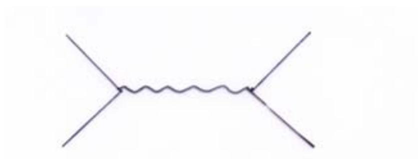
4. Higher orders

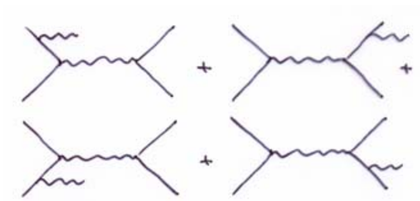
- Prediction of measurable higher order corrections
- Important to compare QED predictions to measurements

4.1 Radiative corrections to "Born" / "tree" level predictions



Born diagram

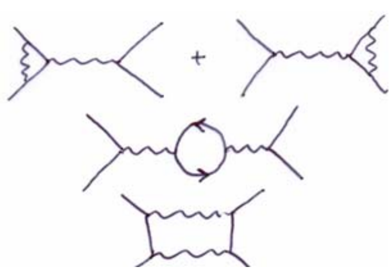
$$\frac{d\sigma^0_{f\bar{f}}}{d\Omega}$$



Bremsstrahlung corrections

$$\frac{d\sigma^{Brems}_{f\bar{f}\gamma}}{d\Omega}$$

Soft and **hard** photons



Virtual corrections:

$$\frac{d\sigma^{Virtual}_{f\bar{f}}}{d\Omega}$$

- Vertex corrections
- Propagator corrections
- Box corrections

Experimental cross section has to be compared to "**Born+corrections**":

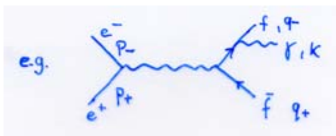
$$\left(\frac{d\sigma_{f\bar{f}(\gamma)}}{d\Omega}\right)_{\exp} \Leftrightarrow \frac{d\sigma^0_{f\bar{f}}}{d\Omega}\left(1+\delta_{Brems}+\delta_{Virtual}\right)$$

Remark: at higher energies also electro-weak corrections are important

4.2 Bremsstrahlung

a.) soft photon radiation: $E_\gamma < \Delta E << \sqrt{s}$

No influence on ff prod./kinematics

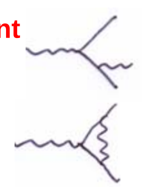


$$\frac{d\sigma_{f\bar{f}\gamma}}{d\Omega} = \frac{d\sigma^0_{f\bar{f}}}{d\Omega} \cdot \underbrace{R(p_+,p_-,q_+,q_-,k)\cdot k^0 dk^0\, d\Omega_\gamma}$$

Radiative corrections factorize:

$$\sim \frac{2\alpha}{\pi}\log(\frac{s}{m_f^2})\frac{dk^0}{k^0}$$

<u>Problems:</u>



- Corrections are divergent for $k^0 \to 0$ ($E_\gamma \to 0$): **infra-red divergent**
- If E_γ < ΔE=detection threshold: $e^+e^- \to f\bar{f}\gamma \Leftrightarrow e^+e^- \to f\bar{f}$
- Vertex corrections to $e^+e^- \to f\bar{f}$ are also divergent

⇒Treat vertex + bremsstrahlung corrections at the same time:

$$\left.\frac{d\sigma_{f\bar{f}}}{d\Omega}\right|^{1st\ order} = \frac{d\sigma^0_{f\bar{f}}}{d\Omega} \cdot \left[\beta(s,m_e,m_f)\cdot\log\frac{\Delta E}{\sqrt{s}} + ...\right]$$

⬅ Divergences cancel in sum

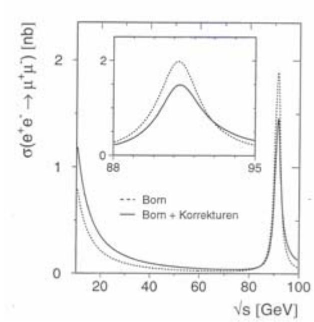
b.) hard photon radiation: $E_\gamma > \Delta E$

Final state with a detectable photon: $f\bar{f}\gamma$

⇒ Photon changes the kinematics and also production cross sections:

<u>Initial state radiation (ISR):</u> ⇒ reduced effective CMS energy s'=zs

$$\sigma_{ff(\gamma)} = \int_{4m_f^2/s}^{1} G(z)\sigma^0_{ff}(zs)\,dz \qquad z = 1 - \frac{2E_\gamma}{\sqrt{s}}$$



Radiator function G(z) describes photon radiation ⇒ large effects if σ^0_{ff} has large s dependence.

