3.6 Rare Kaon Decays

FCNC decays such as $K_L \to \mu^+\mu^-$ or $K \to \pi \nu\bar{\nu}$ are in the SM forbidden at tree-level and can only happen at loop level. Due to the unitarity of the CKM matrix the decays are strongly suppressed and have typical branching ratios in the order of $10^{-9} \ldots 10^{-11}$ thus they are extremly rare!

3.6.1 $K_L \to \mu\mu$ decay

Semi-leptonic kaon decays ($\Delta S = 1$) have BR of $\mathcal{O}(1)$:

$$BR(K^+ \to \mu^+\nu) = 0.64$$

The similar looking „neutral current" decay $K_L \to \mu^+\mu^-$ is however heavily suppressed:

$$BR(K_L \to \mu^+\mu^-) = 7 \cdot 10^{-9}$$

At the end of the 1960's (begining of the 1970s) in the times of the „3 Quark model" this tiny BR was not understandable:

- In the 3-quark model one expects a tree-level $\bar{s}dZ$ coupling with strength $G_F \to BR(K_L \to \mu\mu) \sim BR(K^+ \to \mu\nu)$
- even if the flavor violating Z couplig were forbidden the problem would appear at 1-loop level.

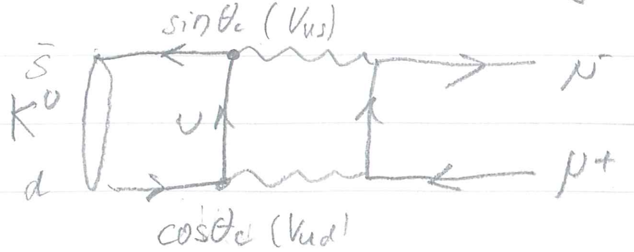
3-Quark Model

$$\begin{pmatrix} |u\rangle \\ |d'\rangle \end{pmatrix} = \begin{pmatrix} |u\rangle \\ \cos\theta_c |d\rangle + \sin\theta_c |s\rangle \end{pmatrix}$$

with $\theta_c \approx 13^0$ = Cabibbo angle

suppression of $s \to u$ decays by $\sin^2\theta_c \approx 0.05$

within this model the decay $K_L \to \mu\mu$ could proceed via loops:



$$A_u \sim \frac{m_u^2}{M_W^2} \ln\left(\frac{M_W}{m_u}\right) \sin\theta_c \cos\theta_c$$

$\rightarrow$ large BR expected

(Note: the 3-quark theory is not renormizable)

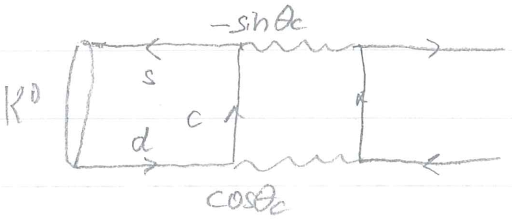
Glashow, Iliopoulos and Maiani (GIM) proposed in 1970 the introduction of a new u-type Quark (c-quark) to solve the problem of large expected BR.

They proposed 2 complete generations of quarks with a 2×2 matrix describing the quark mixing:

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} |d'\rangle \\ |s'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_c |d\rangle + \sin\theta_c |s\rangle \\ -\sin\theta_c |s\rangle + \cos\theta_c |d\rangle \end{pmatrix}$$

The 2×2 unitary matrix is called Cabibbo Matrix

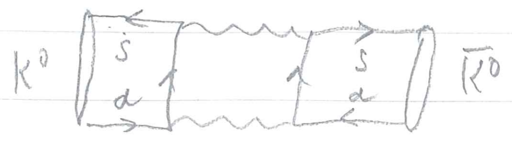
In addition to the amplitude A_u there is now also A_c



$$A_c \sim -\frac{m_c^2}{M_W^2} \cdot \ln\left(\frac{M_W}{m_c}\right) \cdot \sin\theta_c \cos\theta_c$$

For $m_c = m_u$ this additional amplitude leads in the 2 Generation model to a perfect cancellation. For $m_c \neq m_u$ the total amplitude is strongly suppressed: GIM suppression.

It is very interesting to note that already GIM in their original paper stressed the connection to the $K^0\bar{K}^0$ mixing:



where the additional c-quark also strongly influences the observable mixing effect. From the potential influence they concluded on a mass term violating the quark flavor symmetry: $\Delta < 3 \ldots 4$ GeV

A careful analysis of the K^0-mixing in the light of a new up-type quark was performed by Gaillard & Lee (1974):

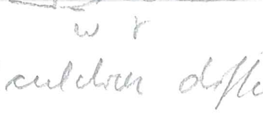
$$\Delta m_K \sim |V_{cs}^* V_{cd}|^2 \cdot S_0\left(\underbrace{\left(\frac{m_c}{M_W}\right)^2}_{\sim m_c^2/M_W^2}\right)$$

They obtained for m_c: $m_c \sim 1.5$ GeV which (accidently) fits very well today's c-quark mass! (*)

The prediction of the 4th quark together with its mass is certainly one of the triumphs of the SM.

