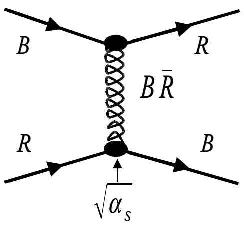
Experimental studies of QCD

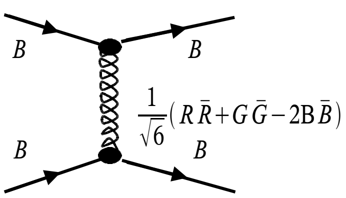
1. Elements of QCD
2. Tests of QCD in e^+e^- annihilation
3. Studies of QCD in DIS
4. QCD in $pp\,(p\bar{p})$ collisions

1. Elements of QCD - SU(3) Theory

(i) Quarks in 3 color states: **R, G, B**

(ii) "colored" gluons as exchange vector boson





$$\mathrm{SU}(3):\ 3\times\bar{3}=8\oplus 1$$

➡ Gluons of color octet:

$$R\bar{B},\ R\bar{G},\ G\bar{B},\ G\bar{R},\ B\bar{G},\ B\bar{R}$$

$$\frac{1}{\sqrt{2}}(R\bar{R}-G\bar{G})$$

$$\frac{1}{\sqrt{6}}(R\bar{R}+G\bar{G}-2B\bar{B})$$

➡ Ninth state = color singlet

does not take part in interaction

$$\frac{1}{\sqrt{3}}(R\bar{R}+G\bar{G}+B\bar{B})$$

Quantum Electrodynamics

To recapitulate the previously discussed material.

Lagrangian for free spin ½ particle:

$$L(x,t) = i\overline{\psi}(x,t)\gamma^{\mu}\partial_{\mu}\psi(x,t) - m\overline{\psi}(x,t)\psi(x,t)$$

Applying the Euler-Lagrange formalism leads to the Dirac equation.

Invariance under <u>Local</u> Gauge Transformation

Demanding invariance under local phase transformation of the free Langrangian (local gauge invariance):

$$\psi(x) \to \psi(x) = e^{i\alpha(x)}\psi(x)$$

requires the substitution:

$$i\partial_{\mu} \to i\partial_{\mu} + eA_{\mu}(x)$$

If one defines the tansformation of A under local gauge transformation as

$$A_{\mu}(x) \to A_{\mu}(x) + \partial_{\mu}\alpha(x)$$

one finds invariance of L:

$$L(x) \xrightarrow{\psi \to \psi\, e^{i\alpha(x)}} L(x)$$

To interpret the introduced field A_μ as photon field requires to complete the Langrangian by the corresponding field energy:

$$\boxed{L = \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi + e\overline{\psi}\gamma^{\mu}\psi A_{\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$$

The requirement of local gauge invariance has automatically led to the interaction of the free electron with a field.

Quantum Chromodynamics – SU(3) Theory

Lagrangian is constructed with quark wave functions $\psi = \begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix}$

Invariance of the Lagrangian under <u>Local</u> <u>SU(3)</u> Gauge Transformation

$$\psi(x) \rightarrow \psi'(x) = U(x)\psi(x) = e^{i\frac{\alpha_k(x)}{2}\lambda_k}\psi(x)$$

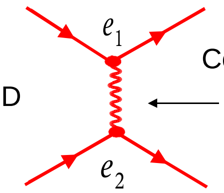
with any unitary (3 x 3) matrix $U(x)$.

$U(x)$ can be given by a linear combination of
8 Gell-Mann matrices $\lambda_1 \ldots \lambda_8$ [SU(3) group generators]

requires interaction fields – 8 gluons corresponding to these matrices

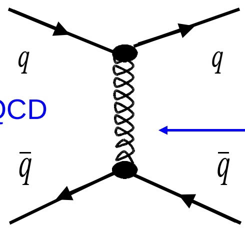
1.1 Color factors

QED



Coupling strength $e_1 e_2\, \alpha$

QCD

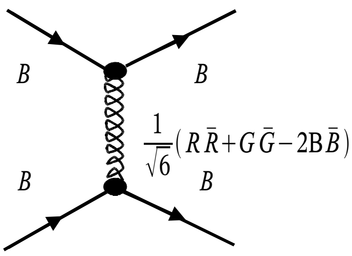


Coupling strength:

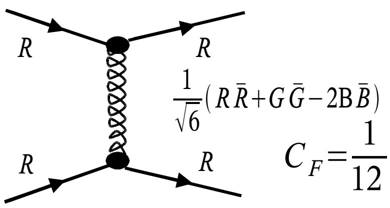
$$\underbrace{\frac{1}{2}c_1 c_2}_{C_F}\, \alpha_s$$

