1.2 Neutrino masses in the SM and Majorana mass terms

The observation of flavor oscillation of neutrinos:

$$\nu_e \leftrightarrow \nu_\mu$$
$$\searrow \nu_\tau \nearrow$$

implies the existence of mass states which are different from flavor states and thus massive neutrinos.

In the SM neutrino masses are set to zero because of the missing RH ν-singlet. The observation of non-vanishing neutrino masses thus indicates physics beyond the SM.

Remark: Nowadays massive neutrinos are often treated as „part of the SM" assuming the existence of RH ν. In a certain sense the additional new particle is not modifying the gauge structure of the theory which is what often is called SM.

a) Dirac mass terms

Neutrino masses can be created in the SM by extending the particle content and by adding ν_R, i.e. RH ν-singlets:

$$-\mathcal{L}_{\text{Yukawa}}^{\text{Neutrino}} = Y_{ij}^{\nu}\ \underbrace{\bar{L}_{Li}}_{\begin{pmatrix}\nu_L \\ \ell_L\end{pmatrix}_i^{"}}\ \tilde{\Phi}\ \nu_{Rj} + h.c.$$

resulting in a Dirac mass term $m\bar{\nu}\nu$:

$$\mathcal{L}_{\text{mass}} = m_\nu\,(\bar{\nu}\nu) = m_\nu(\bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L)$$

The mass term mixes LH $\leftrightarrow$ RH neutrino states

The smallness of the neutrino masses are explained by very small Yukawa-coupling. However it is not clear why compared to the quark sector the differences between ν masses and charged lepton masses are so large. The RH neutrinos have Hypercharge $Y=0$ and $I_3=0$ $\rightarrow$ they would not interact with anything $\Rightarrow$ Sterile neutrinos.

b) Majorana masses

A mass term is a sum of Lorentz invariant products of LH+RH field components. If neutrinos are Majorana Fermions, i.e. they are their own anti-particles, mass terms are possible w/o demanding additional particles.

The reason is that the charge conjugated fields (anti-particle) $(\nu_L)^C = C\bar{\nu}_L^T$ and $(\nu_R)^C = C\bar{\nu}_R^T$ are RH and LH correspondingly

One finds that the unitary matrix for charge conjugation is given by $C = i\gamma^2\gamma^0$ and that C satisfies:

(1) $C^\dagger = C^T = C^{-1} = -C$

(2) $C\gamma^5 = \gamma^5 C \quad C\gamma^0 = -\gamma^0 C$

And further: $\left(\left(\frac{1 \pm \gamma^5}{2}\right)\psi\right)^C = \left(\frac{1 \mp \gamma^5}{2}\right)\psi^C$ i.e. C flips chirality LH $\Leftrightarrow$ RH

or ⊛ $(\psi_{L,R})^C = \psi^C_{R,L}$

Mass terms where both chiralities are needed can therefore expressed with the help of the charge conjugated fields.

with ⊛ For Majorana-Particles: $\psi^C_{R,L} = \psi_{R,L} \Rightarrow (\psi_{L,R})^C = \psi_{R,L}$

→ Majorana Field: $\psi = \psi_L + \psi_L^C$
$\psi^C = \psi_L^C + \psi_L$] C

One can write down a mass term for Majorana-particles:

$$\mathcal{L}_M = \frac{1}{2} m_L \left(\overline{\nu}_L^C \cdot \nu_L + \overline{\nu}_L \cdot \nu_L^C \right)$$
$$= \frac{1}{2} m_L \left(\overline{\nu}_R^C \cdot \nu_L + \overline{\nu}_L \cdot \nu_R^C \right)$$

While Dirac mass terms couple LH and RH components the Majorana mass term couple neutrinos with anti-neutrinos ⇒ <u>Lepton flavor violation by 2 units.</u>

BUT: Mass term cannot be generated within SM

$\nu_L \begin{cases} I_3 = +\frac{1}{2} \\ Y = -1 \end{cases}$ $\quad \overline{\nu}_L^C \nu_L \begin{cases} I_3 = 1 \\ Y = -2 \end{cases}$

Higgs Triplet: $\begin{pmatrix} \Delta^0 \\ \Delta^- \\ \Delta^{--} \end{pmatrix}$ → VeV V_3

to generate the mass term via a Higgs-coupling a Higgs-Triplet with $Y=2$ $I_3 = -1$ is necessary → does not exist in SM.

We are forced to consider existence of an additional RH neutrino field even in case of the Majorana mass terms ☹: ν_R-Singlet $\begin{pmatrix} I_3 = 0 \\ Y = 0 \end{pmatrix}$ can couple to the Higgs

