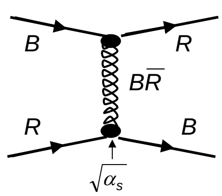
V. Experimental studies of QCD

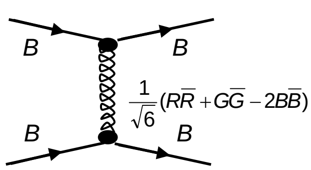
1. Elements of QCD
2. Test of QCD in e+e- annihilation
3. Study of QCD in DIS
4. QCD at LHC

1. Elements of QCD - SU(3) Theory

(i) Quarks in 3 color states: **R**, **G**, **B**

(ii) "colored" gluons (color charge) as exchange vector boson





$SU(3): \quad 3 \times \overline{3} = 8 \oplus 1$

➡ Gluons of color octett:

$R\overline{B}, R\overline{G}, G\overline{B}, G\overline{R}, B\overline{G}, B\overline{R}$

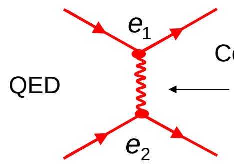
$\frac{1}{\sqrt{2}}(R\overline{R} - G\overline{G})$

$\frac{1}{\sqrt{6}}(R\overline{R} + G\overline{G} - 2B\overline{B})$

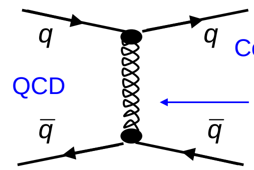
➡ Ninth state = color singlett does not take part in interaction

$\frac{1}{\sqrt{3}}(R\overline{R} + G\overline{G} + B\overline{B})$

1.1 Color factors



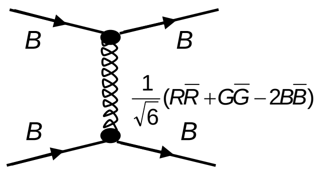
Coupling strength $e_1 e_2 \alpha$



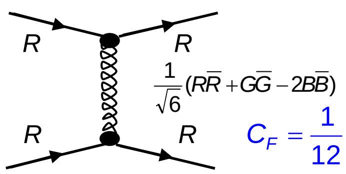
Coupling strength: $\underbrace{\frac{1}{2} c_1 c_2}_{C_F} \alpha_s$

C_F Color factor

Examples:

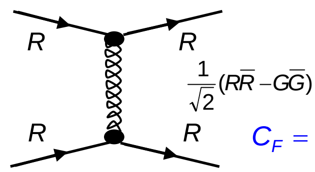


$$C_F = \frac{1}{2}\cdot\frac{2}{\sqrt{6}}\cdot\frac{2}{\sqrt{6}} = \frac{1}{3}$$



$$C_F = \frac{1}{12}$$

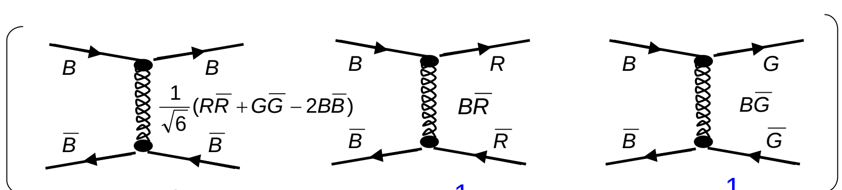
+



$$C_F = \frac{1}{4}$$

Color factor for qq color singlet state (meson): $\frac{1}{\sqrt{3}}(R\bar{R}+G\bar{G}+B\bar{B})$

$3\times$



$C_F = \frac{1}{3}$ $\qquad C_F = \frac{1}{2}$ $\qquad C_F = \frac{1}{2}$

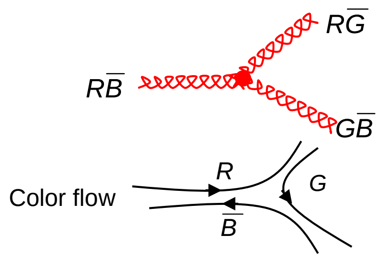
$$\Rightarrow \quad C_F = 3\cdot\left(\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left(\frac{1}{3}+\frac{1}{2}+\frac{1}{2}\right)\right) = \frac{4}{3}$$

