created in the fusion process of the sun are of the order of a few MeV ($\leq$ 20 MeV). Which of the reactions of a) are kinematically possible for neutrinos coming from the sun.

c) Which reactions allow to measure the electron neutrino flux and the total neutrino flux used by SNO.

For further informations about the SNO experiment consult *http://www.sno.phy.queensu.ca/*