4. Neutrino Oscillations

For massive neutrinos one could introduce in analogy to the quark mixing a mixing matrix describing the relation between mass and flavor states:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 3} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Pontecorvo-Maki-Nakagawa-Sakata matrix

$$\underbrace{\nu_e = U_{e1}\nu_1 + U_{e2}\nu_2 + U_{e3}\nu_3}$$

Constant for massless ν: mixing is question of convention

Massive neutrinos develop differently in time.

$$|\nu_i(t)\rangle = |\nu_i(0)\rangle e^{-iE_i t} = |\nu_i(0)\rangle e^{-i(p_i + \frac{m_i^2}{2p_i})}$$

for masses $m_i << E_i$:

$$E_i = \sqrt{p^2 + m_i^2} = p_i + \frac{m_i^2}{2p_i}$$

→ there will be a mixing of the flavor states with time.

$$|\nu(t)\rangle_\alpha = \sum_i U_{\alpha i} e^{-iE_i t} |\nu_i(0)\rangle = \sum_{i,\beta} U_{\alpha i} U^*_{\beta i} e^{-iE_i t} |\nu_\beta\rangle$$

4.1 Two-Flavor mixing (for simplicity)

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Definite momentum p; same for all mass eigenstate components

$$E_i = \sqrt{p^2 + m_i^2} = p + \frac{m_i^2}{2p}$$

$$E_2 - E_1 = \frac{m_1^2 - m_2^2}{2p} \approx \frac{\Delta m^2}{2E}$$

(assuming p_i is the same)

$t = L/\beta$ w/ $\beta \approx 1$:

$$(E_2 - E_1)t = \frac{\Delta m^2}{2E} L$$

Time development for an initially pure $|\nu_\alpha>$ beam:

$$|\nu_\alpha(t)\rangle = \cos\theta\, e^{-iE_1 t}|\nu_1\rangle + \sin\theta\, e^{-iE_2 t}|\nu_2\rangle$$

$$= \left[\cos^2\theta\, e^{-iE_1 t} + \sin^2\theta\, e^{-iE_2 t}\right] \cdot |\nu_\alpha\rangle$$

$$+ \left[\cos\theta \sin\theta (e^{-iE_1 t} - e^{-iE_2 t})\right] \cdot |\nu_\beta\rangle$$

Mixing probability:

$$P(\nu_\alpha \to \nu_\beta, t) = \left|\langle \nu_\beta | \nu_\alpha(t)\rangle\right|^2 = 2(\cos\theta \sin\theta)^2 \left[1 - \cos^2 \frac{E_2 - E_1}{2} t\right]$$

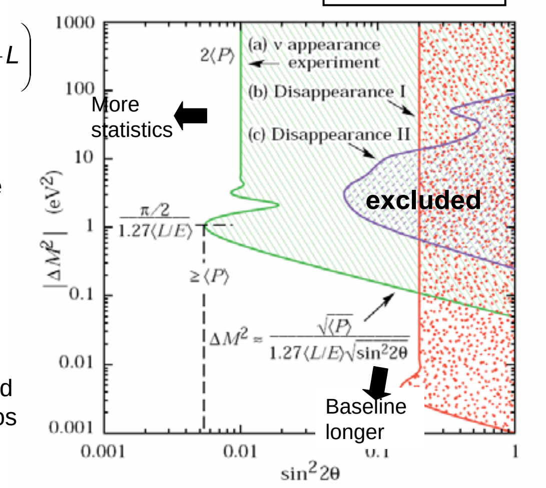
$$P(\nu_\alpha \to \nu_\beta, t) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E} L\right) = \sin^2 2\theta \sin^2\left(\frac{1.27 \cdot \Delta m^2[eV]}{4E[GeV]} L[km]\right)$$

Search for Neutrino Oscillations (PDG 1996)

Exclusion plots

$$P(\nu_\alpha \to \nu_\beta, t) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E} L\right)$$

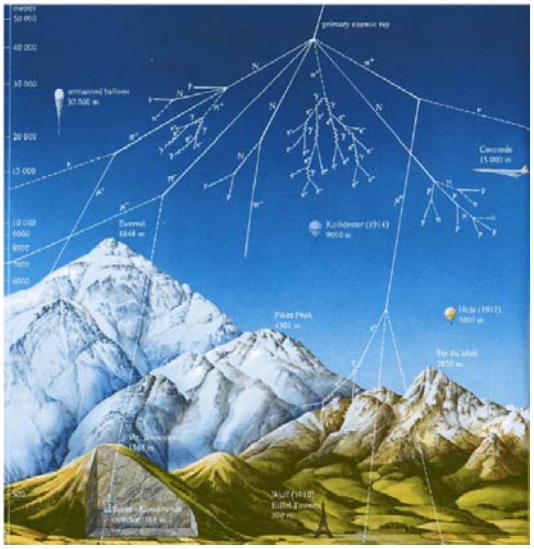
- Disappearance:
  (I) With known neutrino flux: Measurement of flux at distance L: reactor experiments,
  (II) Measure neutrino flux at position 1 and verify flux after distance L.
- Appearance:
  Use neutrino beam of type A and search at distance L for neutrinos of type B.



