3.5 CP Violation in the neutral Kaon system

How can we explain the observation of CPV in
$K_L\ (CP=-1) \to 2\pi\ (CP=+1)$

3.5.1 CP Violation in mixing

The key to the theoretical explaination is the observation
← (**) of the time dependent interference term in $K^0(\bar{K}^0) \to \pi^+\pi^-$ decays*):

$$\Gamma(K^0 \to \pi\pi)(t) - \Gamma(\bar{K}^0 \to \pi\pi)(t) \sim 2|\eta_{+-}|\, e^{-(\Gamma_S+\Gamma_L)t/2} \cos(\Delta m t - \varphi_{+-})$$

$$\text{where } \eta_{+-} = |\eta_{+-}|\, e^{i\phi_{+-}} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$$

(The appearance of the interference term is equivalent with a CPV difference in the mixing probabilities:

$$P(K^0 \to \bar{K}^0) \neq P(\bar{K}^0 \to K^0) \Leftrightarrow |q/p| \neq 1$$
see above

(→ the oscillatory interference term is CP violating and describes the different probabilities that a $K^0 \to \bar{K}^0$ or $\bar{K}^0 \to K^0$)

→ Confirmed also by semileptonic $K \to \pi l \nu$ decays: only CPV in mixing possible

1) with $\lambda_{\pi\pi} = \left(\frac{q}{p}\right)\frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}}$: $\eta_{+-} = \frac{pA_{\pi\pi} - q\bar{A}_{\pi\pi}}{pA_{\pi\pi} + q\bar{A}_{\pi\pi}} = \frac{1-\lambda_{\pi\pi}}{1+\lambda_{\pi\pi}}$

if $\eta_{+-} \neq 0 \Leftrightarrow \lambda_{\pi\pi} \neq 1$ with the time-integrated amplitudes $A_{\pi\pi} = \bar{A}_{\pi\pi}$ follows that if $\eta_{+-} \neq 0 \Leftrightarrow \lambda_{\pi\pi} \neq 1 \Leftrightarrow \left|\frac{q}{p}\right| \neq 1$

2) or with $K_{S,L} = \frac{1}{\sqrt{2}}\left(p \cdot |K^0\rangle \pm q\, |\bar{K}^0\rangle\right)$ and $\frac{A(K_L \to \pi\pi)}{A(K_S \to \pi\pi)} \neq 0$
$\Leftrightarrow |q/p| \neq 1$

*) see the explizit discussion in exercise 6

It is usual to rewrite $q/p = \frac{1-\epsilon}{1+\epsilon}$, ϵ being a complex parameter : $\epsilon = \frac{p-q}{p+q}$ $\Rightarrow$

$$K_{S,L} = \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \left[(1+\epsilon)|K^0\rangle \pm (1-\epsilon)|\bar{K}^0\rangle \right]$$

or equivalently:

$$K_{S} = \frac{1}{\sqrt{1+\epsilon^2}} \left[|K_1\rangle - \epsilon |K_2\rangle \right]$$

$$K_L = \frac{1}{\sqrt{\quad}} \left[|K_2\rangle + \epsilon \cdot |K_1\rangle \right]$$

Which means that K_S, K_L are not equivalent to the CP eigenstates anymore but have very small (ϵ) wrong admixtures.

Using the λ Parameter as introduced in the last section :

$$\lambda_{\pi\pi} = \left(\frac{q}{p}\right)_{K^0} \frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}}$$

One can rewrite the CPV ratio of the amplitudes as :

$$\eta_{+-} = \frac{A(K_L \to \pi\pi)}{A(K_S \to \pi\pi)} = \frac{p\, A_{\pi\pi} - q\, \bar{A}_{\pi\pi}}{p\, A_{\pi\pi} + q\, \bar{A}_{\pi\pi}}$$

$$= \frac{1-\lambda_{\pi\pi}}{1+\lambda_{\pi\pi}} = \frac{1 - q/p}{1 + q/p} = \frac{p-q}{p+q} = \epsilon$$

where the last equality assumes that there is no direct CPV in the decay process itself.

From the time-dependent study of $K^0 \to \pi^+\pi^-$ decays on finds

$$|\eta_{+-}| = (2.236 \pm 0.018)\, 10^{-3}$$
$$\phi_{+-} = (43.4 \pm 1.2)^\circ$$

aus Sozzi S. 349
Ref.: Yao et al. 2006

$\rightarrow$

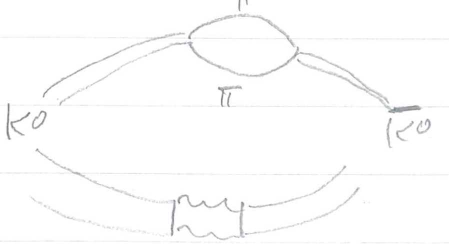
and for the ratio $|q/p| = \left|\frac{1-\varepsilon}{1+\varepsilon}\right| = 0{,}995552 \pm 0{,}000024$ *)

*) includes also the value for
$|\eta_{00}| = (2.232 \pm 0.025) \cdot 10^{-3}$
from $K^0 \to \pi^0\pi^0$
$\rightarrow |\epsilon| = (2.232 \pm 0.007) \cdot 10^{-3}$

Sozzi 349
Yao et al. 2006

Explaination within the Standard Model

CP violating effects require the interference of amplitudes with different weak and strong phases:



$\leftarrow$ no weak phases because π consist only out of u, d

$\leftarrow$ interfering diagrams with diff. weak phase: internal heavy quarks

different strong phases

CP violating weak phases from internal c and t quarks.
$\rightarrow$ Theoretical precision is limited by the knowledge of the hadronic uncertainties.

