VII. Electro-weak unification: Phenomenological approach to the Standard Model (SM)

1. Requisites
2. Weak isospin and weak hypercharge
3. Couplings to gauge fields
4. Feynman rules
5. Higgs boson and the parameters of the SM

1. Requisites

a) Fundamental fermions

Leptons

$$\begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu \\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau \\ \tau^-\end{pmatrix}_L$$

Left-handed doublets

$$e_R^- \quad \mu_R^- \quad \tau_R^-$$

right-handed singlets

Quarks

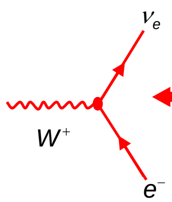
$$\begin{pmatrix}u \\ d'\end{pmatrix}_L \quad \begin{pmatrix}c \\ s'\end{pmatrix}_L \quad \begin{pmatrix}t \\ b'\end{pmatrix}_L$$

$$u_R, d_R \quad c_R, s_R \quad t_R, b_R$$

b) Fundamental interaction

- Charged current interaction: transitions inside LH doublets
- Neutral current interaction: couples to LH and RH fermions
- Electromagnetic interaction couples equally to LH and RH fermions

Charged current weak interaction

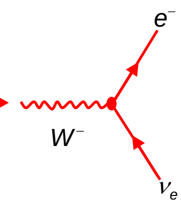


Charge raising current:

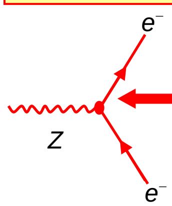
$$J_\mu^+ = \bar{u}_\nu \gamma_\mu \frac{1-\gamma^5}{2} u_e = \bar{\nu}\gamma_\mu \frac{1-\gamma^5}{2} e = \bar{\nu}_L \gamma_\mu e_L$$

Charge lowering current:

$$J_\mu^- = \bar{e}\gamma_\mu \frac{1-\gamma^5}{2}\nu = \bar{e}_L \gamma_\mu \nu_L$$



Neutral current weak interaction



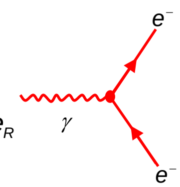
$$J_\mu^{NC} = \bar{e}\gamma_\mu \frac{1}{2}(g_V - g_A\gamma^5)e = \bar{e}\gamma_\mu \left[\underbrace{\frac{1}{2}(g_V + g_A)}_{g_L} \frac{1-\gamma^5}{2} + \underbrace{\frac{1}{2}(g_V - g_A)}_{g_R} \frac{1+\gamma^5}{2} \right]$$

$$= \bar{e}_L g_L \gamma_\mu e_L + \bar{e}_R g_R \gamma_\mu e_R$$

Electromagnetic interaction

units of e ↓

$$J_\mu^{em} = q\bar{e}\gamma_\mu e = q\bar{e}_L\gamma_\mu e_L + q\bar{e}_R\gamma_\mu e_R$$



2. Weak isospin and weak hypercharge

Weak isospin:

In analogy to the strong isospin one can describe the particles of the LH doublets as $T_3=\pm½$ states of a particle with weak isospin T= ½.

$$\chi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \qquad T = \frac{1}{2}, T_3 = \pm\frac{1}{2}$$

As in case of the (iso)spin one can use the raising and lowering operators defined by the Pauli matrices to express state transitions.

$$\tau^\pm = \frac{1}{2}(\tau^1 \pm i\cdot\tau^2)$$

$$\tau^i = \sigma^i = \text{Pauli-matrices}$$

The charge current then can be written in the compact form:

$$J_\mu^\pm = \bar{\chi}_L \gamma_\mu \tau^\pm \chi_L$$

From the SU(2) structure of the isospin formalism one expects that in addition to the currents $J^\pm$ there exists a 3[rd] neutral current J^3 of the form:

$$J_\mu^3 = \bar{\chi}_L \gamma_\mu \cdot T_3\tau^3 \chi_L = \frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma_\mu e_L$$

⇒ Weak isospin Triplett of LH fermion currents

$$J_\mu^i = \bar{\chi}_L \gamma_\mu \frac{1}{2}\tau^i \chi_L$$

Electro-weak unification

The new current J^3 is not equal to the current J^{NC}:
J^{NC} contains LH and RH fermion contributions

Treat both neutral currents, J^{em} and J^{NC}, simultaneously:

As both currents contain RH contributions it should be possible to construct a linear combination which couples only to LH fermions:

two linear combinations of J^{em} and J^{NC}

$$J_\mu^3 = \sin^2\theta_W J_\mu^{em} + J_\mu^{NC}$$

$$\frac{1}{2}J_\mu^Y = \cos^2\theta_W J_\mu^{em} - J_\mu^{NC}$$

Choose θ_W such that RH fermions components in J^3 vanish.