Color factor

Examples:

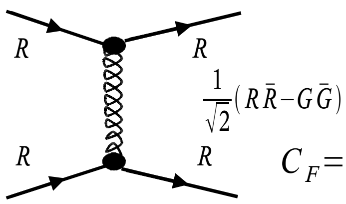


$$C_F = \frac{1}{2}\cdot\frac{2}{\sqrt{6}}\cdot\frac{2}{\sqrt{6}} = \frac{1}{3}$$



$$C_F = \frac{1}{12}$$

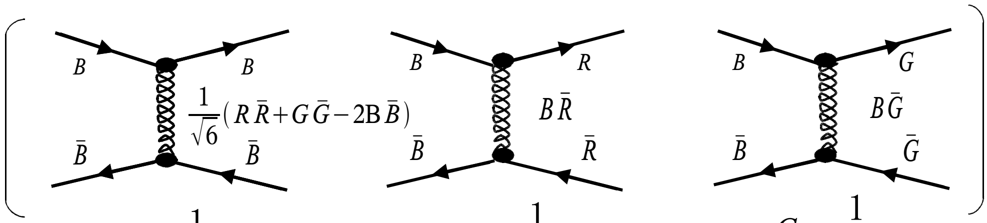
$+$



$$C_F = \frac{1}{4}$$

Color factor for $q\bar{q}$ color singlet state (meson): $\frac{1}{\sqrt{3}}(R\bar{R}+G\bar{G}+B\bar{B})$

$3\times$



$C_F=\frac{1}{3}$ $\qquad$ $C_F=\frac{1}{2}$ $\qquad$ $C_F=\frac{1}{2}$

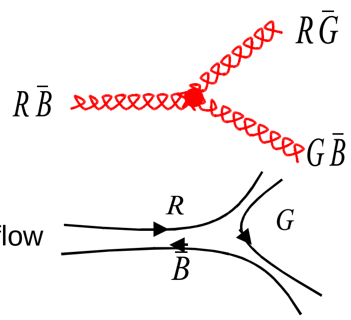
$$\Rightarrow \quad C_F=3\cdot\left(\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\left(\frac{1}{3}+\frac{1}{2}+\frac{1}{2}\right)\right)=\frac{4}{3}$$

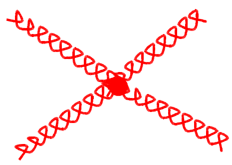
Color singlet meson is composed of 3 different possibilities

In the case of a color singlet, each initial and final state carries a factor $\frac{1}{\sqrt{3}}$

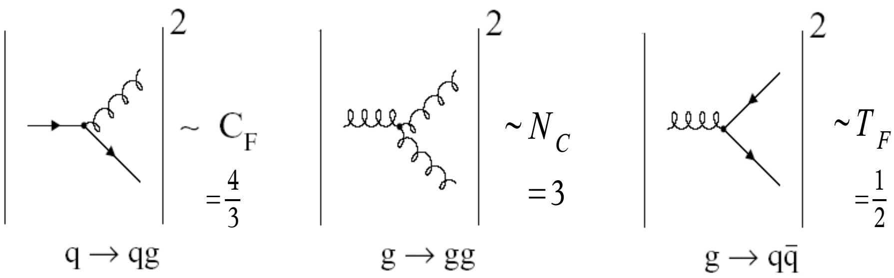
Triple and quadruple gluon Vertex

Gluons carry color charges: important feature of SU(3)





Color factors



1.2 Evidence of colored spin ½ quarks

a) successful classification of mesons and baryons

b) clear two-jet event structure in $e^+e^- \to hadrons\ (q\bar{q})$

$$\frac{d\sigma}{d\Omega} \sim (1+\cos^2\theta)$$

c) $R_{had} = \frac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)}$ indicates fractional charges and N_C=3

d) Further indications for N_C=3:

Δ^{++} (Ω_s) statistic problem:

Spin J(Δ^{++})=3/2 (L=0), quark content |uuu>

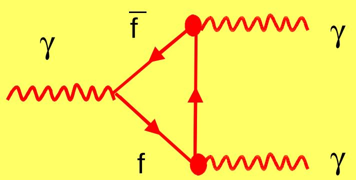
$\rightarrow\ |\Delta^{++}\rangle = |u\uparrow u\uparrow u\uparrow\rangle$ forbidden by Fermi statistics

Solution is additional quantum number for quarks (color)

$$|\Delta^{++}\rangle = \frac{1}{\sqrt{6}} \varepsilon_{ijk} |u_i\uparrow u_j\uparrow u_k\uparrow\rangle \quad i,j,k = \text{color index}$$