(*) Today we know thus the theoretical calculation of the $BR(K_L \to \mu^+\mu^-)$ is difficult. In addition to the short range contributions

Box $\qquad$ Z-Penguin

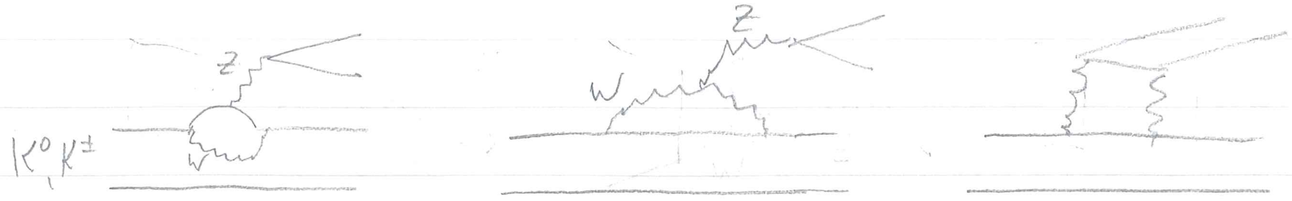
there are contributions from a long-distant amplitude with a 2-γ intermediate state 

$\Rightarrow$ Makes the exact theoretical calculation difficult!

3.6.2 $K \to \pi \nu \nu$

(Rare decay which is theoretical much better under control)

The decay $K \to \pi \nu \nu$ is a genuine loop-process in SM:



Beside the loop- and the helicity suppression there exists a strong GIM suppression $\Rightarrow$ BR $\sim \mathcal{O}(10^{-10})$

Because of the GIM mechanism a very large contribution comes from the top $\Rightarrow$ m_t, V_{ts}, V_{td}.
In addition this decay is very sensitive to new physics effects.

Since $K \to \pi \nu \nu$ is reliably calculable the precise measurement of BR$(K \to \pi \nu \nu)$ provides an excellent test of the flavor sector.
For the amplitude one finds:

$$A \sim \sum_{i=u,c,t} \lambda_i \cdot F(x_i) \underset{\text{unitarity}}{=} \lambda_c \left(F(x_c) - F(x_u) \right) + \lambda_t \left(F(x_t) - F(x_u) \right)$$

$$\text{with } \lambda_i = V_{is}^* V_{id} \quad x_i = m_i^2 / M_W^2 \quad \ldots$$

One finds:

$$F(x_u) \sim 10^{-5} \ll \frac{m_c^2}{M_W^2} \cdot \ln\left(\frac{M_W}{m_c}\right) \sim 10^{-3} \ll F(x_t) \sim \mathcal{O}(1)$$

With the CKM factors λ_i included:

$$\lambda_c \cdot F(x_c) \sim 10^{-4} \qquad \lambda_t \cdot F(x_t) \sim 10^{-4}$$

(and u-quark contribution can be neglected)

A careful analysis gives (see Buchalla)

Channel	$K^+ \to \pi^+ \nu\bar{\nu}$	$K^0 \to \pi^0 \nu\bar{\nu}$
CP	conserving	CP violating
CKM	V_{td}	$Im(V_{ts}^* V_{td}) \sim J_{CP} \sim \eta$
	c & t	only t
BR *)	$(0{,}85 \pm 0{,}07) \times 10^{-10}$	$(2{,}6 \pm 0{,}4)\, 10^{-11}$

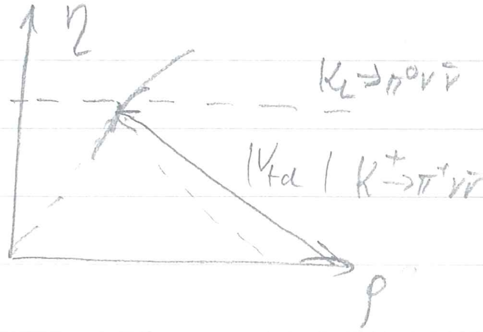
*) Talk by Cecchini + Ref. in this talk

The measurements of $BR(K^+ \to \pi^+ \nu\bar{\nu})$ and $BR(K^0 \to \pi^0 \nu\bar{\nu})$ allow an interesting phenomenological study.

→ the two BR define by their own the UT

$$BR(K_L \to \pi^0 \nu\bar{\nu}) \to Im(V_{ts}^* V_{td})$$

$$\left[\begin{array}{l} J_{CP} \doteq Im(V_{ts}^* V_{td} V_{us}^* V_{ud}) \\ \quad = \lambda (1 - \lambda^2/2)\, Im(V_{ts}^* V_{td}) \end{array}\right.$$



Experimental Status:

$K^+ \to \pi^+ \nu\bar{\nu}$: $1{,}73^{+1.15}_{-1.05}$ (AGS E787/E949, 2008) *)

$K_L^0 \to \pi^0 \nu\bar{\nu}$ $< 2{,}6 \cdot 10^{-8}$ @ 90% CL (KEK E391a, 2009)

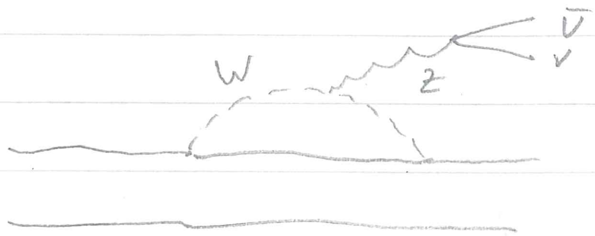
*) observed 2 events

Future Exp: NA62 → K^+
KOTO at JPARC → K^0
} decays in flight!

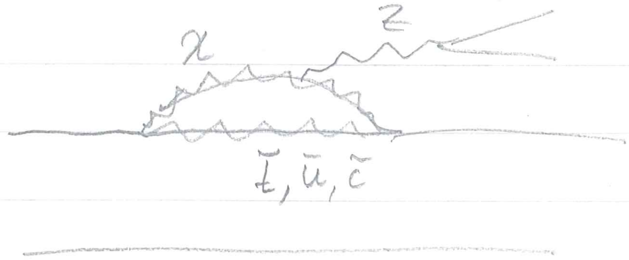
New Physics Sensitivity: see Slide from Augusto.

