

VI. Probing the weak interaction

1. Phenomenology of weak decays
2. Parity violation and neutrino helicity
3. V-A theory
4. Structure of neutral currents

The weak interaction was and is a topic with a lot of surprises:

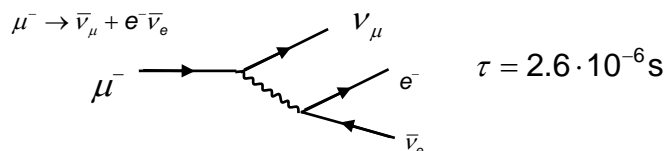
Past: Flavor violation, P and CP violation.

Today: Weak decays used as probes for new physics

1. Phenomenology of weak decays

All particles (except photons and gluons) participate in the weak interaction. At small q^2 weak interaction is shadowed by strong and electro-magnetic effects.

- Observation of weak effects only possible if strong/electro-magnetic processes are forbidden by conservation laws:



Electromagnetic decay $\mu^- \rightarrow e^- \gamma$ forbidden by lepton number conservation

- In addition the observation of interference effects is possible (e.g. atomic parity violation: γ/Z interference).

1.1 Weak hadronic decays

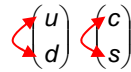
a) Dominant decay modes (quark level)

$$d \rightarrow u \ell^- \bar{\nu}_\ell$$

$$u \rightarrow d \ell^+ \nu_\ell$$

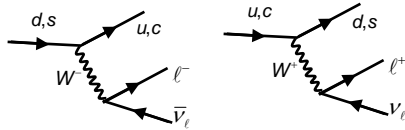
$$s \rightarrow c \ell^- \bar{\nu}_\ell$$

$$c \rightarrow s \ell^+ \nu_\ell$$



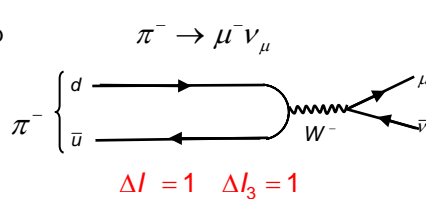
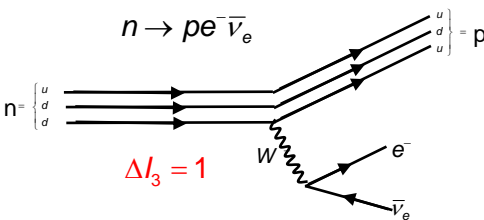
$$M^2 \sim \cos^2 \theta_c \sim 0.95$$

Cabibbo angle: $\theta_c \approx 0.22$



If q^2 is large enough the W can also decay to (u, \bar{d}) or (\bar{u}, d) quark pairs

Using the the “quark level” decay one can describe weak hadron decays (treating the not weakly interacting quarks as spectators)



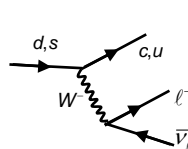
b) suppressed decay modes

$$d \rightarrow c \ell^- \bar{\nu}_\ell$$

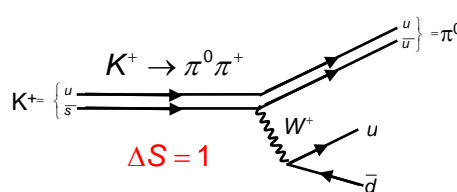
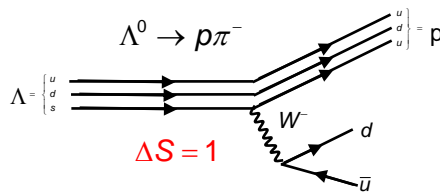
$$u \rightarrow s \ell^+ \nu_\ell$$