Observation of Neutrino Oscillations

Neutrino source	Experiment	Comments
Solar neutrinos	Radio-chemical exp.: Homestake Cl exp., GALLEX, SAGE,	First observation of “neutrino disappearance” dates more than 20 years ago: “Solar neutrino problem”
	Water experiments: (Super)Kamiokande, IMB	Confirm disappearance of solar neutrinos
	Water++: SNO	Ultimate “solar neutrino experiment”: proves the oscillation of solar ν
Atmospheric neutrinos	(Super)Kamiokande	Oscillation signal
Accelerator	LSDN Not confirmed	Much disputed signal
	K2K	Clear disappearance signal
Reactor	KamLAND, CHOOZ	Clear disappearance signal

4.2 Atmospheric neutrino problem



Cosmic radiation: Air shower

$$p + N \to \pi^{\pm}, K^{\pm}$$
$$\pi^{\pm}, K^{\pm} \to \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$$
$$\mu^{\pm} \to e^{\pm} + \nu_e(\bar{\nu}_e) + \bar{\nu}_{\mu}(\nu_{\mu})$$

$$R = \frac{\nu_{\mu} + \bar{\nu}_{\mu}}{\nu_e + \bar{\nu}_e} = 2$$

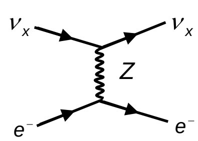
Exact calculation: R=2.1 (E_{ν}<1GeV)

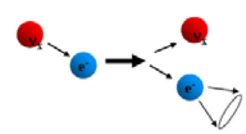
(For larger energies R>2.1)

Neutrino detection with water detectors [E_{ν}~O(GeV)]

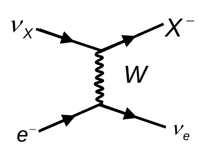
Water = "active target"

Elastic scattering





Charged current



Kinematical limit for ν_{μ}: $E_{\nu}>m_{\mu}$

Detection of Cherenkov photons: Photo multiplier

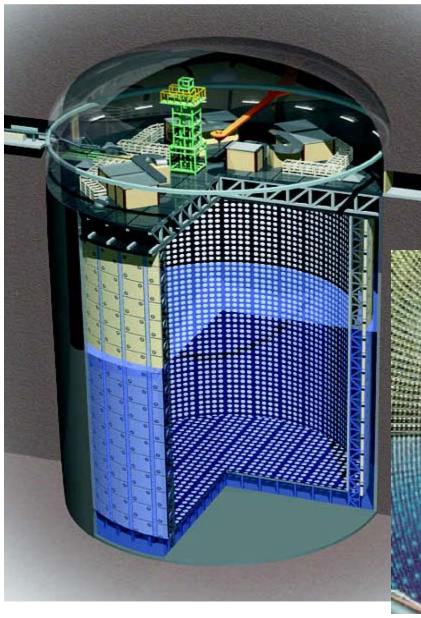
Experiments:

(Super)-Kamiokande

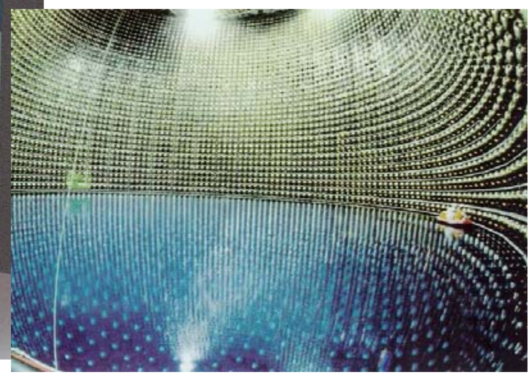
IMB

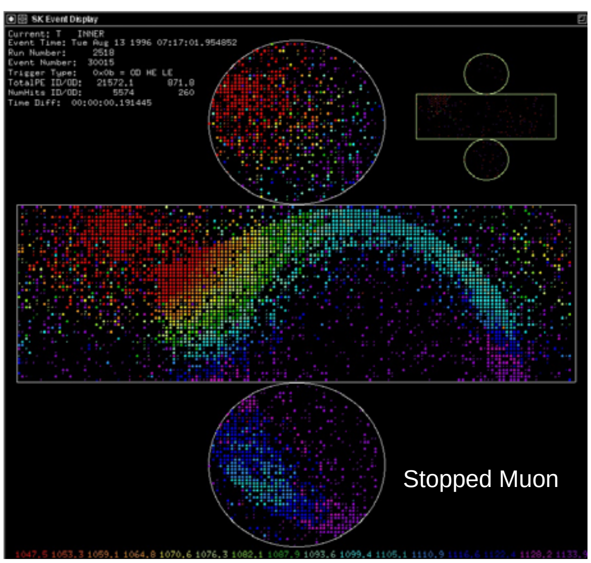
Soudan-2

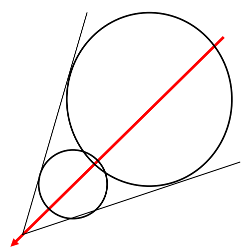
Super-Kamiokande



- Largest artificial water detector (50 kt)
- Until the 2001 accident:
  11000 PMTs (50 cm tubes!): 40% of surface covered with photo-cathode
- Back in operation since 2003







