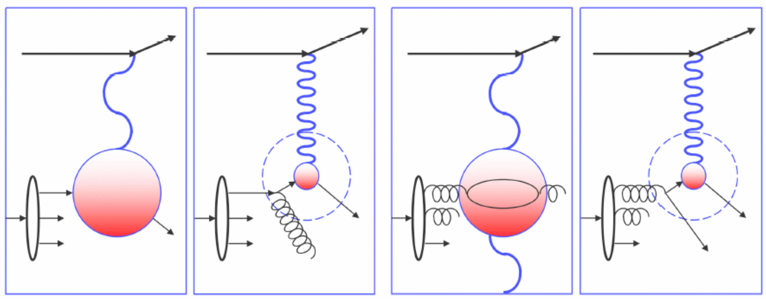
QCD explains observed scaling violation

Large x: valence quarks

Small x: Gluons, sea quarks

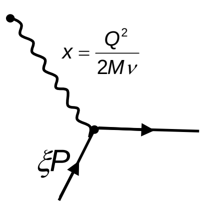


$Q^2 \uparrow \Rightarrow F_2 \downarrow$ for fixed x

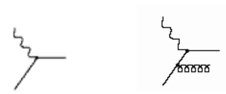
$Q^2 \uparrow \Rightarrow F_2 \uparrow$ for fixed x

Quantitative description of scaling violation

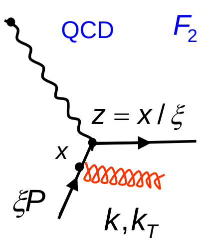
Quark Parton Model



$$F_2(x) = x\sum_i e_i^2 \int_0^1 q_i(\xi)\cdot\delta(x-\xi)d\xi = x\sum_i e_i^2 q_i(x)$$



QCD



$$F_2(x,Q^2) = x\sum_i e_i^2 \int_0^1 \frac{d\xi}{\xi} q_i(\xi)\cdot\left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right)\log\frac{Q^2}{\mu_0^2}\right]$$

$$\hat{\sigma}(\gamma^* q \to qg) \sim \frac{\alpha_s}{2\pi} P_{qq}(z) \int_{\mu_0^2}^{Q^2} \frac{dk_T^2}{k_T^2}$$

$$\sim \frac{\alpha_s}{2\pi} P_{qq}(z) \log\left(\frac{Q^2}{\mu_0^2}\right)$$

P_{qq} probability of a quark to emit gluon and becoming a quark with momentum reduced by fraction z.

μ^0 cutoff parameter

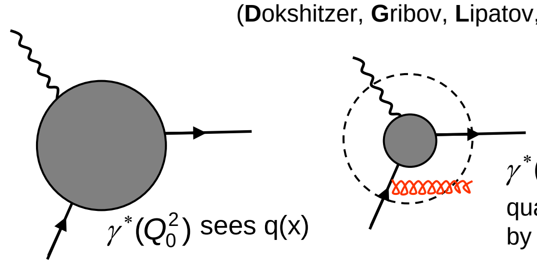
Changing to the quark densities:

$$q_i(x,Q^2) = q_i(x) + \underbrace{\frac{\alpha_s}{2\pi}\log\frac{Q^2}{\mu_0^2}\int_0^1 \frac{d\xi}{\xi} q_i(\xi) P_{qq}\left(\frac{x}{\xi}\right)}_{\Delta q(x,Q^2)}$$

Integro-differential equation for $q(x,Q^2)$:

$$\frac{d}{d\log Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_0^1 \frac{d\xi}{\xi} q(\xi,Q^2) P_{qq}\left(\frac{x}{\xi}\right)$$

DGLAP evolution equation
(**D**okshitzer, **G**ribov, **L**ipatov, **A**ltarelli, **P**arisi, 1972 – 1977)



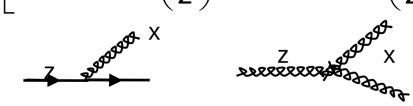
Evolution of parton densities (quarks and gluons)

evolution of quark density with $\ln Q^2$

$$\frac{\partial q(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[q(z,Q^2)P_{qq}\left(\frac{x}{z}\right) + g(z,Q^2)P_{qg}\left(\frac{x}{z}\right)\right]$$