$$s \rightarrow u \ell^- \bar{\nu}_\ell$$

$$c \rightarrow d \ell^+ \nu_\ell$$

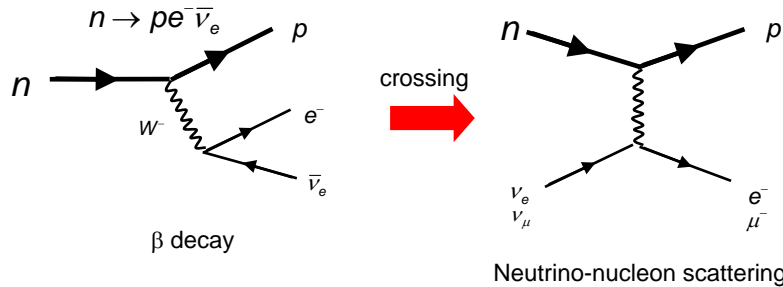


$$M^2 \sim \sin^2 \theta_c \sim 0.05$$



Weak interaction does not conserve strong isospin, strangeness or other quark flavor numbers. Lepton number is conserved.

1.2 Neutrino interactions



Very small cross section for νN scattering: $\sigma(\nu N) \approx E_\nu[\text{GeV}] \times 10^{-38} \text{ cm}^2 = E_\nu[\text{GeV}] \times 10 \text{ fb}$

- intense neutrino beams
 - large instrumented targets
- (see also DIS neutrino nucleon scattering)

2. Parity violation

Reminder: Parity transformations (**P**) = space inversion

$$P\psi(t, \vec{x}) = \psi'(t, \vec{x}) = \psi(t, -\vec{x})$$

⇔ mirroring at plane + rotation around axis perpendicular to plane

⇒ To test P symmetry it is sufficient to study the process in the "mirrored system": physics invariant under rotation

P transformation properties:

$$\begin{aligned}
 P: \quad & \vec{r} \rightarrow -\vec{r} \\
 & t \rightarrow t \\
 & \vec{p} \rightarrow -\vec{p} \\
 & \vec{\ell} = \vec{r} \times \vec{p} \rightarrow \vec{\ell} \quad \text{Axial/pseudo vector}
 \end{aligned}$$

e.g.: Helicity operator

$$H = \frac{\vec{s} \cdot \vec{p}}{|\vec{s}||\vec{p}|} \xrightarrow{P} -\frac{\vec{s} \cdot \vec{p}}{|\vec{s}||\vec{p}|} \quad (\text{pseudo - scalar})$$

2.1 Historical θ/τ puzzle (1956)

Until 1956 parity conservation as well as T and C symmetry was a “dogma”:
 → very little experimental tests done

In 1956 Lee and Yang proposed parity violation in weak processes.

Starting point: Observation of two particles θ^+ and τ^+ with exactly equal mass, charge and strangeness **but** with different parity:

$$\theta^+ \rightarrow \pi^+ \pi^0 \quad w/ \quad P(\theta^+) = P(\pi)^2 (-1)^\ell \rightarrow J^P(\theta^+) = 0^+, 1^-$$

$$\tau^+ \rightarrow \pi^+ \pi^+ \pi^- \quad P(\tau^+) = P(\pi)^3 (-1)^{2\ell} \rightarrow J^P(\tau^+) = 0^-, 2^-$$

Lee + Yang: θ^+ and τ^+ same particle, but decay violates parity

⇒ particle is K^+ :

$$K^+(0^-) \rightarrow \pi^+ \pi^0 \quad P \text{ is violated}$$

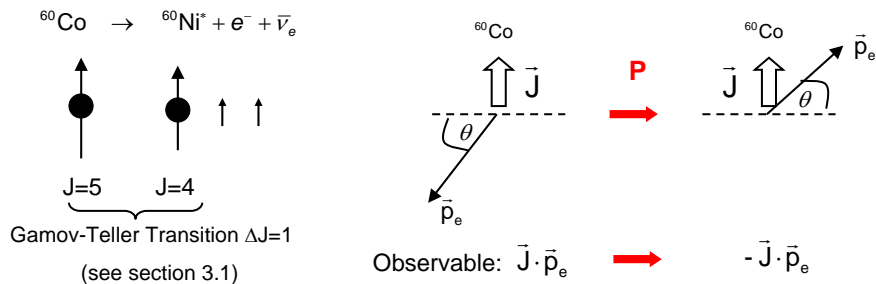
$$K^+(0^-) \rightarrow \pi^+ \pi^+ \pi^- \quad P \text{ is conserved}$$