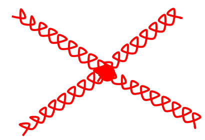
Color singlet meson is composed of 3 different possibilities

In the case of a color singlet, each initial and final state carries a factor $\frac{1}{\sqrt{3}}$

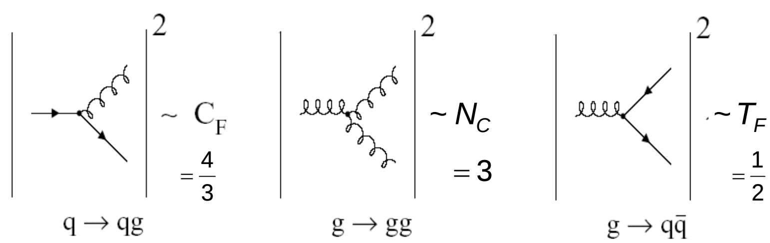
Triple and quadruple gluon Vertex

Gluons carry color charges: important feature of SU(3)





Color factors



1.2 Evidence of colored spin ½ quarks

a) clear two-jet event structure in $e^+e^- \to \; hadrons\,(q\overline{q})$

b) $R_{had} = \dfrac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)}$ indicates fractional charges and N_C=3

c) Further indications for N_C=3:

Δ^{++} statistic problem:

Spin J(Δ^{++})=3/2 (L=0), quark content |uuu>

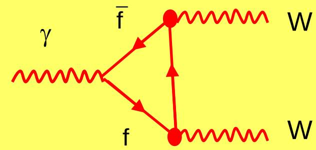
$\to \; \left|\Delta^{++}\right\rangle = \left|u\uparrow u\uparrow u\uparrow\right\rangle$ forbidden by Fermi statistic

Solution is additional quantum number for quarks (color)

$$\left|\Delta^{++}\right\rangle = \frac{1}{\sqrt{6}}\varepsilon_{ijk}\left|u_i\uparrow u_j\uparrow u_k\uparrow\right\rangle \quad i,j,k = \text{color index}$$

- Triangle anomaly

  Divergent fermion loops

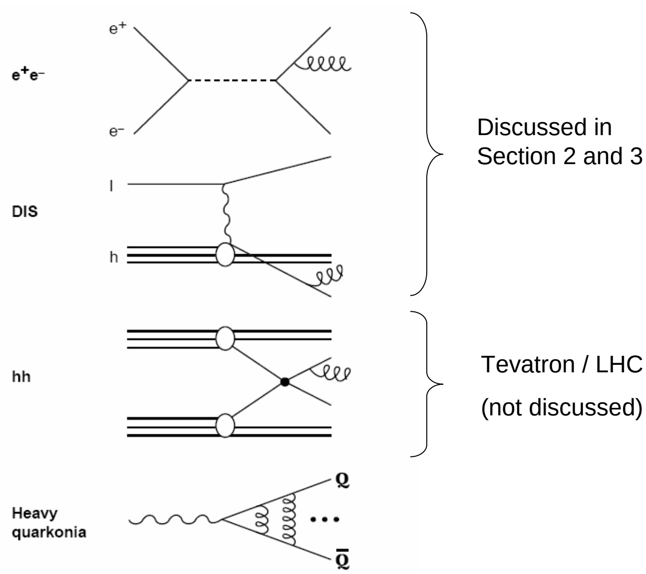


Divergences cancel if $N_C = 3$:

$$0 = \sum_f Q_f = (-1) + (-1) + (-1) + N_C \cdot \left[\left(\frac{2}{3} - \frac{1}{3}\right) \cdot 3 \right]$$