- J^3 completes the isospin triplet J^i
- J^Y is called hypercharge current, couples via hypercharge

Hypercharge

From the above definitions follows:

$$J_\mu^Y = 2J_\mu^{em} - 2J_\mu^3 = \overline{\psi}\gamma_\mu[2Q - 2T_3]\psi = \overline{\psi}\gamma_\mu Y\psi$$

Hypercharge operator: $Y = 2[Q-T_3]$

(Gell-Mann Nishijima Formula)

Electro-weak quantum numbers

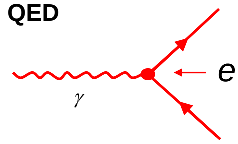
Leptons	T	T_3	Q	Y
ν_e	$1/2$	$+1/2$	0	-1
e_L	$1/2$	$-1/2$	-1	-1
e_R	0	0	-1	-2

Quarks	T	T_3	Q	Y
u_L	$1/2$	$+1/2$	$2/3$	$1/3$
d'_L	$1/2$	$-1/2$	$-1/3$	$1/3$
u_R	0	0	$2/3$	$4/3$
d_R	0	0	$-1/3$	$-2/3$

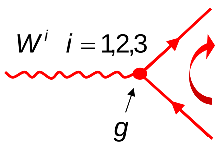
3. Current coupling to the gauge fields/bosons

In the **electro-weak theory** the coupling between boson and fermions is defined in analogy to the coupling of the photon to the fermions currents in QED. There are in total 4 boson fields:

QED



$$\sim -ieJ_\mu^{em} \cdot A^\mu$$

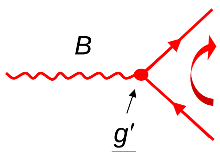


$W^i \quad i = 1,2,3$

$$J_\mu^i \qquad \sim -igJ_\mu^i \cdot W^{i,\mu}$$

Corresponding to $J^\pm$ and J^3 there are fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2) \quad \text{and} \quad W_\mu^3$$



$$J_\mu^Y \qquad \sim -i\frac{g'}{2}J_\mu^Y \cdot B^\mu$$

convention

g, g′ are coupling constants.

Gauge bosons:

While the charged boson fields $W^\pm$ correspond to the observed W bosons, the neutral fields B and W^3 only correspond to linear combinations of the observed photon and Z boson:

$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W \quad \leftarrow \text{massless photon}$$

$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W \quad \leftarrow \text{massive Z boson}$$

$$B_\mu = A_\mu \cos\theta_W - Z_\mu \sin\theta_W$$

$$W_\mu^3 = A_\mu \sin\theta_W + Z_\mu \cos\theta_W$$

The weak mixing angle θ_W (Weinberg angle) is defined by the coupling constants to A^μ and Z^μ.

The fermion coupling to the neutral fields are given by →

$$-igJ_\mu^3 \cdot W^{3,\mu} - i\frac{g'}{2}J_\mu^Y \cdot B^\mu$$
$$= i\left[g\sin\theta_W J_\mu^3 + \frac{g'}{2}\cos\theta_W J_\mu^Y\right]A^\mu$$
$$-i\left[g\cos\theta_W J_\mu^3 - \frac{g'}{2}\sin\theta_W J_\mu^Y\right]Z^\mu$$

Fermion coupling to the photon

$$[\quad]A^\mu \equiv eJ_\mu^{em}A^\mu = e\left[J_\mu^3 + \frac{1}{2}J_\mu^Y\right]A^\mu$$

$$J_\mu^Y = 2\left[J_\mu^{em} - J_\mu^3\right]$$

Comparison of the coefficients gives:

$$e = g\cdot\sin\theta_W \qquad g = \frac{e}{\sin\theta_W} \qquad g' = \frac{e}{\cos\theta_W}$$
$$e = g'\cdot\cos\theta_W$$

⬅ The couplings to the different boson types have similar strength.