To search for possible P violation, a number of experimental tests of parity conservation in weak decays has been proposed:

1957 Observation of P violation in nuclear β decays by Wu et al.

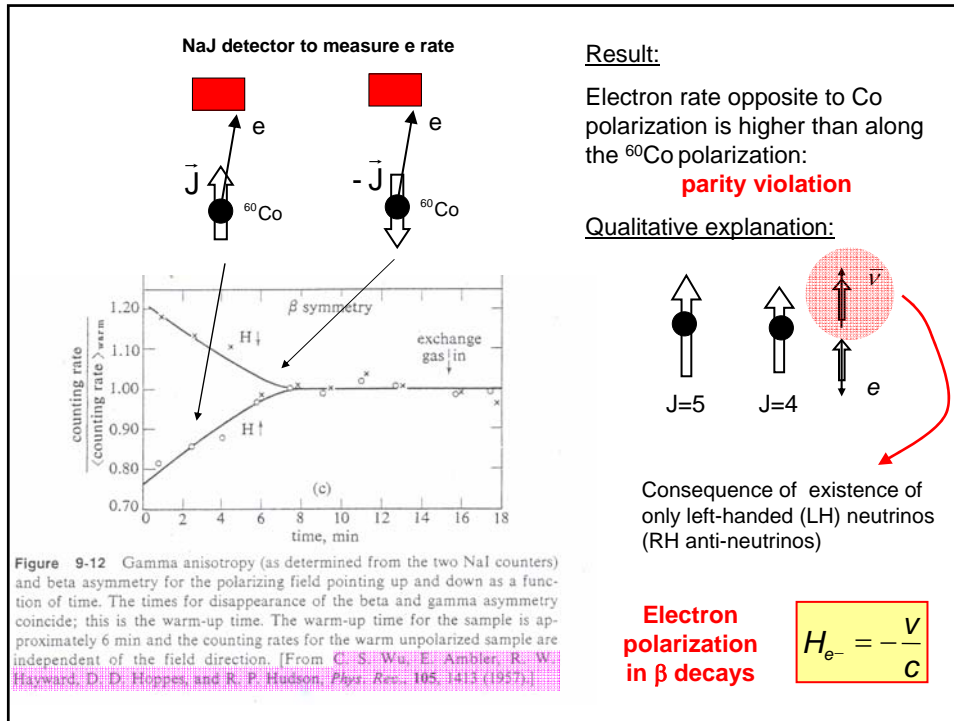
2.2 Observation of parity violation, Wu et al. 1957

Idea: Measurement of the angular distribution of the emitted e^- in the decay of polarized ^{60}Co nuclei



If P is conserved, the angular distribution must be symmetric in θ (symmetric to dashed line): transition rates for $\vec{J} \cdot \vec{p}_e$ and $-\vec{J} \cdot \vec{p}_e$ are identical.

Experiment: Invert Co polarization and compare the rates at the same position θ .



2.3 Determination of the neutrino helicity Goldhaber et al., 1958

Indirect measurement of the neutrino helicity in a K capture reaction:

$^{152}\text{Eu} + e^- \rightarrow ^{152}\text{Sm}^* + \nu_e$
 $E_\nu \approx 950\text{KeV}$

Idea of the experiment:

- Electron capture and ν emission**

^{152}Eu
 $J^P = 0^+$

$+ e^-$

\rightarrow

$\leftarrow \bullet \rightleftarrows + \rightleftarrows$ RH

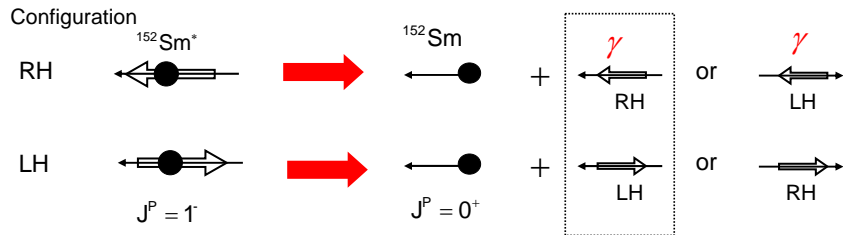
$\leftarrow \bullet \rightleftarrows + \leftarrow \bullet \rightleftarrows$ LH

$^{152}\text{Sm}^*$
 $J^P = 1^-$

$+ \nu_e$

Sm undergoes is small **recoil** ($p_{\text{recoil}} = 950 \text{ KeV}$). Because of angular momentum conservation Spin $J=1$ of Sm^* is opposite to neutrino spin. Important: **neutrino helicity is transferred to the Sm nucleus.**

2. γ emission: $^{152}\text{Sm}^*(J^P = 1^-) \rightarrow ^{152}\text{Sm}(J^P = 0^+) + \gamma$



Photons along the Sm recoil direction carry the polarization of the Sm^* nucleus

- How to select photons along the recoil direction ? \Rightarrow 3
- How to determine the polarization of these photons ? \Rightarrow 4

3. Resonant photon scattering: $\gamma + ^{152}\text{Sm} \rightarrow ^{152}\text{Sm}^* \rightarrow ^{152}\text{Sm} + \gamma$

Resonant scattering:

To compensate the nuclear recoil, the photon energy must be slightly larger than 960 keV.

This is the case for photons which have been emitted in the direction of the $\text{Eu} \rightarrow \text{Sm}$ recoil (Doppler-effect).



Resonant scattering only possible for "forward" emitted photons, which carry the polarization of the Sm^* and thus the polarization of the neutrinos.

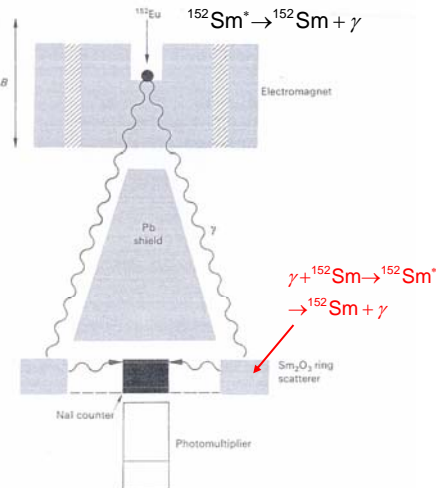


Fig. 7.8. Schematic diagram of the apparatus used by Goldhaber *et al.*, in which γ -rays from the decay of $^{152}\text{Sm}^*$, produced following K-capture in ^{152}Eu , undergo resonance scattering in Sm_2O_3 and are recorded by a sodium iodide scintillator and photomultiplier. The transmission of photons through the iron surrounding the source depends on their helicity and the direction of the magnetic field B .

4. Determination of the photon polarization

Exploit that the transmission index through magnetized iron is polarization dependent: Compton scattering in magnetized iron

LH $\leftarrow \overleftrightarrow{\gamma}$
RH $\leftarrow \overleftrightarrow{\gamma}$

$\overrightarrow{\vec{B}}$
 $\overrightarrow{\vec{B}}$

Polarization of electrons in iron \leftarrow
Polarization of electrons in iron \leftarrow

(to minimize pot. energy)
Absorption leads to spin flip

↓
↓

LH photons cannot be absorbed: Good transmission
RH photons undergo Compton scattering: Bad transmission

Photons w/ polarization anti-parallel to magnetization undergo less absorption

Experiment

Sm^* emitted photons pass through the magnetized iron. Resonant scattering allows the photon detection by a NaJ scintillation counter. The counting rate difference for the two possible magnetizations measure the polarization of the photons and thus the helicity of the neutrinos.