Cherenkov cone:

$$\cos\theta = \frac{1}{\beta n}$$

$$\Leftrightarrow \theta = 42^o \ (\beta = 1)$$

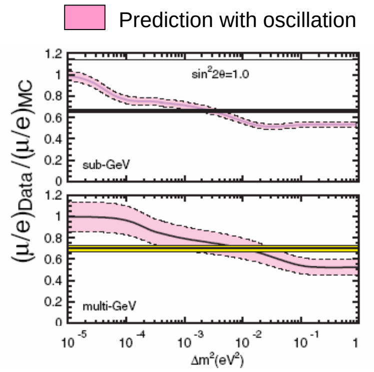
Experiment can distinguish electron and muon events, can measure energy

Ratio of muon to electron neutrinos

$$R \equiv \frac{(\mu/e)_{DATA}}{(\mu/e)_{M.C.}}$$

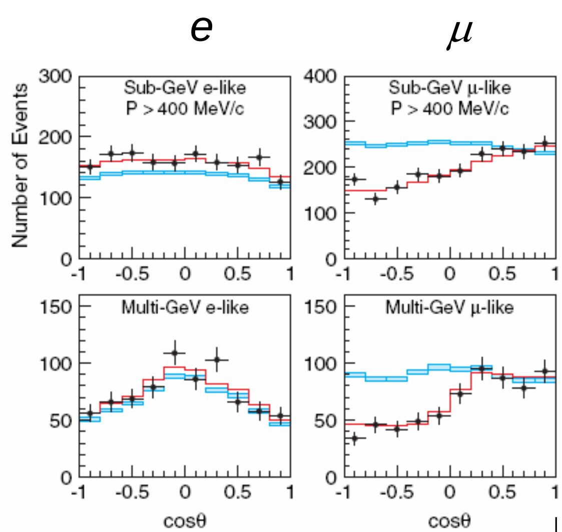
$$R_{sub-GeV} = 0.658 \pm 0.016(stat) \pm 0.032(sys)$$

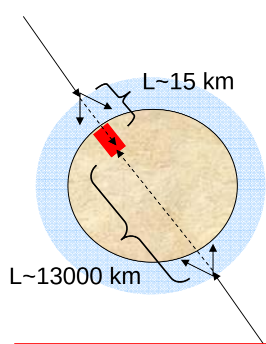
$$R_{multi-GeV} = 0.702^{+0.032}_{-0.030}(stat) \pm 0.099(sys)$$

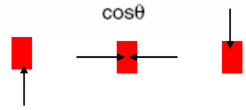


- Too few muon neutrinos observed
- Can be explained by oscillation.