3 generations of u/d-type quark

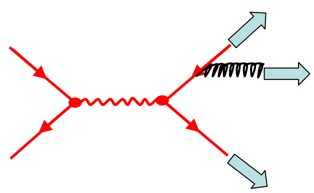
1.3 Test of QCD in different processes



2. Test of QCD in e^+e^- annihilation

2.1 Discovery of the gluon

Discovery of 3-jet events by the TASSO collaboration (PETRA) in 1977:



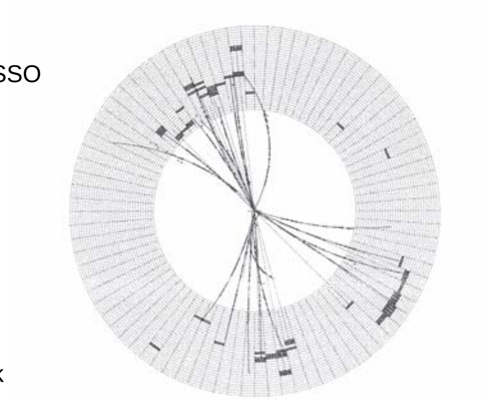


Fig. 11.12 A three-jet event observed by the JADE detector at PETRA.

3-jet events are interpreted as quark pairs with an additional hard gluon.

$$\frac{\#3-\text{jet events}}{\#2-\text{jet events}} \approx 0.15 \sim \alpha_s$$

at $\sqrt{s}$=20 GeV

→ α_s is large

2.2 Spin of the gluon

Ellis-Karlinger angle

Ordering of 3 jets: $E_1>E_2>E_3$

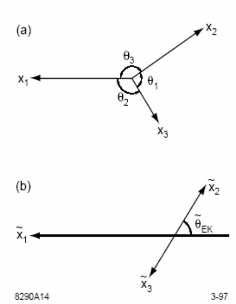


Figure 8: (a) Representation of the momentum vectors in a three-jet event, and (b) definition of the Ellis-Karliner angle.

Measure direction of jet-1 in the rest frame of jet-2 and jet-3: θ_{EK}

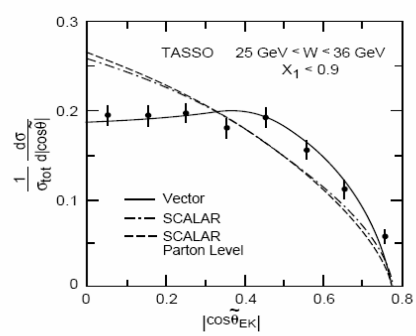


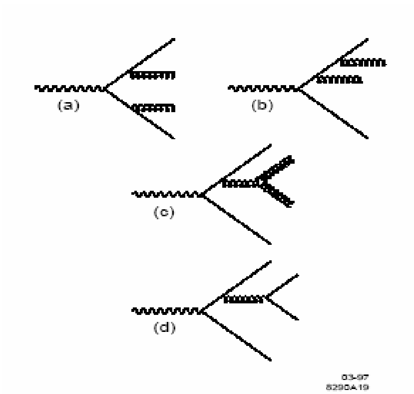
Figure 9: The Ellis-Karliner angle distribution of three-jet events recorded by TASSO at $Q \sim 30$ GeV [18]; the data favour spin-1 (vector) gluons.

Gluon spin J=1

2.3 Multi-jet events and gluon self coupling

Non-abelian gauge theory (SU(3))

4-jet events



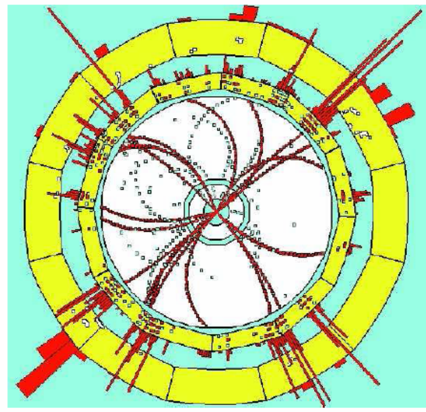


Figure 1: Hadronic event of the type $e^+e^- \to 4$ jets recorded with the ALEPH detector at LEP-I.