The decay $K \to \pi \nu \bar{\nu}$ is an ideal place to look for new physics. New particles (e.g. SUSY) could significantly enhance / modify the BR :

SM



SUSY



NP can alter the helicity suppression / add additional amplitudes.

~~$K \to \pi \nu \bar{\nu}$ decays~~

4 B - Mesons

	$\tau = 1/\Gamma$	Δm	$x = \frac{\Delta m}{\Gamma}$	$y = \frac{\Delta\Gamma}{2\Gamma}$
K^0-System	$0.26 \cdot 10^{-9}\,s$	$5.29\ ns^{-1}$	0.477	-1
D^0-System	$0.4 \cdot 10^{-12}\,s$	$0.0024\ ps^{-1}$	0.0097	0.0078
B^0-System	$1.5 \cdot 10^{-12}\,s$	$0.507\ ps^{-1}$	0.78	0.0015
B_s System	$1.5 \cdot 10^{-12}\,s$	$17.7\ ps^{-1}$	26.8	0.07

Remember the Wolfenstein parametrization of V_{CKM}:

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \cdot e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| e^{-i\beta} & -|V_{ts}| \cdot e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

$\rightarrow$ usefull to evaluate the expected CP asym. in B-decays

4.1 Measurement of CKM phase β in $B^0 \to J/\psi K^0_S$ decays

The channel $B^0 \to J/\psi K_S$ allows an interference of the decay w/ and w/o mixing:

$$\begin{tikzcd} B^0 \arrow[rr, bend left] \arrow[dr] & & J/\psi K_S \\ & \bar{B}^0 \arrow[ur] & \end{tikzcd}$$

With the master Eq. from Chap. 3.3E one obtains for small $\Delta\Gamma$ and for negligible (no) direct CPV (i.e. $|A_f| = |\bar{A}_f|$):

$$A_{CP}(t) = \frac{\Gamma(B^0 \to f)(t) - \Gamma(\bar{B}^0 \to f)(t)}{\Gamma(\quad)(t) + \Gamma(\quad)(t)} = -\,\mathrm{Im}(\lambda_f)\,\sin(\Delta m t)$$

For the final-state $f = f_{CP}$ with $CP|f_{CP}\rangle = \eta_{CP}|f_{CP}\rangle$ and $\eta_{CP} = \pm 1$

one obtains for λ_{fcp}

$$\lambda_{fcp} = \frac{q}{p} \cdot \frac{\bar{A}_{fcp}}{A_{fcp}}$$

where we have rewritten λ_{fcp} ...

$\bar{A}_{fcp}$ and A_{fcp} are related ... only in the signs of the weak phases ... $\bar{A}_{fcp}$ from A_{fcp} by inverting the phases ...

For the final-state $J/\psi K_S$ ...

Moreover for this final ...

following amplitudes ...

$A(B^0 \to J/\psi K$ ...

one obtains for $\lambda_{f_{CP}}$:

$$\lambda_{f_{CP}} = \frac{q}{p} \cdot \frac{\bar{A}_{f_{CP}}}{A_{f_{CP}}} = \frac{q}{p} \cdot \eta_{CP} \cdot \frac{\bar{A}_{\bar{f}_{CP}}}{A_{f_{CP}}}$$

where we have rewritten $\lambda_{f_{CP}}$ as a function of $\bar{A}_{\bar{f}_{CP}}$. $\bar{A}_{\bar{f}_{CP}}$ and $A_{f_{CP}}$ are related be CP conjugation and differ only in the signs of the weak phases → $\bar{A}_{\bar{f}_{CP}}$ can be calculated from $A_{f_{CP}}$ by inverting the phases!

For the final-state $J/\psi K_S$: $\eta_{CP} = -1$
Moreover for this finalstate one has to consider the following amplitudes:

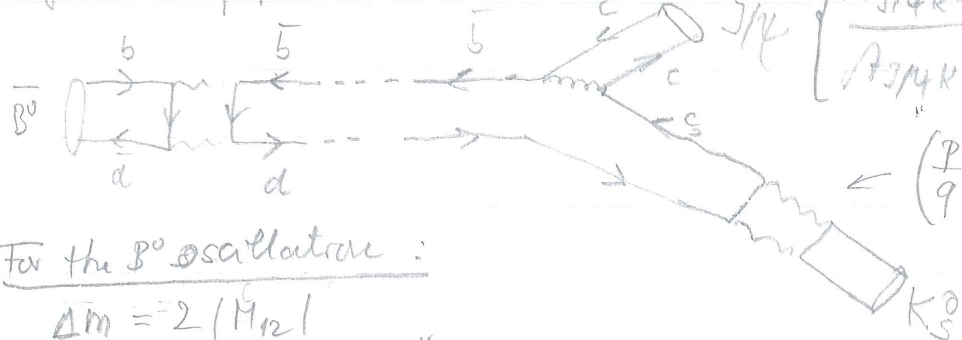
$$A(B^0 \to J/\psi K_S) = A(B^0 \to J/\psi K^0) \times A(K^0 \leftrightarrow K_S)$$
$$A(\bar{B}^0 \to J/\psi K_S) = A(\bar{B}^0 \to J/\psi \bar{K}^0) \times \underbrace{A(\bar{K}^0 \leftrightarrow K_S)}$$