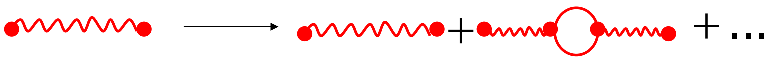
<u>Final state radiation (FSR):</u>

$$\sigma_{ff(\gamma)} = \sigma^0_{ff}(1 + \frac{3\alpha}{4\pi} + ...) \approx 1.0017 \cdot \sigma^0_{ff}$$

4.3 Propagator corrections and running couplings

$\rightarrow$ Leads to a change of the effective coupling constant

Propagator



$$\frac{e^2}{q^2} \longrightarrow \frac{e^2}{q^2}\left[1-\Pi^{\gamma}(q^2)\right]$$

$$e \longrightarrow e\left[1-\frac{1}{2}\Pi^{\gamma}(q^2)\right]$$

divergent

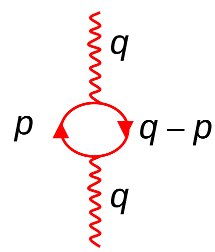
keeping terms in same O(α)

In the Thomson limit $q^2 \rightarrow 0$ effective charge should be equal to "e" but vacuum polarization leads to divergent correction $-\frac{1}{2}e\,\Pi^{\gamma}(0)$

$\Rightarrow$ Redefine the charge used in Feynman rules as "bare" charge $\mathbf{e_0}$ which is not measurable. $\mathbf{e_0}$ related to the physical charge **e**:

$e_0 = e + \delta e$ with renormalization condition $\delta e = \frac{1}{2}e\,\Pi^{\gamma}(0)$

Remark: Divergent Loop Integrals



$$-i\frac{g_{\mu\nu}}{q^2} \rightarrow -i\frac{g_{\mu\nu}}{q^2}+\frac{(-i)}{q^2}I_{\mu\nu}\frac{(-i)}{q^2}$$

$$I_{\mu\nu}(q^2)=(-1)^n\int\frac{d^4p}{(2\pi)^4}\,\mathrm{Trace}\left\{(\mathrm{i}e\gamma^{\mu})\frac{i(\not p+m)}{p^2-m^2}(\mathrm{i}e\gamma^{\mu})\frac{i(\not p-\not q+m)}{(p-q)^2-m^2}\right\}$$

Integral over all possible momenta p:

Logarithmically divergent

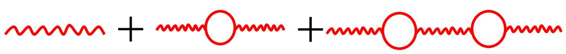
$$\frac{e_0^2}{q^2} \longrightarrow \frac{e^2}{q^2}\left[1+\Pi^\gamma(0)-\Pi^\gamma(q^2)\right] \quad \text{(to 1st order in } \Pi(q^2))$$

$$= \frac{e^2}{q^2}\left[1-\widehat{\Pi}^\gamma(q^2)\right] \quad \text{with} \quad \widehat{\Pi}^\gamma(q^2) = \Pi^\gamma(q^2) - \Pi^\gamma(0)$$

finite: $\widehat{\Pi}^\gamma(q^2)$; divergent: $\Pi^\gamma(q^2)$, $\Pi^\gamma(0)$

For $q^2 >> m^2$ (fermion masses)

$$\widehat{\Pi}^\gamma(q^2) = -\frac{\alpha}{3\pi}\sum_f Q_f^2 \log\frac{q^2}{m_f^2}$$



$$\left[1-\widehat{\Pi}^\gamma(q^2)+[\widehat{\Pi}^\gamma(q^2)]^2-\ldots\right]=\left[\frac{1}{1+\widehat{\Pi}^\gamma(q^2)}\right]$$