Multiple jets and jet algorithm

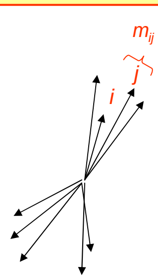
Jet Algorithm

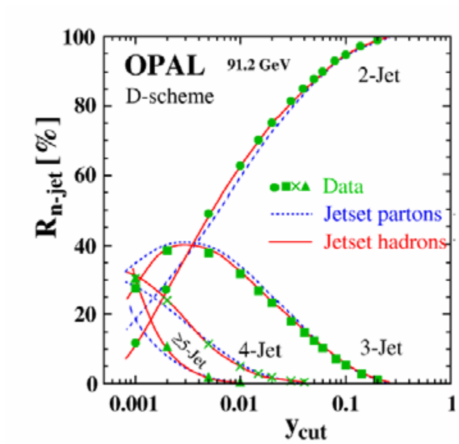
Hadronic particles are i and j grouped to a pseudo particle k as long as the invariant mass is smaller than the **jet resolution parameter**:

$$\frac{m_{ij}^2}{s} < y_{cut}$$

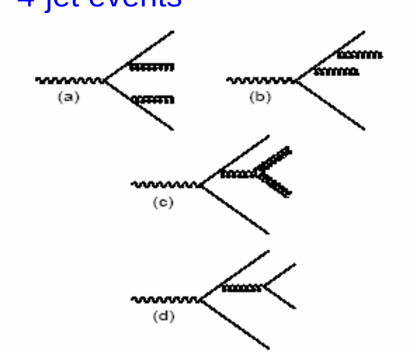
m_{ij} is the invariant mass of i and j.

Remaining pseudo particles are **jets.**

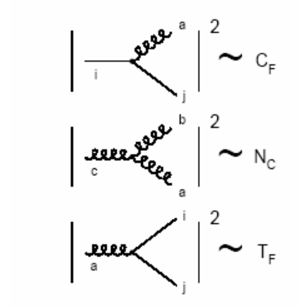




4-jet events



Color factors:



4-jet cross section:

$$\frac{1}{\sigma_0}d\sigma^4 = \left(\frac{\alpha_s C_F}{\pi}\right)^2 \left[F_A + \left(1 - \frac{1}{2}\frac{N_C}{C_F}\right) F_B + \frac{N_C}{C_F} F_C\right]$$

$$+ \left(\frac{\alpha_s C_F}{\pi}\right)^2 \left[\frac{T_F}{C_F} N_f F_D + \left(1 - \frac{1}{2}\frac{N_C}{C_F}\right) F_E\right]$$

$F_{A,B,C,D,E}$ are kinematic functions

Group	N_C	C_F	T_F
U(1)	0	1	1
$U(1)_3$	0	1	3
SU(N)	N	$(N^2-1)/2N$	1/2
SU(3)	3	4/3	1/2

Angular correlation of jets in 4-jet events

Exploiting the angular distribution of 4-jets:

- Bengston-Zerwas angle

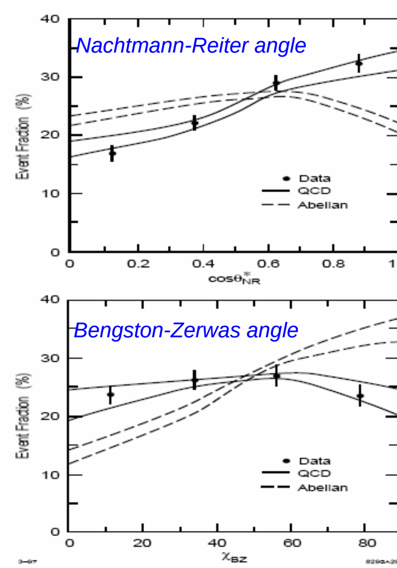
  $\cos\chi_{BZ} \propto (\vec{p}_1 \times \vec{p}_2)\cdot(\vec{p}_3 \times \vec{p}_4)$