Remember: $|K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$

Considering the K^0 mixing one obtains:

$$\lambda_{J/\psi K_S} = \left(\frac{q}{p}\right)_{B^0} \cdot \eta_{CP} \frac{\bar{A}_{J/\psi K_S}}{A_{J/\psi K_S}} = \left(\frac{q}{p}\right)_{B^0} \eta_{CP} \frac{\bar{A}_{J/\psi \bar{K}^0}}{A_{J/\psi K^0}} \cdot \left(\frac{p}{q}\right)_{K^0}$$

Feynman-Graph:



$$\frac{\bar{A}_{J/\psi \bar{K}^0}}{A_{J/\psi K^0}} = \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}$$

$$\left(\frac{p}{q}\right)_{K^0} = \sqrt{\frac{M_{12}^*}{M_{12}}}\Bigg|_{K} = \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}$$

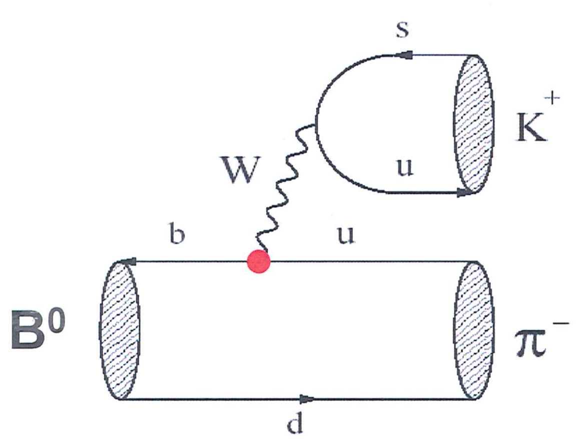
<u>For the B^0 oscillation:</u>

$$\Delta m = 2|M_{12}|$$

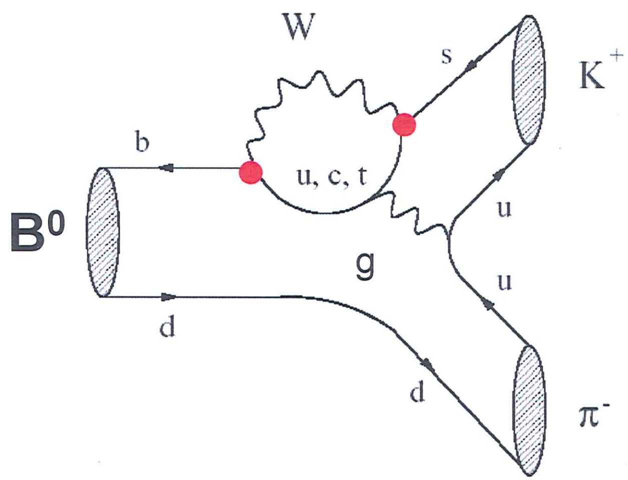
$$\underbrace{q/p}_{\text{a pure phase, } |q/p| \approx 1} = \sqrt{\frac{M_{12}^*}{M_{12}}} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} = e^{-i2\beta}$$