(geometric progression)

Physical charge at $q^2=0$

$$e^2(q^2) = \frac{e^2}{1+\widehat{\Pi}^\gamma(q^2)}$$

(including higher orders)

Running coupling α

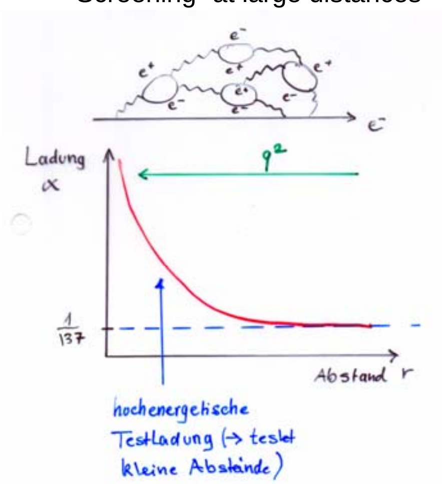
$$\alpha(q^2) = \frac{\alpha}{1+\widehat{\Pi}^\gamma(q^2)} = \frac{\alpha}{1-\frac{\alpha}{3\pi}\cdot\sum_f Q_f^2\cdot\log\frac{q^2}{m_f^2}}$$

$\alpha(q^2=0) = \frac{1}{137}$ (Thomson limit)

$\alpha(q^2=45^2 GeV^2) = \frac{1}{129}$ (PETRA)

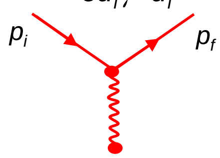
$\alpha(q^2=91^2 GeV^2) = \frac{1}{128}$ (LEP)

"Screening" at large distances



4.4 Vertex corrections and anomalous magnetic moment

$-e\bar{u}_f \gamma^\mu u_i$



$=$

Gordon decomposition:

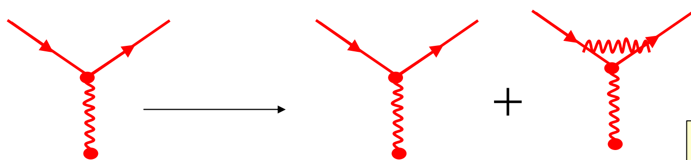
$$\frac{1}{2m}\bar{u}_f\left((p_f + p_i)^\mu + i\sigma^{\mu\nu}(p_f - p_i)^\nu\right)$$

Interaction of spinless charges

Interaction due to spin

Magnetic moment in the non-relativistic limit: $\vec{\mu} = -g \cdot \mu_B \cdot \vec{s} \quad g = 2$

Vertex corrections:



$\vec{\mu} = -2 \cdot \mu_B \cdot \vec{s}$

$\vec{\mu} = -(2 + \frac{\alpha}{\pi}) \cdot \mu_B \cdot \vec{s}$

$g = 2 + \frac{\alpha}{\pi}$

$a = \frac{g-2}{2} = \frac{\alpha}{2\pi}$

Higher order corrections to g-2

Radiative corrections g-2 are calculated to the 4-loop level:

Feynman Graphs	
O(α)	1
O(α²)	7
O(α³)	72
O(α⁴)	891
til O(α⁴)	971

Fig. 8.2 The Feynman graphs which have to be evaluated in computing the α^3 corrections the lepton magnetic moments (after Lautrup *et al.* 1972).

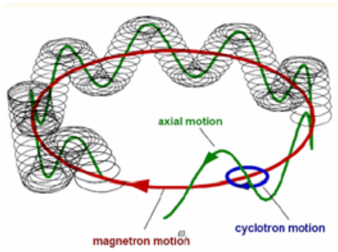
Most precise QED prediction.

Kinoshita et al.