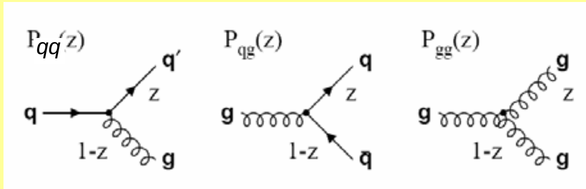
$$\frac{\partial g(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[q(z,Q^2)P_{gq}\left(\frac{x}{z}\right) + g(z,Q^2)P_{gg}\left(\frac{x}{z}\right)\right]$$

evolution of gluon density with $\ln Q^2$



Splitting functions: Probability that a parton (quark or gluon) emits a parton **(q, g)** with momentum fraction $\varepsilon = x/z$ of the parent parton.

Splitting functions are calculated as power series in α_s up to a given order:



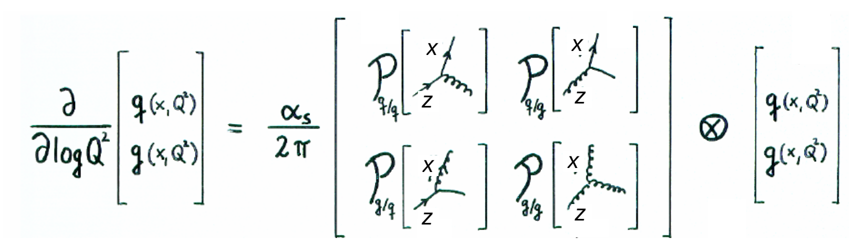
$$P_{ij}(z,\alpha_s) = P_{ij}^0(z) + \frac{\alpha_s}{2\pi} P_{ij}^1(z) + \dots$$

In leading order: $P_{ij}(z,\alpha_s) \equiv P_{ij}^0(z)$

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z} \qquad P_{gq}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}$$

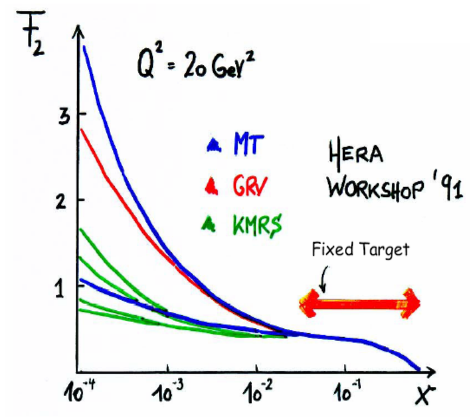
$$P_{qg}(z) = \frac{z^2+(1-z)^2}{2} \qquad P_{gg}(z) = 6\left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right)$$

DGLAP Evolution ("symbolic"):



$$P \otimes f(x,Q^2) = \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}\right) f(z,Q^2)$$

Bjorken x dependence of parton densities:



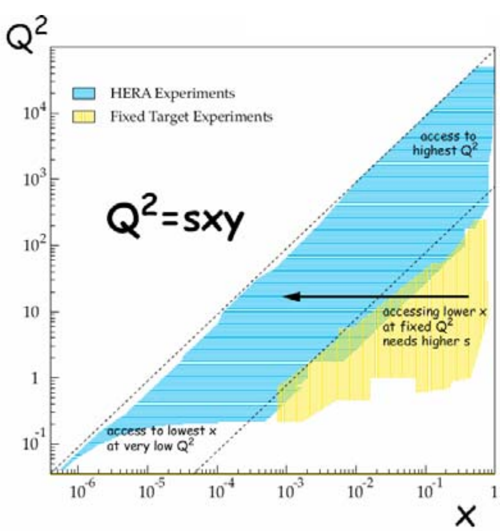
DGLAP:

No prediction for the x dependence of the parton densities.

Status in 1991 (pre HERA): Data limited to a small x region. Models to extrapolate to smaller x differ significantly.

Measure structure functions (parton densities) at low x.