Results: $P_\gamma = -0.66 \pm 0.14$

→ photons from Sm^* are left-handed. The measured photon polarization is compatible with a neutrino polarization of $H=-1$.

From a calculation with 100% photon polarization one expects a measurable value $P_\gamma \sim 0.75$. Reason is the finite angular acceptance.
 → Also not exactly forward-going γ 's can lead to resonant scattering.

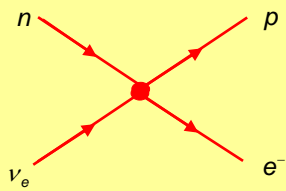
→ Summary: Lepton polarization in β decays

	e^-	e^+	ν	$\bar{\nu}$
H	$-v/c$	$+v/c$	-1	+1

3. "V-A Theory" for charged current weak interactions

3.1 Nuclear β decay

Fermi's original ansatz for $n \rightarrow p e^- \bar{\nu}_e$



4-fermion "point" interaction of fermion vector currents:

$$M = \frac{G_F}{\sqrt{2}} \cdot J_{N,\mu} \cdot J_e^{\mu+} = \frac{G_F}{\sqrt{2}} \cdot (\bar{u}_p \gamma_\mu u_n) \cdot (\bar{u}_e \gamma_\mu \nu_e)$$

\rightarrow Fermi coupling constant

Problem: ansatz cannot explain parity violation

More general ansatz by Gamov & Teller:

(Lorentz invariant current current form)

$$M = \sum_i C_i (\bar{u}_p \Gamma_i u_n) \cdot (\bar{u}_e \Gamma_i \nu_e)$$

$i = S, P, V, A, T$

- $\bar{u}_p \Gamma_i u_n$
- S: $\bar{u}_p u_n$
- P: $\bar{u}_p \gamma^5 u_n$
- V: $\bar{u}_p \gamma^\mu u_n$
- A: $\bar{u}_p \gamma^5 \gamma^\mu u_n$
- T: $\bar{u}_p \sigma^{\mu\nu} u_n$

Nuclear transitions in non-relativistic limit:

- S: $\bar{u}_p u_n \rightarrow u_p^+ u_n \quad \leftarrow$ No spin change
- P: $\bar{u}_p \gamma^5 u_n \rightarrow 0$
- V: $\bar{u}_p \gamma^\mu u_n \rightarrow u_p^+ u_n$ if $\mu=0$, else $=0 \quad \leftarrow$ No spin change
- A: $\bar{u}_p \gamma^5 \gamma^\mu u_n \rightarrow u_p^+ \sigma^i u_n$ if $\mu=i=1, \dots, 3$, else $=0 \quad \leftarrow$ spin change
- T: $\bar{u}_p \sigma^{\mu\nu} u_n \rightarrow u_p^+ \sigma^i u_n$ if $\mu=j, \nu=k, k=1, \dots, 3$, and i, j, k cyclic, else $=0 \quad \leftarrow$ spin change

Fermi-Transitions: e.g. $^{14}\text{O} \rightarrow ^{14}\text{N} + e^+ + \nu$ ($0^+ \rightarrow 0^+$) ⇨ S or V

Gamov-Teller - Transitions: e.g. $^6\text{He} \rightarrow ^6\text{Li} + e^- + \bar{\nu}$ ($0^+ \rightarrow 1^+$) ⇨ A or T

+ Parity violation
 + neutrino helicity
 + muon decay properties together with universality

⇨ V - A Theory

$$M = \frac{G_F}{\sqrt{2}} (\bar{u}_p \gamma^\mu (c_V - c_A \gamma^5) u_n) \cdot (\bar{u}_e \gamma^\mu (1 - \gamma^5) \nu_e)$$

c_V, c_A vector and axial-vector couplings of nucleons:
 $c_A/c_V = 1.262 \pm 0.005$