Direct CP Violation in $B \to K\pi$

Tree

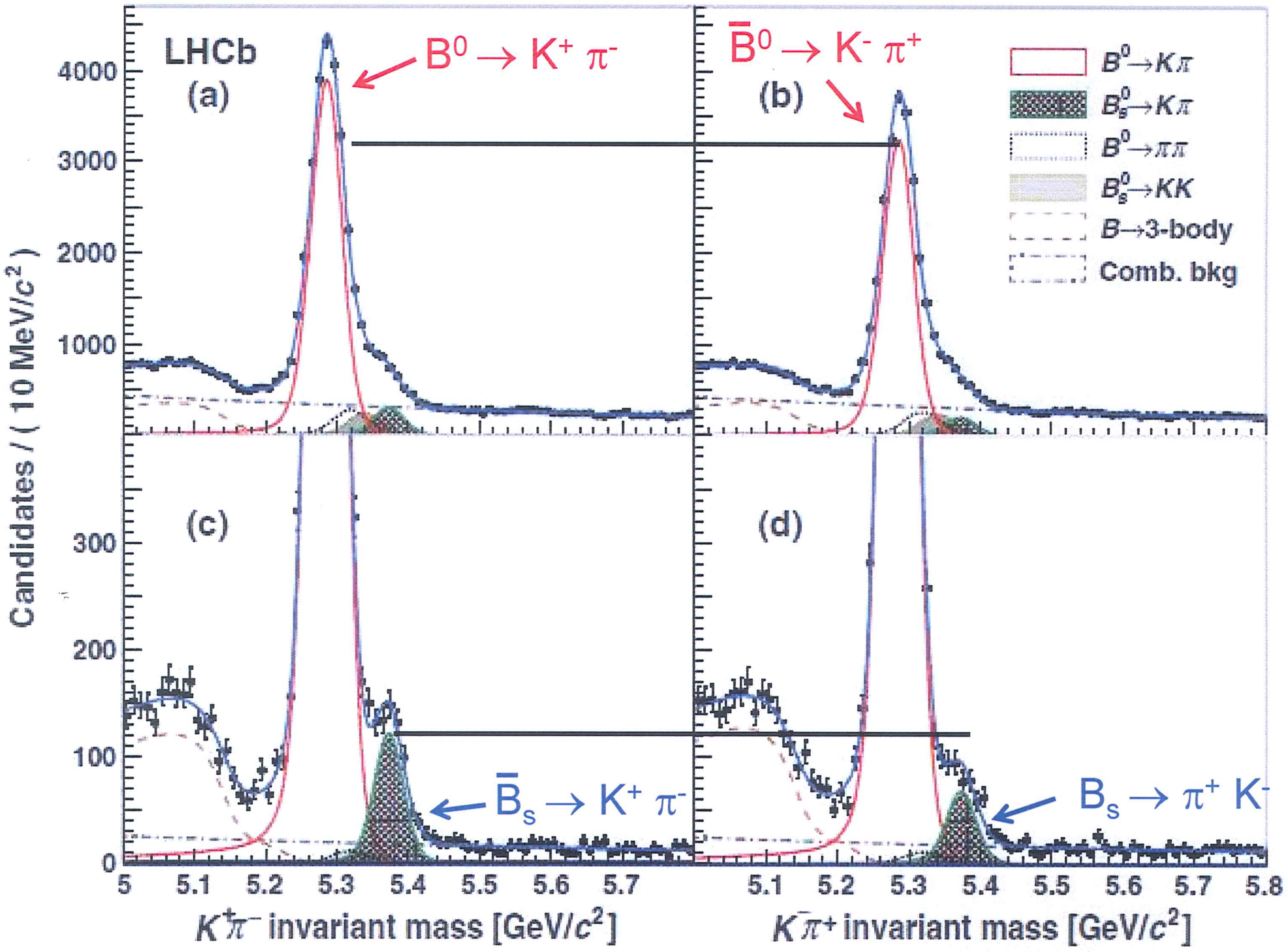


Penguin

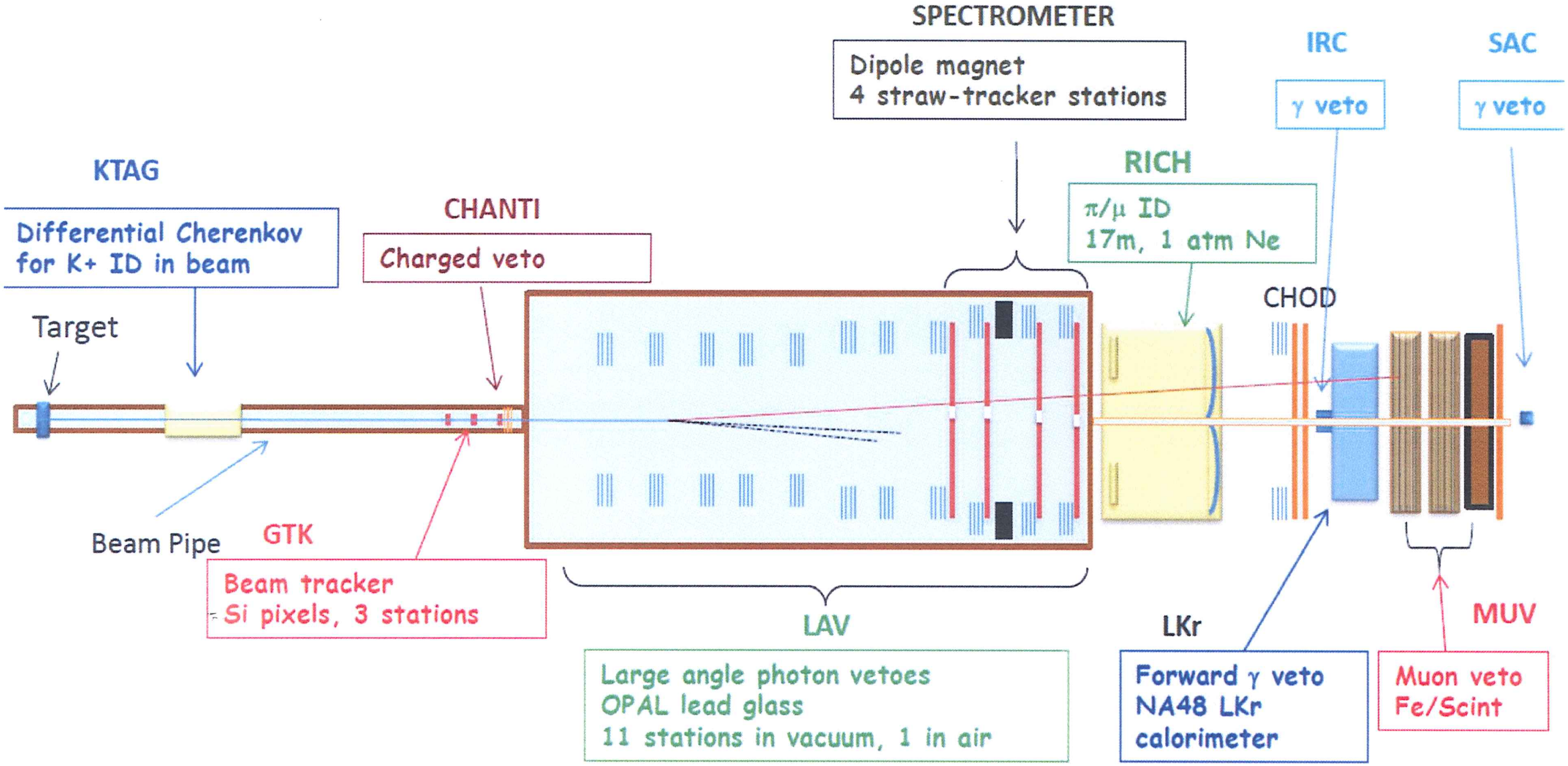


Direct CP asymmetries for $B^0_{d,s} \to K\pi$

PRL 110, 221601 (2013)