Zenith angle dependence of the neutrino flux





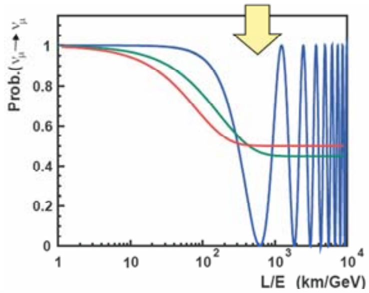


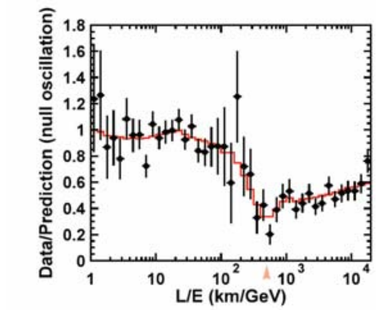
ν_μ deficit depends on angle

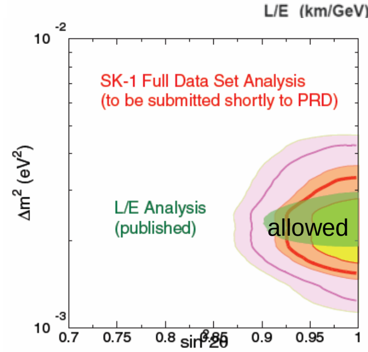
ν_e flux okay

Oscillation: $\nu_\mu \leftrightarrow \nu_\tau$

Oscillation pattern of atmospheric neutrinos







Neutrino 2004

oscillation dip seen at ~500 km/GeV

$\nu_\mu \leftrightarrow \nu_\tau$ mixing of atmos. neutrinos

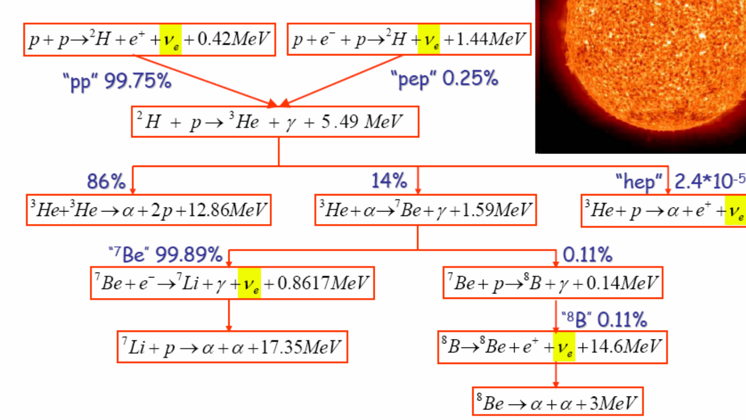
$$\Delta m^2 = (2.4 \pm 0.4) \times 10^{-3}\,\text{eV}$$

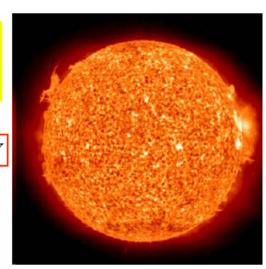
$$\sin^2 2\theta > 0.92 \;@\, 90\%\; C.L.$$

4.3 Solar neutrino problem

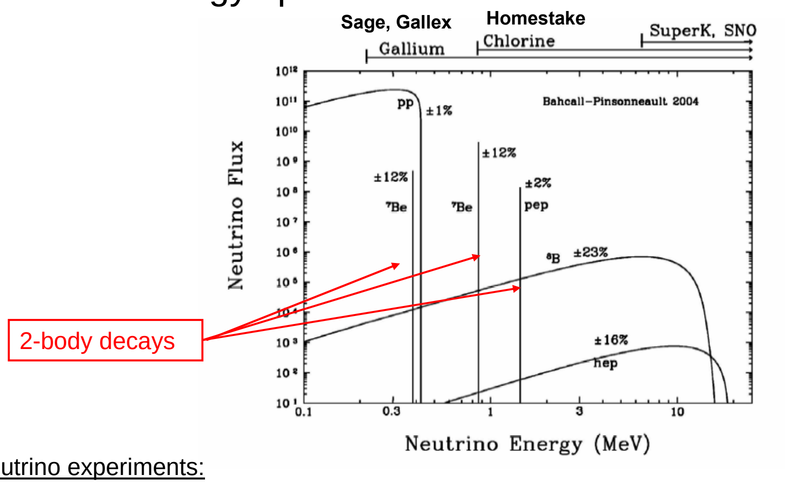
Neutrino production

ν_e are abundant by-products of nuclear fusion in the sun





Neutrino energy spectrum



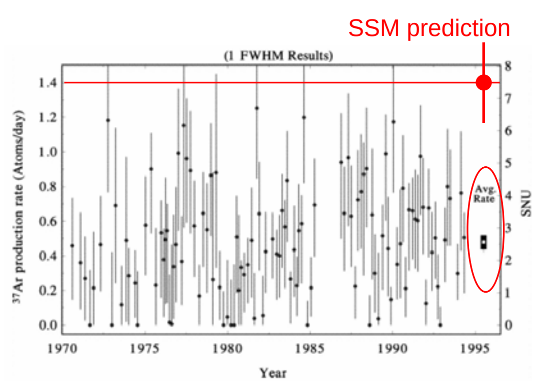
<u>Neutrino experiments:</u>

Cl_2 detectors	$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e$, ${}^{37}Ar \rightarrow {}^{37}Cl$ (EC)	$E_\nu > 0.8$ MeV
Ga detectors	$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e$	$E_\nu > 0.2$ MeV
H_2O detectors	Elastic scattering: $\nu_e + e \rightarrow \nu_e + e$	$E_\nu > 5$ MeV (detection)

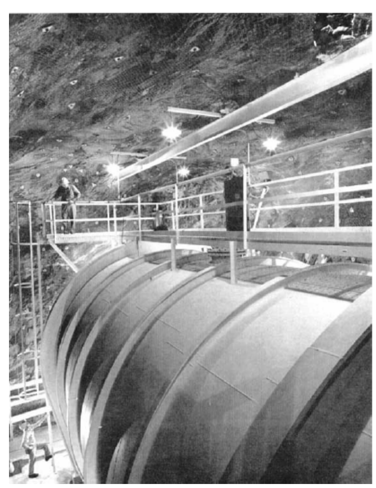
Radio-chemical experiments:

Homestake, SAGE, GALLEX

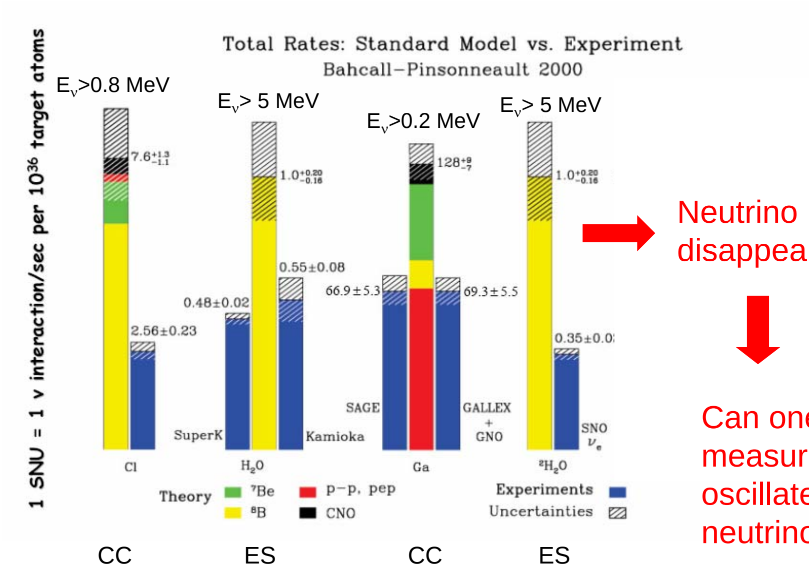
- Homestake mine, 1400 m underground
- 615 t of C_2Cl_4 (perchloroethilene) = 2.2×10^{30} atoms of ${}^{37}Cl$
- Use ${}^{36}Ar$ and ${}^{38}Ar$ to carry-out the few atoms of ${}^{37}Ar$ (~ 1 atom/day)
- Count radioactive ${}^{37}Ar$ decays



Homestake Cl_2 experiment



Solar Neutrino Problem: Experimental summary



Neutrino disappearance

Can one measure the oscillated neutrinos ??

The Nobel Prize in Physics 2002

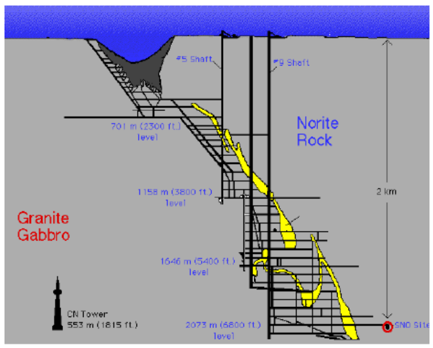
Raymond Davis Jr.	Masatoshi Koshiba	Riccardo Giaconi

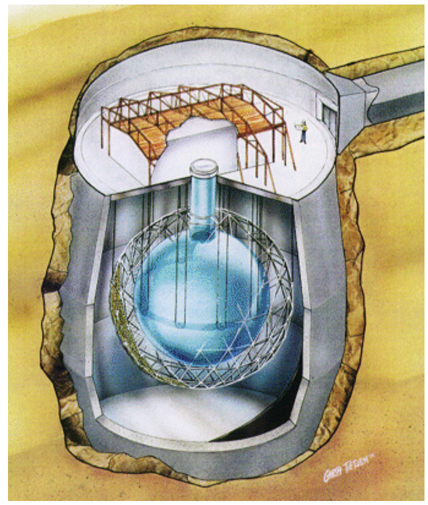
"for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos"

"for pioneering contributions to astrophysics, which have led to the discovery of cosmic X-ray sources"

Sudbury Neutrino Observatory

- 6 m radius transparent acrylic vessel
- 1000 t of heavy water (D_2O)
- 9456 inward looking photo multipliers
- Add 2 t of NaCl to detect neutrons



