Problem sheet 11 - Physics V - WS 2006/2007

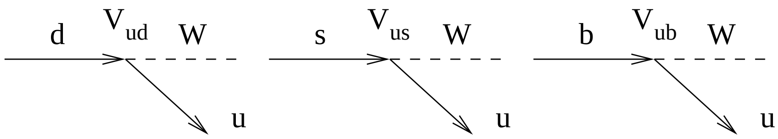
Due: 18 January 2007

Problem 11.1 CKM Matrix (30P)

The CKM Matrix V_{CKM} is a SU(3) matrix (complex 3x3 matrix with $V \cdot V^\dagger = 1$ and det(V)=1) which relates the mass eigenstate of the (d, s, b) quarks to the states which couple to the W bosons (weak eigenstates) (d', s', b').

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The W boson couples for example to a u and a $d' = V_{ud}d + V_{us}s + V_{ub}b$. The coupling strength to e.g. the (u, d) quarks (mass eigenstates) is therefore modified by the factor V_{ud} and accordingly for the s and b quarks.



a) (30P) In 1973, the b and t quarks had not been discovered and V_{CKM} was only a SU(2) matrix. Show that in this case, V_{CKM} can be chosen to be real, i.e. V_{CKM} is a simple rotation by an angle called the Cabbibo angle. Use in the proof that the phase between two quarks is arbitrary, i.e. measurement results do not change if e.g. $d \to d \cdot e^{i\,\phi}$ for any angle ϕ.

Hint: Start with the number of real parameters for a general complex 2x2 matrix and show that after all constraints only one parameter is left. A change of the $d(u)$ quark phase is equivalent to the multiplication of the first column (row) of V_{CKM} with $e^{i\,\phi}$.

b) (voluntary part!) Show in the same way that for three quark families, 4 real parameters are needed and that V_{CKM} is in general complex.

Note: The coupling strength of W bosons to anti-particles is given by V^*_{xy}, i.e. it is the complex conjugate of the one for particles. Consequently, weak interactions are in general different for particles and anti-particles. This is the only source of CP violation in the Standard Model.

Problem 11.2 Cabbibo suppressed decay (30P)

Draw the Feynman diagrams for the weak decay (via W exchange) of $D^0 = |c\bar{u}> \to K^-\pi^+$ and for $D^0 \to K^+\pi^-$. Estimate the ratio of the two branching ratios.

For an estimate of the size of V_{CKM} matrix elements use the approximation by Wolfenstein. This is an approximation of V_{CKM} in powers of $\lambda \sim \sin\theta_c$ (θ_c is the Cabbibo angle) which works very well:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ \lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

with $\lambda = 0.23$, $A = 0.82$, $\rho = 0.22$ and $\eta = 0.34$. Note: V_{CKM} has to $O(\lambda^3)$ a 2x2 unitary sub-matrix and only the smallest off-diagonal elements are complex. Hence CP violation in K_L decays is small.

Problem 11.3 Tau lepton decay (40P)

Tau leptons decay to both (light) leptons and hadrons and therefore present an excellent system for studying properties of weak and strong interactions. Note, that the lightest charmed hadron (D meson) has a mass of 1.865 GeV.

a) Draw Feyman graphs for all possible τ^- decays at the parton level (no QCD contributions like gluon bremsstrahlung, quarks are considered as free final state particles). Add the coupling strengths described by the CKM matrix elements V_{ud} and V_{us} to the quark vertices in each graph.

b) Neglecting the mass of the decay products, estimate the τ^- branching fractions to electon, muon and hadrons. For the size of V_{CKM} matrix elements use above mentioned Wolfenstein parameterization.

Hint: If masses are neglected and the number of final state partons is the same, what can you conclude on the phase space factors? How do the quark contributions with their different coupling strength in a) add to the total hadronic decay width of the tau? Do not forget the different color charges.

c) Find an equation which relates $\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$ to the muon lifetime.

Hint: Use the Sarget rule which states that for the weak decay of a particle of mass M into 3 particles of negligible mass the (partial) decay width is $\Gamma \propto M^5$.

d) Express the τ^- lifetime as a function of the partial decay width $\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$. Use the result of c) and the muon lifetime to estimate the τ lifetime. Compare your results with the experimental value listed by the PDG.