Accessing the low x region:



HERA: $y = 1$

$s \sim 10^5\,GeV^2$

Fixed Target: $y = 1$

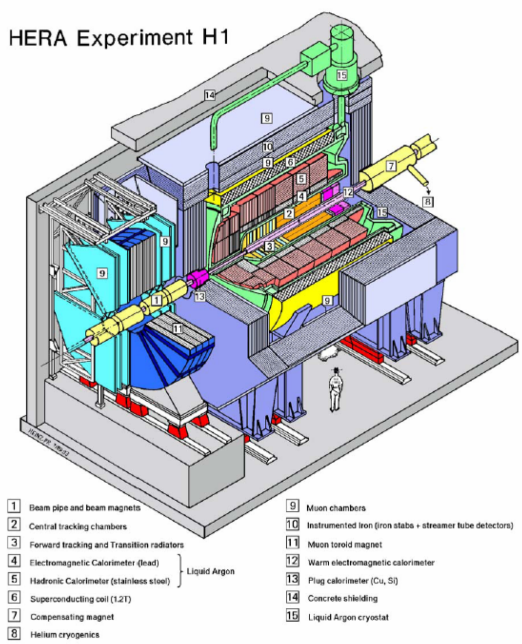
$s < 10^3\,GeV^2$

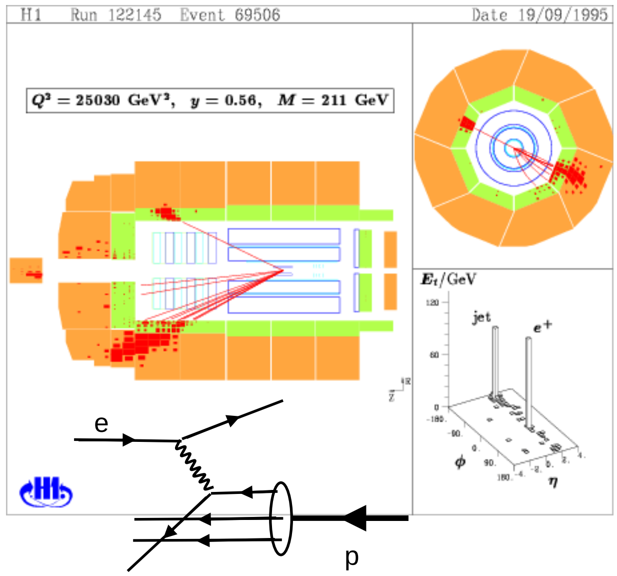
HERA

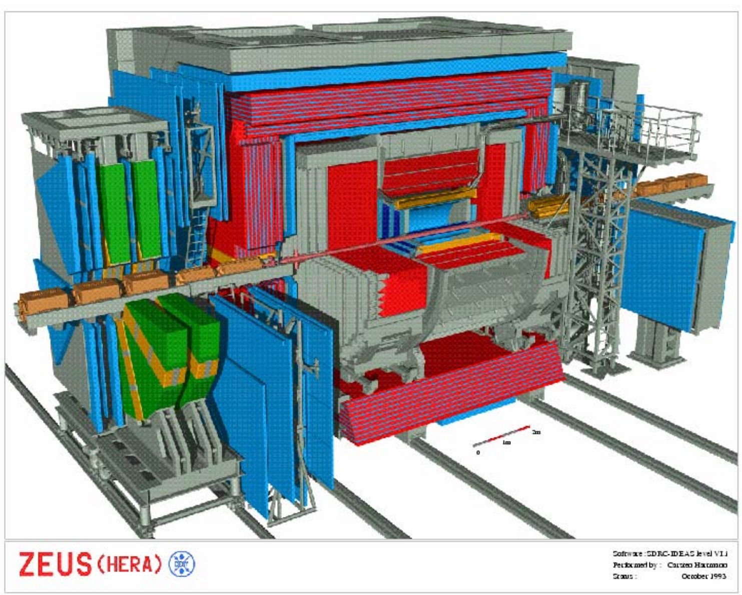
$e^{\pm}$ 30 GeV → ← p 900 GeV

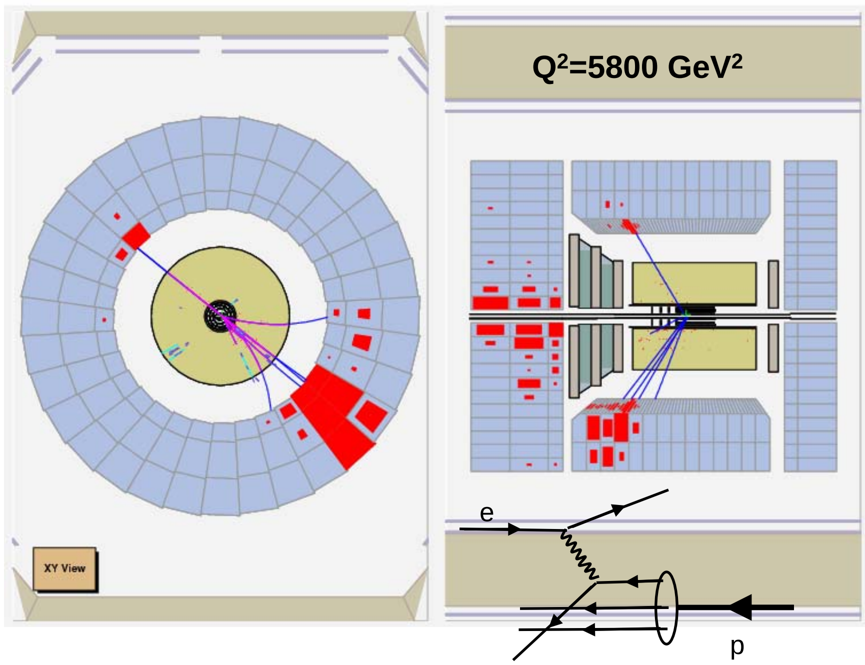
$$s = 4E_eE_p \approx 10^5\,GeV^2$$


HERA
PETRA


HERA Experiment H1
1 Beam pipe and beam magnets
2 Central tracking chambers
3 Forward tracking and Transition radiators
4 Electromagnetic Calorimeter (lead)