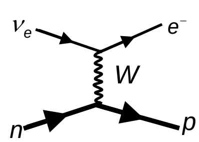


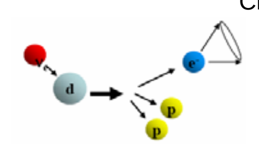
Neutrino detection with SNO

Charged current

$\sigma(\nu_\mu) = \sigma(\nu_\tau) = 0$

$\phi_\nu = \phi_{\nu_e}$

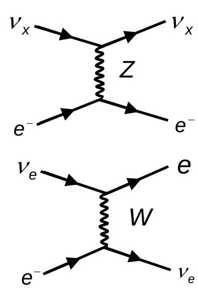


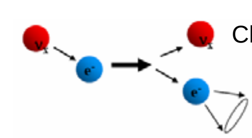


Elastic scattering

$0.154 \cdot \sigma(\nu_e) = \sigma(\nu_\mu) = \sigma(\nu_\tau)$

$\phi_\nu = \phi_{\nu_e} + (\phi_{\nu_\mu} + \phi_{\nu_\tau})/6$

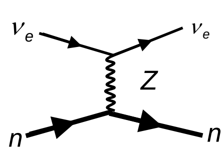


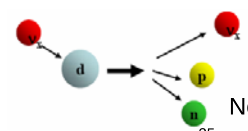


Neutral current

$\sigma(\nu_e) = \sigma(\nu_\mu) = \sigma(\nu_\tau)$

$\phi_\nu = \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}$

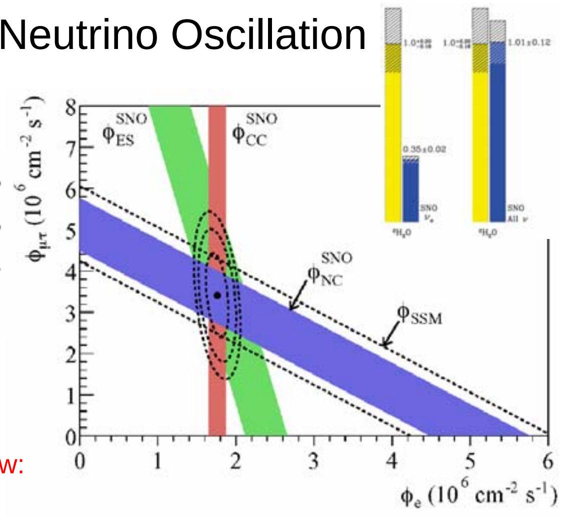




SNO Evidence for Neutrino Oscillation

$$\phi_{CC}^{SNO} = 1.76^{+0.06}_{-0.05}(\text{stat})^{+0.09}_{-0.09}(\text{syst}),$$
$$\phi_{ES}^{SNO} = 2.39^{+0.24}_{-0.23}(\text{stat})^{+0.12}_{-0.12}(\text{syst}),$$
$$\phi_{NC}^{SNO} = 5.09^{+0.44}_{-0.43}(\text{stat})^{+0.46}_{-0.43}(\text{syst}).$$

Phys. Rev. Lett. 89 (2002) 011301



Electron neutrino flux is too low:

$P_{\nu_e \nu_e} = (35 \pm 2)\%$

Total flux of neutrinos is correct. → Interpreted as $\nu_e \leftrightarrow \nu_\mu$ or ν_τ oscillation

But in case of simple "vacuum oszillation": $P_{\nu_e \nu_e} \geq 1 - \frac{1}{2}\sin^2 2\theta = 50\%$?

Neutrino oscillations in matter: MSW-effect

Mikhaev, Smirnov (1986), Wolfenstein (1976)

Neutrino oscillation in vacuum:

time development of mass eigenstates

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \frac{1}{2p}\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

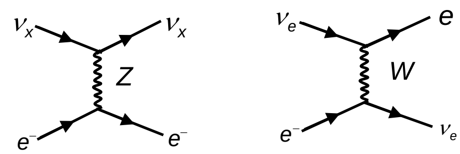
With unitary transformation U one obtains for the flavor oscillation in vaccum:

$$\mathbf{U}_\theta = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$$

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \mathbf{UMU}^\mathbf{T}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \frac{\mathbf{M}^2}{2p}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

$$\underbrace{\mathbf{UMU}^\mathbf{T}}_{\frac{\mathbf{M}^2}{2p}} = \frac{\Delta m^2}{4p}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}$$

Neutrinos in matter:



Electron neutrinos suffer an additional potential V_e affecting the forward scattering amplitude which leads to change in the effective mass for ν_e:

$$V_e = G_F \sqrt{2}\, N_e$$ N_e=electron density

$$m^2 = E^2 - p^2 \rightarrow (E + V_e)^2 - p^2 \approx m^2 + 2EV_e$$

$$\Delta m_M^2 = 2\sqrt{2}\, G_F N_e E$$

Neutrino oscillation in matter:

$$\Delta m_M^2 = 2\sqrt{2}\, G_F N_e E$$

$$\mathbf{M^2} \rightarrow \mathbf{M_M^2} = \frac{\Delta m^2}{2}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} \Delta m_M^2 & 0 \\ 0 & -\Delta m_M^2 \end{pmatrix}$$

Go the opposite direction…

one define the matter mass eigenstates which one obtains by diagonalizing $\mathbf{M_M^2}$

$$\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = \mathbf{U_{\theta_m}^T}\begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix}$$

$$\mathbf{U_{\theta_m}^T M^2 U_{\theta_m}} = \frac{1}{2}(m_1^2 + m_1^2)\begin{pmatrix} -\Delta_m & 0 \\ 0 & \Delta_m \end{pmatrix}$$

$$\Delta_m = \Delta m^2 \sqrt{(a - \cos 2\theta)^2 + \sin^2 2\theta}$$

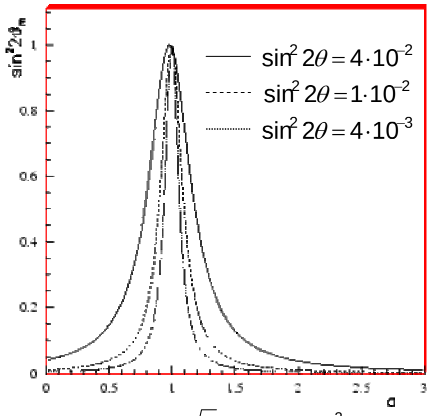
$$a = 2\sqrt{2}\, E G_F N_e \big/ \Delta m^2$$

$$\mathbf{U}_{\theta_m} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \quad \text{with}$$