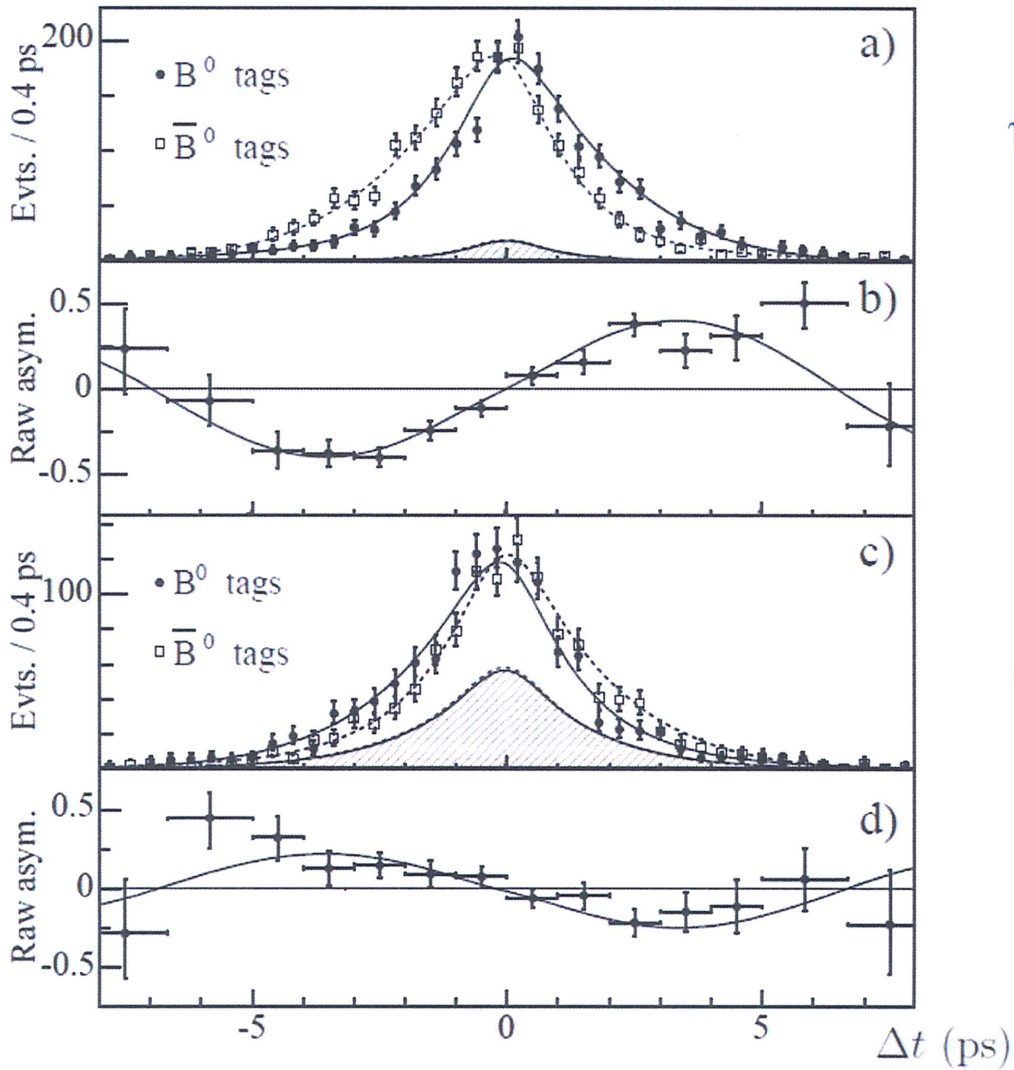
NA62 Detector: $K^+ \rightarrow \pi^+ \nu\nu$



<u>High intensity kaon beam:</u>
SPS proton beam @ 400 GeV
Proton-on-target: 3×10^{12} / pulse

Decay rates: 10 MHz mainly K^+
$\rightarrow 4.5 \times 10^{12}$ K+ decays / year:
Expect to detect O(100) decays

Time dependent CPV in $B^0 \rightarrow J/\psi K_s$



$\eta_{CP} = -1$

$\eta_{CP} = +1$

Putting everything together one obtains for $\lambda_{J/\psi K_S}$:

$$\lambda_{J/\psi K_S} = -\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)\cdot\left(\frac{V_{cb} \cdot V_{cs}^*}{V_{cb}^* \; V_{cs}}\right)\left(\frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}}\right)$$

$$= -\left(\frac{V_{tb}^* V_{td} \;\; V_{cb} V_{cd}^*}{V_{tb} V_{td}^* \;\; V_{cb}^* V_{cd}}\right)$$

$$\mathrm{Im}\left(\lambda_{J/\psi K_S}\right) = -\sin\left(\arg\left(\ldots\right)\right) = -\sin\left(2\arg\left(\frac{V_{cb} V_{cd}^*}{V_{tb} \; V_{td}^*}\right)\right)$$

$$= -\sin 2\beta$$

$$\rightarrow \quad A_{CP}(t) = -\underbrace{\sin(2\beta)}_{2\times \text{ phase of } V_{td}} \sin(\Delta m t)$$

One often short cuts the argumentation by saying that the $B^0 \to \bar{B}^0$ mixing leads to the factor $(q/p)_{B^0} = e^{-2i\beta}$ and all other amplitudes are essentially real!