Fermion coupling to the Z boson

From $-i[\quad]Z^\mu$ follows with

$$\begin{cases} J_\mu^Y = 2\left[J_\mu^{em} - J_\mu^3\right] \\ g\cdot\sin\theta_W = g'\cdot\cos\theta_W \end{cases}$$

$$-i[\quad]Z^\mu = -i\frac{g}{\cos\theta_W}\left[J_\mu^3 - \sin^2\theta_W J_\mu^{em}\right]Z^\mu$$
$$= -i\frac{g}{\cos\theta_W}J_\mu^{NC}Z^\mu$$

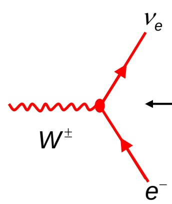
Using
$$\begin{cases} J_\mu^3 = \bar{\chi}_L\gamma_\mu\cdot T_3\tau^3\chi_L \\ J_\mu^{em} = \bar{e}\gamma_\mu Qe \end{cases}$$
one finds

$$-i[\quad]Z^\mu = -i\frac{g}{\cos\theta_W}\left[T_3\cdot\bar{e}\gamma_\mu\frac{1-\gamma^5}{2}e - q\cdot\sin^2\theta_w\,\bar{e}\gamma_\mu e\right]Z^\mu$$
$$-i[\quad]Z^\mu = -i\frac{g}{\cos\theta_W}\left[\bar{e}\gamma_\mu\frac{1}{2}[g_V - g_A\gamma^5]e\right]Z^\mu$$

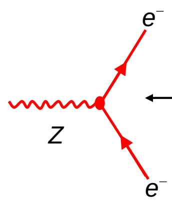
with $g_V = T_3 - 2q\sin^2\theta_W$ and $g_A = T_3$

4. Feyman rules

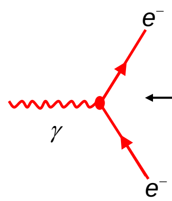
Vertex factors | Propagator



$$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1}{2}(1-\gamma^5) \qquad \frac{g_{\mu\nu}-q_\mu q_\nu/M_W^2}{q^2-M_W^2}$$

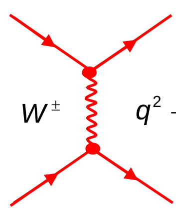


$$-i\frac{g}{\cos\theta_W}\gamma_\mu\frac{1}{2}(g_V-g_A\gamma^5) \qquad \frac{g_{\mu\nu}-q_\mu q_\nu/M_Z^2}{q^2-M_Z^2}$$

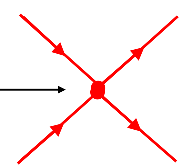


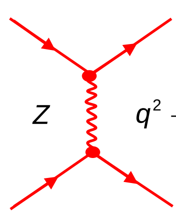
$$-ie\gamma_\mu \qquad \frac{1}{q^2}$$

Comparison of the $q^2 \to 0$ limit with the 4-femion ansatz

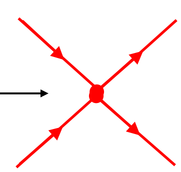


$$q^2 \to 0: \quad \frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$$





$$q^2 \to 0: \quad \frac{g^2}{8\cos^2\theta_W M_Z^2} = \frac{G_{NC}}{\sqrt{2}}$$



For $\frac{G_F}{\sqrt{2}} \equiv \frac{G_{NC}}{\sqrt{2}}$ follows $\cos^2\theta_W = \frac{M_W^2}{M_Z^2}$ ⇐ follows also from Higgs mechanism

5. Higgs boson and the parameters of the SM

To generate boson and fermion masses in an gauge invariant way the Standard Model uses the Higgs-mechanism.

A scalar field with a non-vanishing vacuum expectation value υ, couples to the boson and fermion fields and generates the particle masses through these couplings.

For the boson masses one finds:

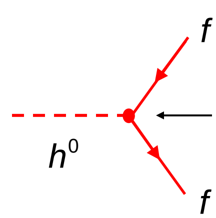
$$M_W = \frac{1}{2}\upsilon g \qquad \frac{M_W}{M_Z} = \frac{g}{\sqrt{g^2 + g'^2}} = \cos\theta_w$$

$$M_Z = \frac{1}{2}\upsilon\sqrt{g^2 + g'^2} \qquad g\sin\theta_w = g'\cos\theta_w$$

For the Higgs-Mass itself one finds $M_H = 2\upsilon^2\lambda$

Parameter λ describes the Higgs potential and as the vacuum expectation value cannot be predicted by theory

Higgs coupling to fermions and fermion masses



$$-i\frac{g}{2}\frac{m_f}{M_W}$$

with fermion masses $m_f = \frac{G_f \upsilon}{\sqrt{2}}$

G_f is an unknown fermion dependent (Yukawa) coupling constant.

Standard Model Parameter

- e (α_{QED})
- (G_F and $\sin\theta_W$) or (M_W and M_Z) or …
- α_s (strong coupling const.)
- 9 fermion masses, neutrinos are massless
- 4 quark mixing parameters (CKM matrix)
- M_H

} 18 parameters

Some people think that this is too much !!