5 Hadronic Calorimeter (stainless steel)
Liquid Argon
6 Superconducting coil (1.2T)
7 Compensating magnet
8 Helium cryogenics
9 Muon chambers
10 Instrumented Iron (iron stabs + streamer tube detectors)
11 Muon toroid magnet
12 Warm electromagnetic calorimeter
13 Plug calorimeter (Cu, Si)
14 Concrete shielding
15 Liquid Argon cryostat


H1 Run 122145 Event 69506
Date 19/09/1995
$Q^2 = 25030$ GeV^2, $y = 0.56$, $M = 211$ GeV
E_t/GeV
jet
e^+
ϕ
η
e
p


ZEUS (HERA)



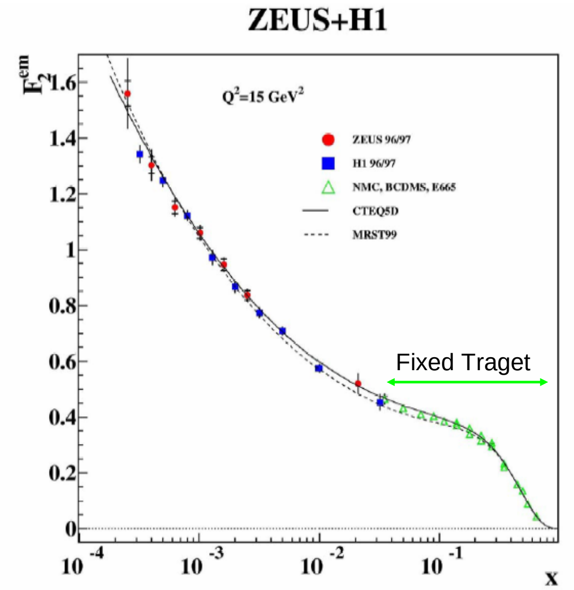
Measurement of the parton densities / F_2