Electron g-2 measurement

H. Dehmelt et al., 1987

G. Gabrielse et al., 2006



Cyclotron frequency $\omega_C = 2\frac{eB}{2mc}$

Lamor / spin prescession frequency $\omega_s = g\frac{eB}{2mc}$

Experimental method:

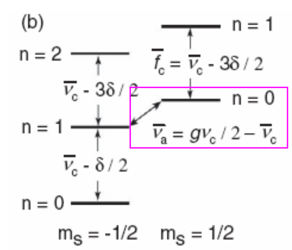
Storage of **single** electrons in a Penning trap (electrical quadrupole + axial B field)

⇒ complicated electron movement (cyclotron and magnetron precessions).

Idea: bound electron (**geonium**):

$$E(n, m_s) = \frac{g}{2}h\nu_c m_s + \left(n+\frac{1}{2}\right)h\bar{\nu}_c - \frac{1}{2}h\delta\left(n+\frac{1}{2}+m_s\right)^2$$

Energy levels of single electron



Trigger RF induced transitions (ω_a) between different n states or spin flips:

$$\omega_a = \omega_s - \omega_c = (g-2)\mu_B B$$

$$a = \frac{g-2}{2} = \frac{\omega_s - \omega_c}{\omega_c}$$

$a_{e^-} = 0.001159\,652\,188\,4\,(43)$

$a_{e^+} = 0.001159\,652\,187\,9\,(43)$

R.S. van Dyck et al. 1987

$a_e = 0.001159\,652\,180\,85\,(76)$

G. Gabrielse et al. 2006

⇒ most precise value of α:

$\alpha^{-1}(a_e) = 137.035\,999\,710\,(96)$

For comparison α from Quanten Hall

$\alpha^{-1}(qH) = 137.036\,003\,00\,(270)$

$$a_e = \frac{\alpha}{2\pi} - 0.328...\left(\frac{\alpha}{\pi}\right)^2 + 1.182....\left(\frac{\alpha}{\pi}\right)^3 - 1.505....\left(\frac{\alpha}{\pi}\right)^4$$

Theory

$a_e = 0.001159\,652\,133\,(290)$

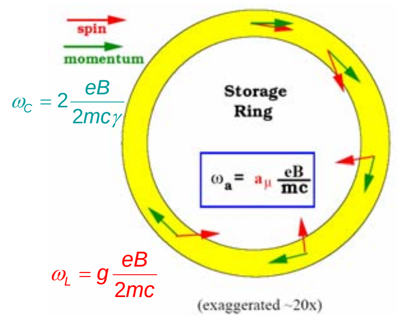
$a_e = 0.001159\,652\,180\,85\,(76)$

Phys. Rev. Lett. ***97****, 030801 (2006)*
Phys. Rev. Lett. ***97****, 030802 (2006)*

Muon g-2 experiment

Principle:

- store polarized muons in a storage ring; revolution with cyclotron frequency ω_c
- measure spin precession around the magnetic dipole field relative to the direction of cyclotron motion



Precession:

$$\vec{\omega}_a = -\frac{e}{m_\mu c}\left[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})\vec{\beta}\times\vec{E}\right]$$

Difference between Lamor and cyclotron frequency

Effect of electrical focussing fields (relativistic effect).

$= 0$ for $\gamma = 29.3$

$\Leftrightarrow p_\mu = 3.094$ GeV/c

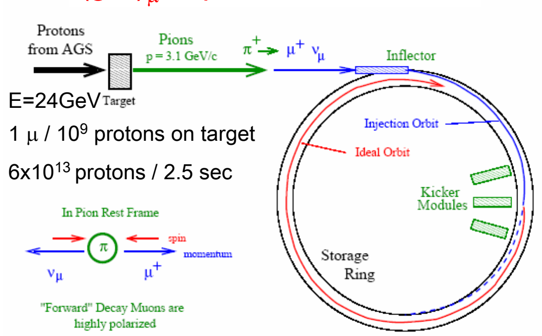
First measurements:

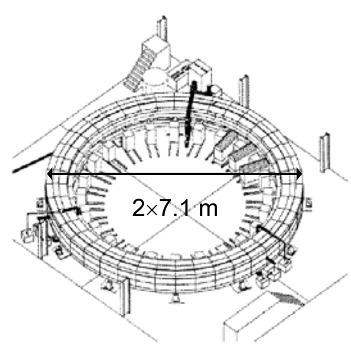
CERN 70s

$a_{\mu^-} = 0.001165937(12)$

$a_{\mu^+} = 0.001165911(11)$

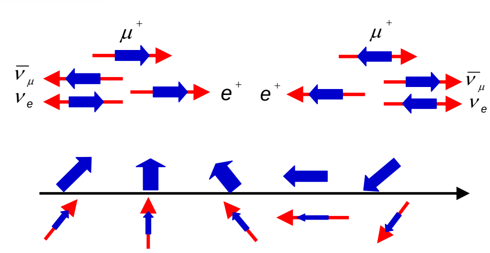
$(g\text{-}2)_\mu$ Experiment at BNL

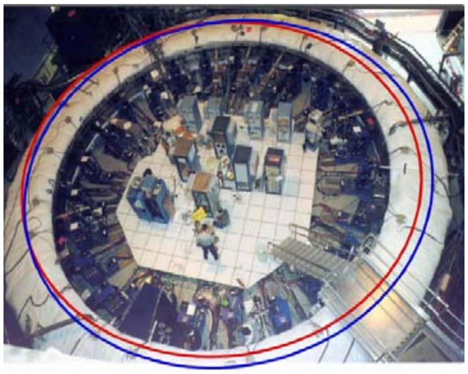




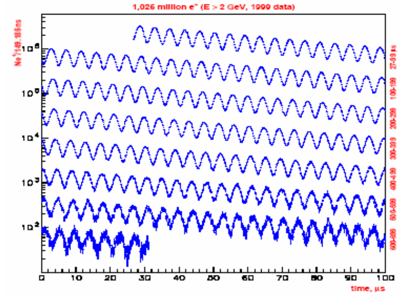
V-A structure of weak decay:

Use high-energy e^+ from muon decay to measure the muon polarization

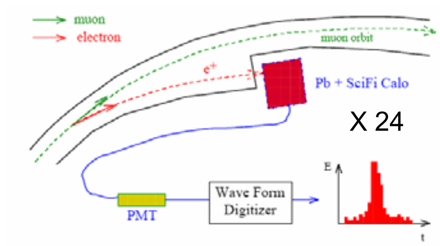




Measure electron rate:

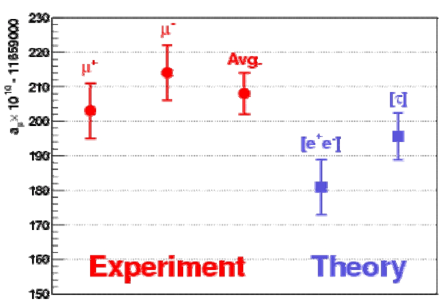


$$N(t) = N_0\, e^{-\lambda t}\left[1 + A\cos(\omega_a t + \varphi)\right]$$



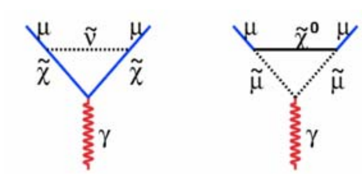
$$\frac{\omega_a}{2\pi} = 229023.59(16)\mathrm{Hz}$$

$$a_{\mu^+} = 11\,659\,203(8)\times 10^{-10}(0.7ppm)$$
$$a_{\mu^-} = 11\,659\,214(8)\times 10^{-10}(0.7ppm)$$
$$a_{\mu} = 11\,659\,208(6)\times 10^{-10}(0.5ppm)$$



About 2.6σ deviation:

- Often interpreted as sign of New Physics: SUSY contributions
- careful: “Theory” has uncertainties!



Potential SUSY contributions:

Remarks: Theoretical prediction of a_μ

Beside pure QED corrections there are weak corrections (W, Z) exchange and „hadronic corrections“

$$a_\mu = a_\mu^{QED} + a_\mu^{Had} + a_\mu^{EW}$$

(For the electron with much lower mass the hadronic and weak corrections are suppressed, and can be neglected.)

→ Determination of hadronic corrections is difficult and is in addition based on data: hot discussion amongst theoreticians how to correctly use the data.

Hadronic corrections