- Nachtmann-Reiter angle

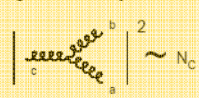
  $\cos\theta_{NR} \propto (\vec{p}_1 - \vec{p}_2)\cdot(\vec{p}_3 - \vec{p}_4)$

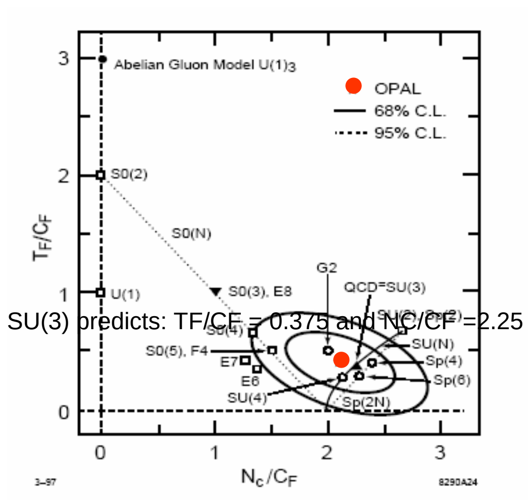
Allows to measure the ratios T_F/C_F and N_C/C_F

SU(3) predicts: $T_F/C_F = 0.375$ and $N_C/C_F = 2.25$



If $N_C/C_F \neq 0$ $\rightarrow$ contribution from gluon self-coupling in the 4-jet events





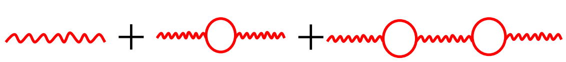
SU(3) predicts: TF/CF = 0.375 and NC/CF=2.25

Confirms QCD prediction (SU(3)) and gluon self-coupling:

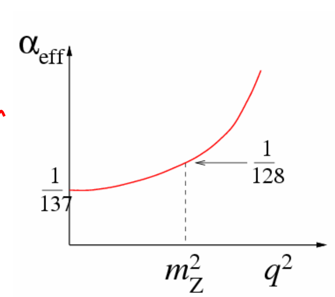
$T_F/C_F = 0.375$ and $N_C/C_F = 2.25$

2.4 Strong coupling constant α_s

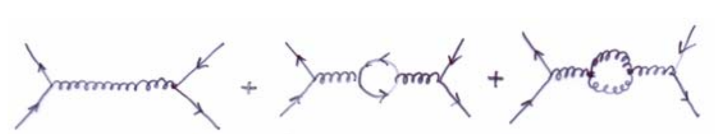
QED: Running coupling constant



$$\alpha(q^2) = \frac{\alpha}{1 - \frac{\alpha}{3\pi} \cdot \sum_f Q_f^2 \cdot \log \frac{q^2}{m_f^2}}$$

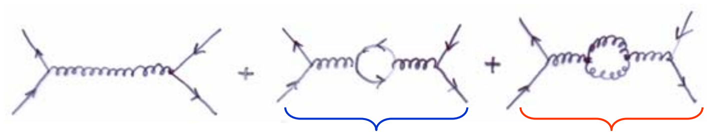


QCD:



Running ??

Propagator corrections:



Effective strong coupling $\alpha_s(Q^2)$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\frac{1}{12\pi}(33 - 2n_f)\log\frac{Q^2}{\mu^2}}$$

n_f = active quark flavors
μ^2 = renormalization scale
conventionally $\mu^2 = M_Z^2$

$$\beta_0 = \frac{1}{12\pi}(33 - 2n_f)$$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\beta_0 \log\frac{Q^2}{\mu^2}}$$

For $Q^2 \to \infty$ $\alpha_s \to 0$:
At large Q^2 quarks are asymptotically free