$$\frac{d^2\sigma}{dx\,dQ^2} = \left(\frac{2\pi\alpha^2}{xQ^4}\right)\cdot\left(2\cdot(1-y)F_2(x,Q^2)+y^2F_2(x,Q^2)\right)$$

for y=1

$$\frac{d^2\sigma}{dx\,dQ^2} = \left(\frac{2\pi\alpha^2}{xQ^4}\right)\cdot F_2(x,Q^2)$$

$$F_2(x,Q^2) = x\sum_q e_q^2\left[q(x,Q^2)+\bar{q}(x,Q^2)\right]$$



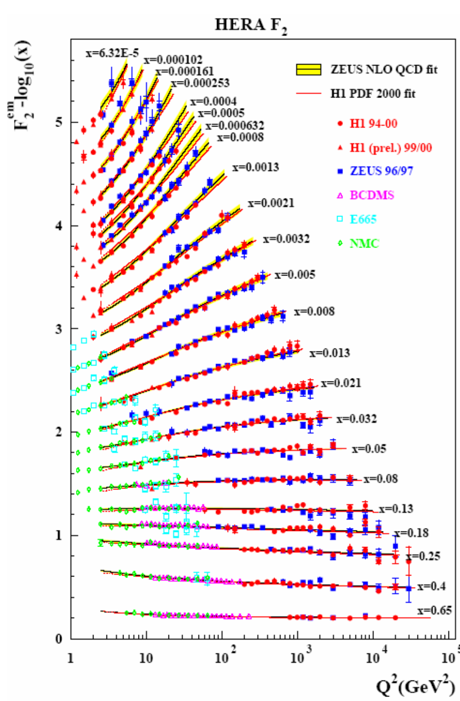
$F_2(x)$

Large increase of $F_2(x)$ for very small x

When does the rise stop ??

$F_2(x,Q^2)$

Described by QCD evolution



Experimental determination of the gluon density

Using the DGLAP evolution eq. one finds for $F_2(x,Q^2)$:

$$\frac{dF_2(x,Q^2)}{d\ln Q^2} = x\sum_i e_i^2 \frac{\alpha_s(Q^2)}{2\pi}\cdot\int_x^1 \frac{dz}{z}\left[P_{qq}(\frac{x}{z})q_i(z,Q^2) + P_{qg}(\frac{x}{z})g(z,Q^2)\right]$$

For small x ($x<10^{-2}$):

quark pair production through gluon splitting dominant (1/x gluon spectrum):

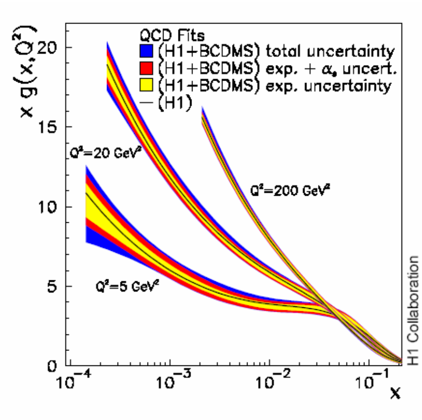
$$\rightarrow \quad P_{qg}(\frac{x}{z})g(z,Q^2) \quad \text{dominant}$$

As an approximation one finds:

$$x\cdot g(x,Q^2) \approx \frac{27\pi}{10\alpha_s(Q^2)}\cdot\frac{dF_2(x,Q^2)}{d\ln Q^2}$$

i.e. scaling violation of F_2 at small x measures the gluon density.

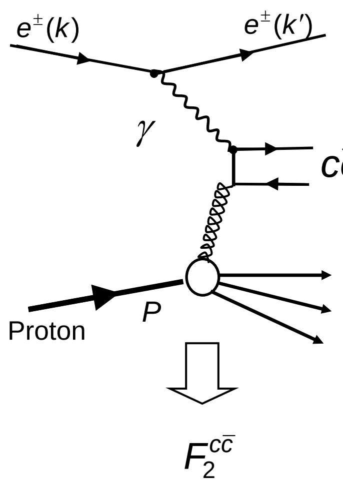
Gluon density $g(x,Q^2)$:



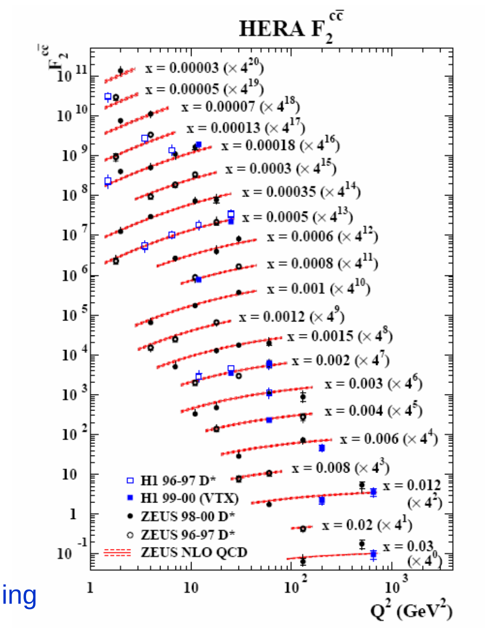
In practice one makes a global DGLAP fit to the measured $F_2(x,Q^2)$

$$\alpha_s(M_Z^2) = 0.115$$

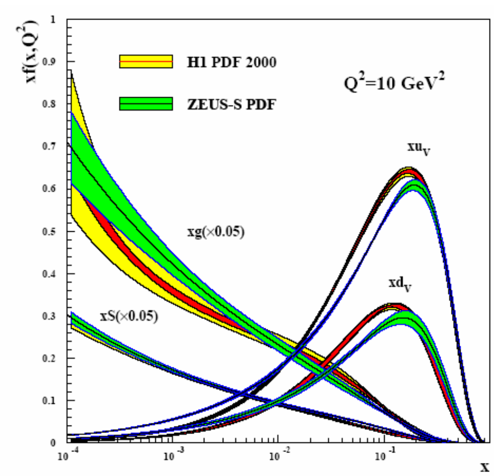
Heavy quark density



(measured by using jets containing D* mesons)

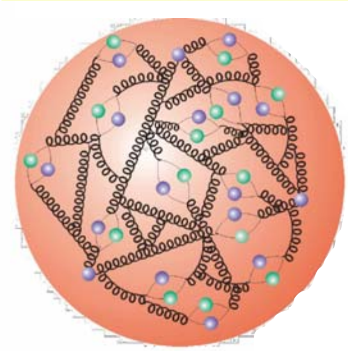


Structure of the proton as seen by HERA

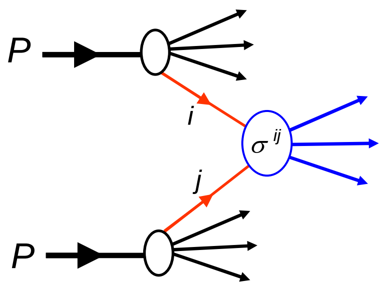


Valenzquarks = $\int u_v(x) + d_v(x)\, dx = 3$

Gluonen = $\int g(x)\, dx > 30$



4. Factorization – QCD at LHC



Generalized cross section for n parton production can be **factorized**:

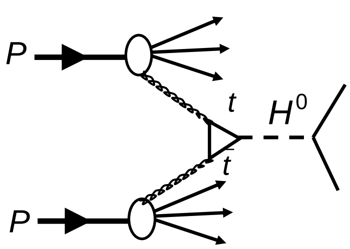
- PDFs $f_i(x,\mu^2)$ and $f_j(x,\mu^2)$
- “short distance” cross section, $\sigma^{ij\to k1\ldots k2}$, pertubatively calculable

$$\sigma^n = \sum_{i,j,k_1,\ldots k_n=q,\bar{q},g} \int dx_1 dx_2\, f_i(x_1,\mu^2)\, f_j(x_2,\mu^2)\, \hat{\sigma}^{ij\to k_1,\ldots k_n}$$

Factorization scale is the value μ at which the structure functions are defined and at which the distinction between short//long range parts is done.

Parton densities important to predict signal and background at LHC:

Higgs-production at LHC



(main production process)

Can we extrapolate from HERA?