3.2 V-A ansatz for fundamental fermions

J_A and J_B are lepton and quark currents

Reminder

$$u_L = \frac{1}{2}(1 - \gamma^5)u$$

$$u_R = \frac{1}{2}(1 + \gamma^5)u$$

$$J_\ell^\mu = \bar{u}_\ell \gamma^\mu (1 - \gamma^5) u_\nu$$

$$J_q^\mu = \bar{u}_u \gamma^\mu (1 - \gamma^5) u_d$$

$$M = \frac{G_F}{\sqrt{2}} \cdot J_{A,\mu} \cdot J_B^{\mu+}$$

According to today's understanding of the 4-fermion coupling is the $q^2 \rightarrow 0$ limit of W propagator:

$g_w =$ coupling for weak interaction

$$M = \frac{g_w}{\sqrt{2}} \cdot \frac{1}{2} J_{A,\mu} \cdot \frac{(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2})}{q^2 - M_W^2} \cdot \frac{g_w}{\sqrt{2}} \cdot \frac{1}{2} J_B^{\mu+}$$

for $q^2 \rightarrow 0$: $= \frac{1}{M_W^2}$

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$$

With $G_F \approx 1.16 \times 10^{-5} \text{GeV}^{-2}$ follows w $M_W \approx 80 \text{GeV}$: $g_w \approx 0.65$

3.3 Helicity and chirality

solution spin \uparrow

i.e. helicity $\lambda = +\frac{1}{2}$

$$u_1(p) = \sqrt{E+m} \cdot \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x + ip_y}{E+m} \end{pmatrix}$$

\vec{p} along z

$$u_1(p) = \sqrt{E+m} \cdot \begin{pmatrix} 1 \\ 0 \\ p/(E+m) \\ 0 \end{pmatrix}$$

Helicity operator $\frac{1}{2} \frac{\vec{\Sigma} \cdot \vec{p}}{|\vec{p}|}$

solution spin \downarrow

i.e. helicity $\lambda = -\frac{1}{2}$

$$u_2(p) = \sqrt{E+m} \cdot \begin{pmatrix} 0 \\ 1 \\ \frac{p_x - ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}$$

\vec{p} along z

$$u_2(p) = \sqrt{E+m} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ -p/(E+m) \end{pmatrix}$$

Measurable quantity !

Chirality operator:

Projection of left- and right-handed components of spinor u

$$u_L = \frac{1}{2}(1 - \gamma^5)u$$

$$u_R = \frac{1}{2}(1 + \gamma^5)u$$

$$\gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Not directly measurable.

$$\frac{1 - \gamma^5}{2} u_1 = \frac{1}{2} \sqrt{E+m} \cdot \underbrace{\left(1 - \frac{p}{E+m}\right)}_{\approx 0 \text{ for } E \gg m} \cdot \begin{pmatrix} 1 \\ 0 \\ p/(E+m) \\ 0 \end{pmatrix} \rightarrow 0 \text{ for } E \gg m$$

$$\frac{1 - \gamma^5}{2} u_2 = \frac{1}{2} \sqrt{E+m} \cdot \underbrace{\left(1 + \frac{p}{E+m}\right)}_{\approx \sqrt{E} \text{ for } E \gg m} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ p/(E+m) \end{pmatrix} \rightarrow u_2 \text{ for } E \gg m$$

In the relativistic limit helicity states are also chirality eigenstates.

Polarization for particles with finite mass

Left handed spinor component $u_L = \frac{1-\gamma^5}{2}(u_1 + u_2)$
 $u_1, u_2 \rightarrow u_L, u_R$

$$Pol = \frac{P(\lambda = +1/2) - P(\lambda = -1/2)}{P(\lambda = +1/2) + P(\lambda = -1/2)} = \frac{(1 - p/(E+m))^2 - (1 + p/(E+m))^2}{(1 - p/(E+m))^2 + (1 + p/(E+m))^2}$$

$$= -\frac{p}{E} = -\frac{v}{c}$$

i.e. the LH spinor component for a particle with finite mass is not fully in the helicity state "spin down" ($\lambda = -1/2$)

Measurable quantity !

Probability for "wrong" helicity: $= 1 - \frac{v}{c}$