Introduce scale Λ_{QCD} at which (perturbative) solutions diverge:

$$\frac{1}{\alpha_s(Q^2)} - 0 = \beta_0 \ln(Q^2/\Lambda_{QCD}^2)$$

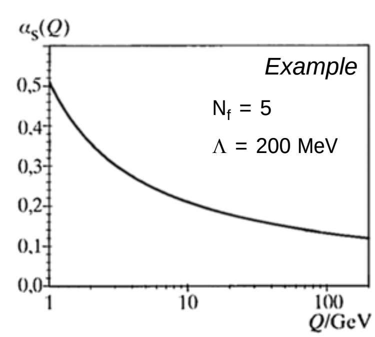
$\Lambda_{QCD} \approx 200\,\text{MeV}$
(parameter, must be determined experimentally)

$$\Rightarrow \quad \alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda_{QCD}^2)}$$

→ Asymptotic freedom for large Q^2

For large Q^2 quarks can be treated as free particles: → Quark Parton Model

Gross&Wilczek (1973), Politzer (1974)



The Nobel Prize in Physics 2004

David J. Gross	H. David Politzer	Frank Wilczek

"for the discovery of asymptotic freedom in the theory of the strong interaction"

2.5 Measurement of strong coupling α_s

➡ α_s measurements are done at fixed scale Q^2: $\alpha_s(Q^2)$

a) α_s from total hadronic cross section

$$\sigma_{had}(s) = \sigma_{had}^{QED}(s)\left[1 + \frac{\alpha_s(s)}{\pi} + 1.411 \cdot \frac{\alpha_s(s)^2}{\pi^2} + \ldots\right]$$

$$R_{had} = \frac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)} = 3\sum Q_q^2 \left\{1 + \frac{\alpha_s}{\pi} + 1.411\frac{\alpha_s^2}{\pi^2} + \ldots\right\}$$

Not very precise.

➡ $\alpha_s(s)$

b) α_s from hadronic event shape variables

3-jet rate: $R_3 \equiv \frac{\sigma_{3-jet}}{\sigma_{had}}$ depends on α_s

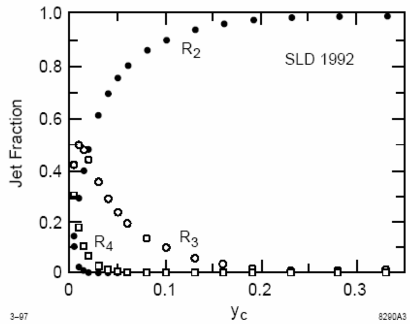
3-jet rate is measured as function of a jet resolution parameter y_{cut}

QCD calculation provides a theoretical prediction for $R_3^{theo}(\alpha_s, y_{cut})$

$\rightarrow$ fit $R_3^{theo}(\alpha_s, y_{cut})$ to the data to determine α_s

Similarly other event shape variables (sphericity, thrust,…) can be used to obtain a prediction for α_s

➡ $\alpha_s(s)$

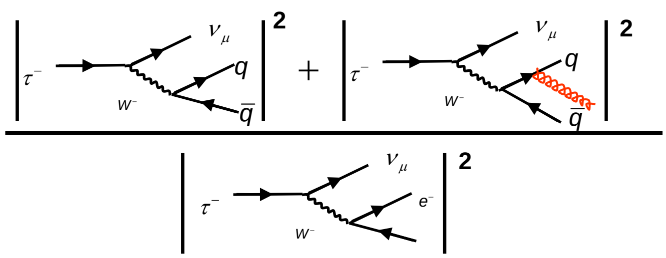


n-jet rate as function of y_{cut}

c) α_s from hadronic τ decays

$$R_{had}^{\tau} = \frac{\Gamma(\tau \to \nu_\tau + Hadrons)}{\Gamma(\tau \to \nu_\tau + e\bar{\upsilon}_e)} \sim f(\alpha_s)$$

$R_{had}^{\tau} =$



$$R_{had}^{\tau} = R_{had}^{\tau,0}\left(1 + \frac{\alpha_s(m_\tau^2)}{\pi} + \ldots\right)$$

d) α_s from DIS (deep inelastic scattering)

Running of α_s and asymptotic freedom

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda_{QCD}^2)}$$