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(\cos 2\theta - a)^2 + \sin^2 2\theta}$$

Matter mixing angle can go through a resonance:

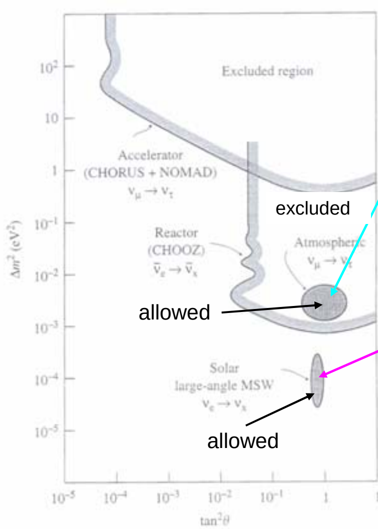
$$\cos 2\theta - a = 0 \quad \text{i.e.} \quad N_e = \Delta m^2 \frac{\cos 2\theta}{2\sqrt{2}EG_F}$$



As in the core of the sun, N_e is larger than the critical density the resonance condition will always be fulfilled: oscillation largely modified by matter.

Explains $P_{\nu_e\nu_e} = (35 \pm 2)\%$ observed with SNO

Status of oscillation measurements



Atmos
$\nu_\mu \to \nu_x$

$\Delta m^2 = (2.4 \pm 0.4) \times 10^{-3}$ eV
$\sin^2 2\theta > 0.92$ @ 90% C.L.

Solar+KamLAND
$\nu_e \to \nu_x$
LMA = large mixing angle: MSW effect necessary

Long baseline "many" reactors experiment

$\Delta m^2 = (8.2 \pm 0.6) \times 10^{-5}$ eV
$\sin^2 2\theta \approx 0.83$

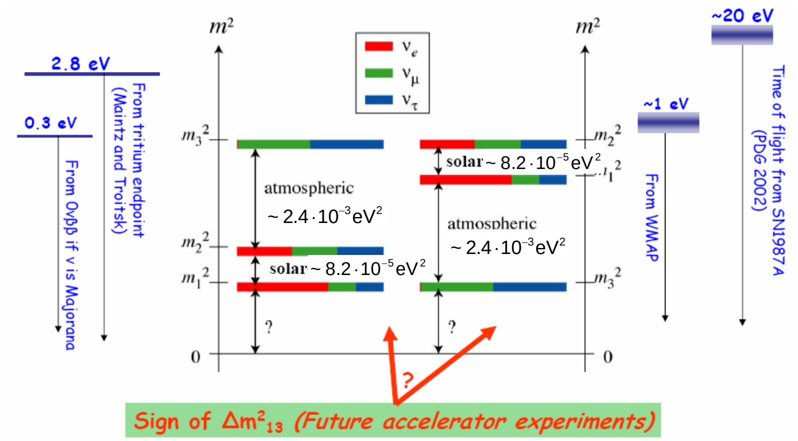
⇨ Different oscillation pattern for different neutrinos – what can we learn about the masses ??

4.4 Neutrino masses

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$

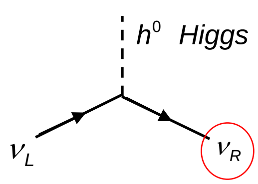
$\theta_{12} \equiv \theta_{sol}$ $\theta_{23} \equiv \theta_{atm}$ $\theta_{13} \approx 0$



Absolute neutrino masses are not known !

Neutrino masses in the Standard Model

- Neutrino Mass term: the same as for charged leptons: $\sim m\bar{\psi}_R\psi_L$



Dirac mass term

$\langle m \rangle = m_D(\bar{L}_\nu R_\nu + \bar{R}_\nu L_\nu)$

Masses of neutrinos through Yukawa coupling to Higgs: $m_\nu = \frac{\lambda_\nu \upsilon}{\sqrt{2}}$

From the vacuum expectation value of the Higgs, υ≈246 GeV, follows that the Yukawa coupling must be extremely small ($<10^{-11}$) to generate the small neutrino masses. → unnatural

- Dirac mass terms imply existence of right (left) -handed (anti) neutrinos.

  Minimal extension of the Standard Model:

  Introduce singlets of right (left)-handed (anti)neutrinos which do not couple to charged and neutral currents.

- Lepton numbers: L_e L_μ and L_τ are not conserved. L is conserved !!
- But why are the neutrino masses so small ?