- Triangle anomaly

  Divergent fermion loops



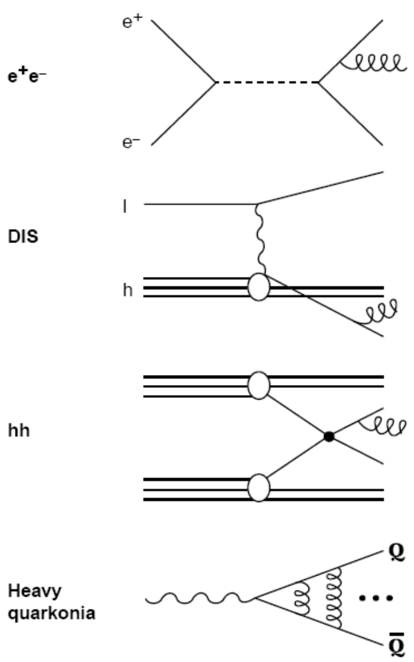
Divergence

$$\sim \sum_f Q_f = \underbrace{(-1)+(-1)+(-1)}_{\text{leptons}} + \underbrace{N_C \cdot \left[\left(\frac{2}{3} - \frac{1}{3}\right) \cdot 3 \right]}_{\text{quarks}}$$

3 generations of u/d-type quark

➡ cancels if N_C = 3

1.3 Tests of QCD in different processes



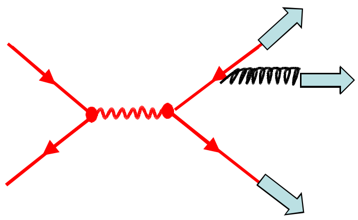
Discussed in Section 2 and 3

Tevatron / LHC (optional)

2. Test of QCD in e^+e^- annihilation

2.1 Discovery of the gluon

Discovery of 3-jet events by the TASSO collaboration (PETRA) in 1977:



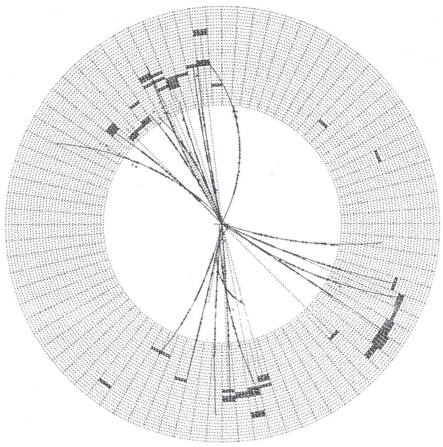


Fig. 11.12 A three-jet event observed by the JADE detector at PETRA.

3-jet events are interpreted as quark pairs with an additional hard gluon.

$$\frac{\#\ \text{3-jet events}}{\#\ \text{2-jet events}} \approx 0.15 \sim \alpha_s$$ ⇒ α_s is large

at $\sqrt{s}$=20 GeV

2.2 Spin of the gluon

Ellis-Karlinger angle

Ordering of 3 jets: $E_1 > E_2 > E_3$

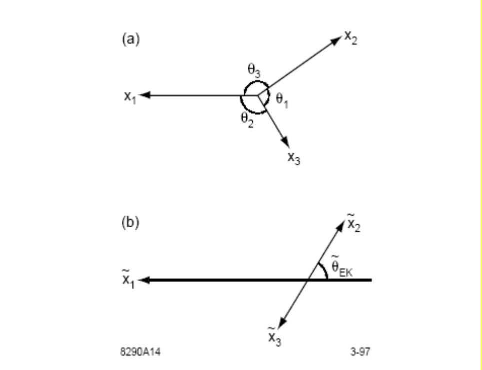


Figure 8: (a) Representation of the momentum vectors in a three-jet event, and (b) definition of the Ellis-Karliner angle.

Measure direction of jet-1 in the rest frame of jet-2 and jet-3: θ_{EK}

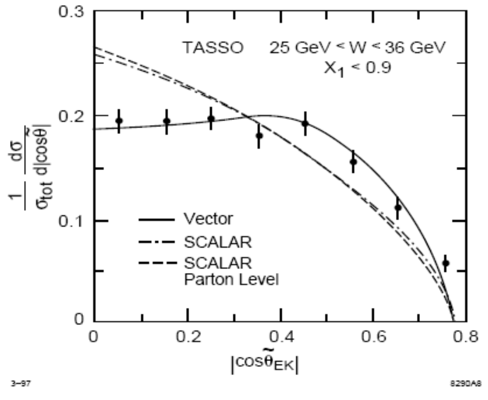


Figure 9: The Ellis-Karliner angle distribution of three-jet events recorded by TASSO at $Q \sim 30$ GeV [18]; the data favour spin-1 (vector) gluons.

Gluon spin J=1