For different final states e.g. $B^0 \to J/\psi K_L$ one has to consider the different CP eigenvalues. In general one therefore often writes:

$$A_{CP}(t) = \eta_{CP} \cdot \sin(2\beta) \sin(\Delta m t)$$

The measurement of 2β was performed by BABAR and Belle, both experiments at a high-luminosity e^+e^- B-factory: e^+e^- B-factories operate at the center-of-mass energy of the $\Upsilon(4S)$ resonance which decays nearly entirely to $B^0\bar{B}^0$ or B^+B^-: $e^+e^- \to \Upsilon(4S) \to B^0\bar{B}^0, B^+B^-$ ($\sqrt{s} = 10.58$ GeV)

Measurement of $\sin(2\beta)$ was the first observation of CPV outside K^0-system $\rightarrow$ correctly predicted by SM.

Experimental problems of the measurement:

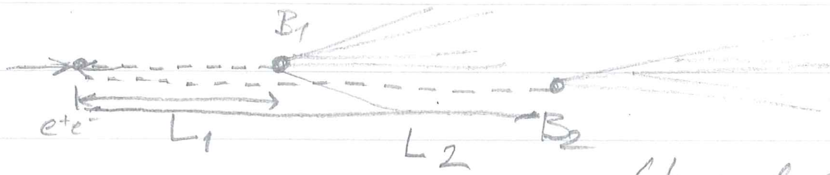
1) $M_{\Upsilon(4S)} = 10.56\ \mathrm{GeV}/c^2 \approx 2 m_B$

→ If the $\Upsilon(4S)$ is produced at rest the 2 B's have very little momentum. A lifetime measurement which is based on the measurement of the flight-length is not possible (B's don't fly!).

To overcome this problems the $\Upsilon(4S)$ is instead produced with a boost:

$$\underset{3.1\,\mathrm{GeV}}{\longrightarrow} e^+ + e^- \underset{9\,\mathrm{GeV}}{\longleftarrow} \Rightarrow \beta\gamma = 0.56$$

Decay kinematics:



$\left\{ \begin{array}{l} \text{typical } \beta\gamma c\tau \approx 250\,\mu m \\ \text{w/ } \tau \approx 1.5\,ps \end{array} \right.$

2) Flavor-tagging:

To measure $A_{CP}(t)$ one has to know whether a $B^0 \to J/\psi K_S$ was produced originally as B^0 or as $\overline{B^0}$.

In general one uses the "2nd B" in the event which always contain two B's w/ opposite flavor:

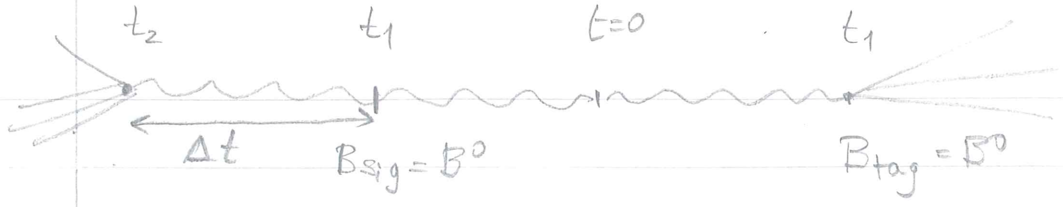
$$B_{tag} = B/b \text{ or } B/\bar{b} \Longrightarrow B_{signal} = B^0/\bar{b} \text{ or } \overline{B^0}/b$$

However for $B_{tag} = B^0$ one has the problem of oscillation, i.e. when decaying the B_{tag} might have changed its flavor!

→ wrong tag!!

At $\Upsilon(4S)$ the problem can be solved:

Both B-mesons are produced coherently and are entangled: One does not know which one is $B^0/\overline{B^0}$.



If one of the two decays in a given state it defines at that time also the flavor of the other one!
(opposite)

↳ After the decay of the tagging B (e.g. $B_{tag} = B^0$) the clock of the Signal B ($B_{sig} = \bar{B}^0$ at t_1) starts to tick ($\Delta t = t - t_1$).
Instead of measuring A_{CP} as function of the absolute time t, it is measured as function of Δt (time after the tagging B has decayed!):

$$A_{CP}(\Delta t) = \frac{\Gamma(B^0(\Delta t) \to f) - \Gamma(\bar{B}^0(\Delta t) \to f)}{\Gamma \qquad + \Gamma}$$

$$= \frac{\Gamma(B_{tag} = \bar{B}^0; B \to f) - \Gamma(B_{tag} = B^0; B \to f)}{\cdots} (\Delta t)$$

$$= -\sin(2\beta)\sin(\Delta m t)$$

Of course the signal B can also decay before the tagging B. In this case $\Delta t < 0$.

BABAR plots for $\sin(2\beta)$ in $J/\psi K_S$ and $J/\psi K_L$!

2. Measurement of β_s in $B_s \to J/\psi\,\phi$

Changing spectator quark $d \to s$ one obtains the B_s counter-part:

$$\begin{tikzcd} B_s^0 \arrow[rr, bend left] \arrow[dr, "(q/p)_{B_s^0}"'] & & J/\psi\,\phi \\ & \bar{B}_s^0 \arrow[ur] & \end{tikzcd} \qquad \phi = |s\bar{s}\rangle$$

$\boxed{\text{Differences w/r to } J/\psi K_S}$

1) $$\left(q/p\right)_{B_s^0} = \left(\frac{V_{ts} V_{tb}^*}{V_{ts}^* V_{tb}}\right) \approx e^{i2\beta_s}$$

$$\text{w/ } \beta_s = \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right) \approx \arg(-V_{ts})$$

$$\to \quad \mathrm{Im}(\lambda_{J/\psi\phi}) = \sin(-2\beta_s)$$

$2\beta_s$ ist very small $\sim \eta\lambda^2$
$\to$ very small CPV

2) ϕ is not oscillating

3) $\Delta\Gamma \neq 0$: $$A_{CP}(t) = \frac{\Gamma(B_s \to J/\psi\,\phi)(t) - \Gamma(\bar{B}_s \to J/\psi\,\phi)}{\Gamma(\quad) + \Gamma(\quad)}$$

$$= \frac{-\mathrm{Im}(\lambda_{J/\psi\phi})\sin(\Delta m_s t)}{\cosh\left(\frac{\Delta\Gamma}{2}t\right) + \mathrm{Re}(\lambda_{J/\psi\phi})\sinh\left(\frac{\Delta\Gamma}{2}t\right)}$$