<u>i.e:</u> Neutrino mass term (Dirac or Majorana) requires physics beyond the SM:
ν_R or Higgs-Triplet or new mass generation

c) Most general mass terms and seesaw model

It is instructive to consider the simplest case of one flavor and two neutrino fields ν_L and $(\nu_R)^c$.
Most general mass term considers Dirac and Majorana terms:

$$\mathcal{L}^{D+M} = -\frac{1}{2} m_L \bar{\nu}_L (\nu_L)^c - \frac{1}{2} m_D \left(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L \right) - \frac{1}{2} m_R \overline{(\nu_R)^c} \nu_R + h.c.$$

which can be rewritten if one uses:

$$n_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} = \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} \quad \text{with } (n_L)^c = \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix} = \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix} = n_R^c$$

$$\Rightarrow \quad \mathcal{L}^{D+M} = -\frac{1}{2} \bar{n}_L \cdot \underbrace{\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}}_{M^{D+M}} (n_L)^c + h.c.$$

$$\begin{tikzcd}
\nu_L \arrow[rr, leftrightarrow, "m_L^M"] & & (\nu_L)^c \\
& m_D \arrow[ul] \arrow[ur] \arrow[dl] \arrow[dr] & \\
(\nu_R)^c \arrow[rr, leftrightarrow, "m_R^M"'] & & \nu_R
\end{tikzcd}$$

Coupling scheme for neutrinos via masses

The helicity fields ν_L and $(\nu_R)^c = \nu_L^c$ are not the mass eigenstates – these are found by diagonalizing the mass matrix M^{D+M} $\rightarrow$ can be easily diagonalized using the orthogonal matrix O

$$O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad M^{D+M} = O M' O^T$$

$$M' = \mathrm{diag}(m_1', m_2')$$

with $\tan 2\theta = \frac{2 m_D}{m_R - m_L}$

and $m'_{1,2} = \frac{1}{2}(m_R + m_L) \mp \frac{1}{2}\sqrt{(m_R - m_L)^2 + 4 m_D^2}$

where $m'_{1,2}$ can be positive and negative

$\rightarrow$ rewrite $m'_i = |m'_i| \cdot \eta_i = m_i \cdot \eta_i$ w/ $\eta_i = \pm 1$

Taking this into account one can express the diagonalization of M^{D+M} as:

$$(*) \quad M^{D+M} = \mathcal{O}\, M \cdot \eta\, \mathcal{O}^T = U\, M \cdot U^T$$

with $M = \mathrm{diag}(m_1, m_2)$
and $U = \mathcal{O} \cdot \sqrt{\eta}$ = unitary matrix.

For the neutrino mass eigenstates one finds from $(*)$:

$$(**) \quad \nu^M = U^+ n_L + (U^+ n_L)^c = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\rightarrow \mathcal{L}^{D+M_\nu} = -\frac{1}{2} \bar{\nu}^M \cdot M \cdot \nu^M = -\frac{1}{2} \sum_{i=1,2} m_i \bar{\nu}_i \nu_i$$

Evidently $(\nu_i)^c = \nu_i \Rightarrow$ Mass eigenstates are Majorana ν.

From $(**)$ one obtains the following mixing eq:

$$\nu_L = \cos\theta \sqrt{\eta_1} \cdot \nu_{1L} + \sin\theta \sqrt{\eta_2} \cdot \nu_{2L}$$
$$(\nu_R)^c = -\sin\theta \sqrt{\eta_1} \cdot \nu_{1L} + \cos\theta \sqrt{\eta_2} \cdot \nu_{2L}$$

The parameters η_i determine the CP parity of the Majorana ν_i

Seesaw (simplest case for 1 family)

The seesaw mechanism was proposed at the end of the 1970ies and is based on the Dirac and Majorana mass terms. It is a natural and viable mechanism to generate neutrino masses.

The three parameters m_L, m_D and m_R characterize left-handed, Dirac and RH-Majorana mass terms. The mass eigenstates characterized by m_1, m_2 are Majorana states (see above).

Assumptions:

1) no LH Majorana mass term $m_L = 0$
2) Dirac mass term generated by SM Higgs-coupling
   $\rightarrow$ m_D is of the order of a lepton or quark mass.
3) RH Majorana mass term $\neq 0$ breaks Lepton number conservation: We assume that this happens at a mass scale much larger than the e.w. scale.
   $\rightarrow$ $m_R \equiv M_R \gg m_D$

One obtains for the mass eigenvalues:

$$m_1 \simeq \frac{m_D^2}{m_R} = \frac{m_D^2}{M_R} \ll m_D$$

$$m_2 \simeq M_R \gg m_D$$

Mixing angle $\theta \simeq \frac{m_D}{M_R} \ll 1$ and w/ $\eta_1 = -1$ $\eta_2 = +1$:

$\rightarrow$ Mixing relations: $\nu_L = i\,\nu_{1L} + \frac{m_D}{M_R}\cdot \nu_{2L}$

$$(\nu_R)^c = -i\,\frac{m_D}{M_R}\cdot \nu_{1L} + \nu_{2L}$$

Expressed as corresponding Majorana fields:

$\nu_1 \simeq \nu_L - \nu_R^c$ = LH component w/ low mass $\rightarrow$ active

$\nu_2 \simeq \nu_L^c + \nu_R$ RH component w/ very high mass $\rightarrow$ sterile

Estimation of M_R:

$m_D \lesssim m_t \simeq 170\,\mathrm{GeV}$

$m_1 \simeq \sqrt{\Delta m^2}\,\big|_{\text{heaviest neutrino}} \simeq 5\cdot 10^{-2}\,\mathrm{eV}$

$\Big\}$ $M_R \simeq \frac{m_D^2}{m_1} \simeq 10^{15}\,\mathrm{GeV}$