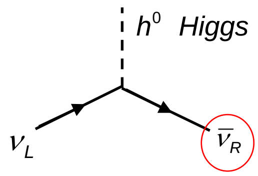
Majorana Neutrinos

- Unlike the charged leptons, neutrinos could be their own anti-particles:

Majorana Neutrinos

$$\nu = \bar{\nu} \quad \begin{cases} \bar{\nu}_L = \nu_L \\ \nu_R = \bar{\nu}_R \end{cases}$$

- Majorana-mass terms in addition to Dirac mass terms possible:

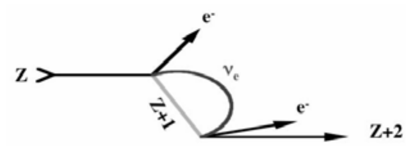


Majorana mass term

$$\langle m \rangle = \left(\bar{L}_\nu, \bar{L}_{\bar{\nu}}\right) \begin{pmatrix} m_{M,L} & 0 \\ 0 & m_{M,R} \end{pmatrix} \begin{pmatrix} R_{\bar{\nu}} \\ R_\nu \end{pmatrix} + c.c.$$

Mass term violates Lepton flavor conservation: $\Delta L = \pm 2$

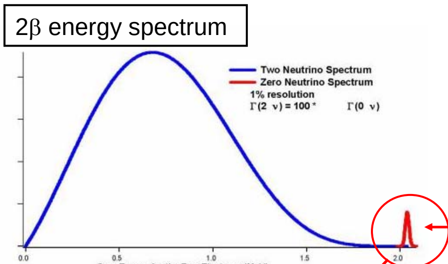
- Majorana character can be checked in neutrinoless double beta decay ($0\nu2\beta$):



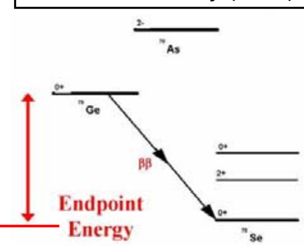
- Can we prove that neutrino is a Dirac particle

Search for $0\nu2\beta$

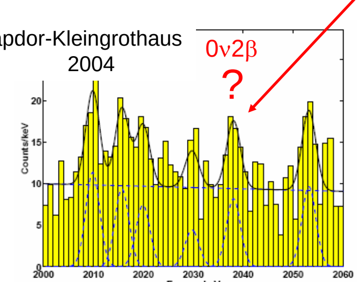
2β energy spectrum



Germanium decay (example)



Klapdor-Kleingrothaus 2004



Isotope	Half-life (years)	$m_{\beta\beta}$ (meV)	Reference
^{48}Ca	$>1.4 \times 10^{22}$	<7200-44700	[7]
^{76}Ge	$>1.9 \times 10^{25}$	<350	[8]
^{76}Ge	$>1.6 \times 10^{25}$	<330-1350	[9]
^{76}Ge	$\approx 1.2 \times 10^{25}$	≈440	[10]
^{82}Se	$>2.7 \times 10^{22}$ (68%)	<5000	[11]
^{100}Mo	$>5.5 \times 10^{22}$	<2100	[12]
^{116}Cd	$>1.7 \times 10^{23}$	<1700	[13]
^{128}Te	$>7.7 \times 10^{24}$ (geochem)	<1100-1500	[14]
^{130}Te	$>5.5 \times 10^{23}$	<370-1900	[15]
^{136}Xe	$>4.4 \times 10^{23}$	<1800-5200	[16]
^{150}Nd	$>1.2 \times 10^{21}$	<3000	[17]

Seesaw mechanism to generate light neutrinos

- If neutrinos are Majorana particles:

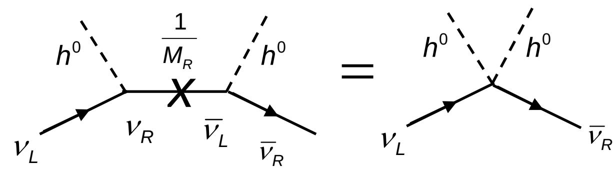
  Introduce in addition to the Dirac mass term also a Majorana mass term for the right-handed neutrino singlet:

$$\langle m \rangle = \left(\bar{L}_\nu, \bar{L}_{\bar{\nu}}\right)\begin{pmatrix} m_{M,L} & m_D \\ m_D & m_{M,R} \end{pmatrix}\begin{pmatrix} R_{\bar{\nu}} \\ R_\nu \end{pmatrix} + c.c.$$

  Seesaw Model: Assume Majorana mass M_R of the right-handed neutrino very heavy, Majorana mass M_L of left handed neutrino =0.

  Solving the mass matrix one obtains a small mass m_ν for LH neutrinos

$$m_\nu = \frac{m_D^2}{M_R}$$



Seesaw mass term for light neutrino

- Small neutrino masses can be explained … but how large is M_R (10^{10}…10^{15} GeV) ?

The End