4) Additional problem:
$J/\psi\,\phi$ is not a pure CP eigenstate but a mixture of $\eta_{CP} = \pm 1$ states.
Reason: J/ψ and ϕ are both vector (Spin=1) particle

$$\begin{array}{cccccc} & B_s & \longrightarrow & J/\psi & + & \phi \\ J^{PC} = & 0^{-+} & & 1^{--} & & 1^{--} \end{array}$$

" implies relative angular momenta $l = 0, 1, 2$ possible

$$\Rightarrow \quad \eta_{CP}(J/\psi\,\phi) = \underbrace{\eta_{CP}(J/\psi)}_{(+1)} \cdot \underbrace{\eta_{CP}(\phi)}_{(+1)} \underbrace{(-1)^l}_{(-1)^l}$$

$l = 1 \to$ CP-odd
$l = 0, 2 \to$ CP-even

If one does not separate the $\ell=0,1,2$ states from each other one might not observe any CPV even if there are large effects.

→ Solved by performing an angular analysis of the final states which allow to separate the CP=+1/−1 states

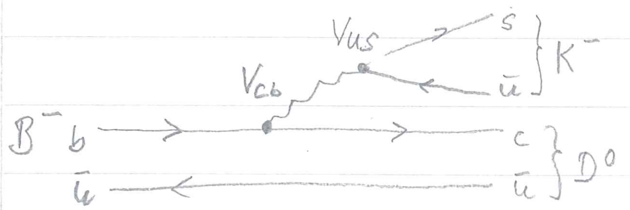
Measurement : LHCb 2003 $\phi_M = -2\beta_s = 0.01 \pm 0.07$

SM $\phi_s^{SM} = -2\beta_s = -0.036 \pm 0.002$

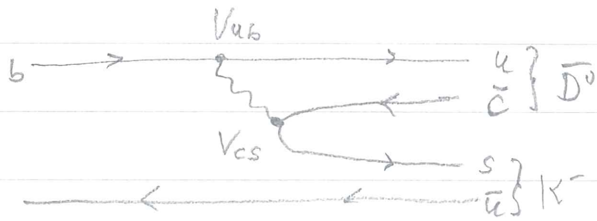
4.3 Measurement of the CKM phase γ using $B^\pm \to \overset{(-)}{D^0} K^\pm$

To measure γ one exploits „direct CPV", i.e. CPV in the decay. However to get the interference is a bit more tricky.

Cabibbo favored $B^- \to D^0 K^-$



Cabibbo suppressed $B^- \to \bar{D}^0 K^-$



Erinnerung: $V_{ub} = |V_{ub}| e^{-i\gamma}$

In general the 2 amplitude lead to a different final states and thus cannot interfere, However if D^0 and $\bar{D}^0$ decay to the same final state:

e.g. $D^0/\bar{D}^0 \to KK, \pi\pi$ (there are others)

the interference of the 2 amplitude can result in CPV and allows to measure the weak phase $\gamma = -\arg(V_{ub})$

The extraction of the weak phase γ from the CPV in $B^\pm \to \overset{(-)}{D^0} K^\pm$ decays is rather complicated.

The most precise determination of γ is from LHCb:

$$\text{LHCb 2013:} \quad \gamma = (67 \pm 12)^\circ$$

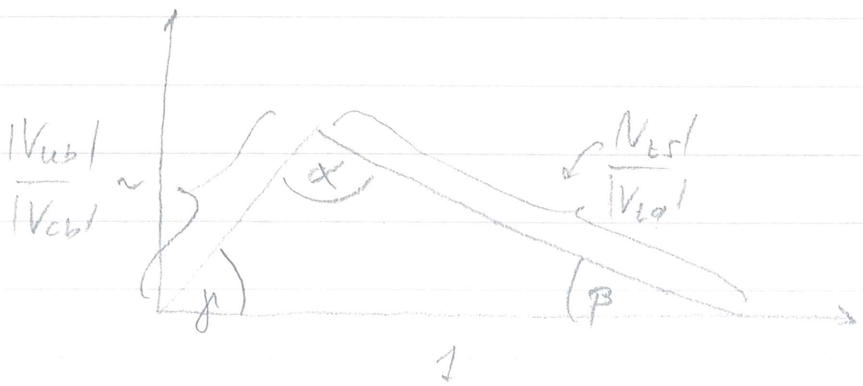
4.4 The unitarity triangle:

CKM phase β: $B^0 \to J/\psi K_S$
CKM phase γ: $B^\pm \to \overset{(-)}{D}{}^0 K^\pm$
CKM phase β_s: $B_s^0 \to J/\psi\, \phi$
(CKM phase α: $B^0 \to \pi^+\pi^-$: Measures $\pi - \beta - \gamma = \alpha$)
not discussed

The sides:

$V_{cb}, V_{ub} \to$ measured from semilept. BR
$b \to c l \nu \quad b \to u l \nu$

$V_{ts}/V_{td} \to$ measured from oscillation



Experimental Test of UT