(see CKM-Fitter)

$$|\epsilon| = \frac{G_F^2 m_W^2 m_K f_K^2}{12\sqrt{2}\pi^2 \Delta m_K} \cdot B_K \cdot \Big(\eta_{cc}\, S_0(x_c, x_c)\, \mathrm{Im}\left[(V_{cs}V_{cd}^*)^2\right]$$
$$+ \eta_{tt}\, S_0(x_t, x_t)\, \mathrm{Im}\left[(V_{ts}V_{td}^*)^2\right]$$
$$+ 2\eta_{ct}\, S_0(x_c, x_t)\, \mathrm{Im}\left[(V_{cs}V_{cd}^* V_{ts}V_{td}^*)\right] \Big)$$

$$x_i = \left(\frac{m_i}{m_W}\right)^2$$

η_{ij} NLO QCD corrections

S_0 = Inami-Lim Fkt.

3.5.2 Direct CP violation

A long standing question of kaon physics was whether the CP asymmetry is also violated in the decay process itself, i.e. whether

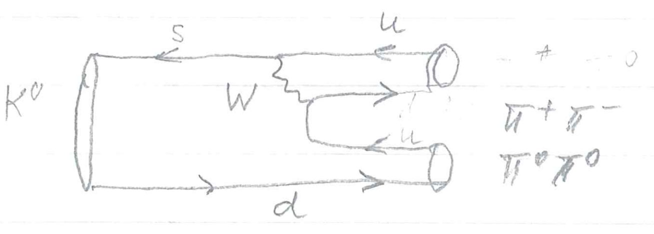
e.g. $$\frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}} = \frac{A(\bar{K}^0 \to \pi\pi)}{A(K^0 \to \pi\pi)} \neq 1$$

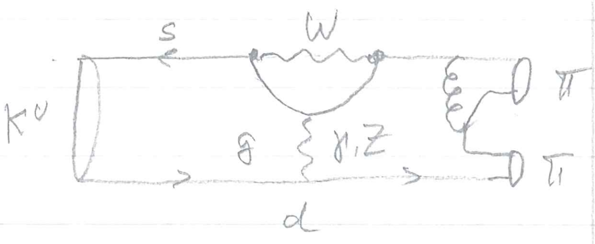
If this is the case the amount of CP violation will depend on the specific decay process (final state) and might be different for $\overset{(-)}{K^0} \to \pi^+\pi^-$ and $\overset{(-)}{K^0} \to \pi^0\pi^0$ decays:

$$\eta_{+-} = \frac{A(K_L^0 \to \pi^+\pi^-)}{A(K_S^0 \to \pi^+\pi^-)} \neq \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}$$

$$\left(\quad \approx \epsilon + \epsilon' \qquad\qquad\qquad \approx \epsilon - 2\epsilon' \quad \right)$$

Direct CPV requires the interference of at least 2 amplitudes with diff. weak and strong phases. For the process $K^0 \to \pi\pi$ there exists beside the dominating Tree-amplitude also a penguin contribution:





no weak phase

ππ System always in Isospin I=0 State

internal quarks → weak phase

ππ system can be in I=0,2 state

Gluon contribution → only I=0

γ, Z —"— → I=0,2

→ different strong phase

Since the pion has Isospin $I=1$ the $\pi\pi$-System can be in a $I=0$ and a $I=2$ state. A Clebsch-Gordan Spin Composition gives:

$$\left.\begin{aligned} |\pi^+\pi^-\rangle &= \sqrt{\tfrac{2}{3}}\,|\pi\pi; I=0\rangle + \sqrt{\tfrac{1}{3}}\cdot|\pi\pi; I=2\rangle \\ |\pi^0\pi^0\rangle &= -\tfrac{1}{\sqrt{3}}\,|\pi\pi; I=0\rangle + \sqrt{\tfrac{2}{3}}\,|\pi\pi, I=2\rangle \end{aligned}\right\} I_3=0$$

($\pi\pi$ obtained from K^0 with $\Delta I_3 = \frac{1}{2}$ rule)

For the decay amplitudes one obtains:

$$A(K^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{3}}\left(\sqrt{2}\,A_0 + A_2\right)$$

$$A(K^0 \to \pi^0\pi^0) = -\frac{1}{\sqrt{3}}\left(A_0 - \sqrt{2}\,A_2\right)$$

The two pions in the final state interact differently if they are in the $I=0$ or $I=2$ state. This introduces a strong phase difference:

$$A_I = a_I \cdot e^{i\delta_I} \quad \text{or} \quad \bar{A}_I = a_I^* \, e^{i\delta_I}$$

The ~~above~~ penguin diagrams with amplitudes which have different CKM phases and different strong phases can lead to direct CPV.

A careful evaluation of the $K_S, K_L \to \pi\pi$ decay amplitude ratio leads to:

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \epsilon + \frac{\epsilon'}{1+\Delta}$$

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \epsilon - \frac{2\epsilon'}{1-2\Delta}$$

with $F = e^{i(\delta_2 - \delta_0)}$ $\quad \Delta = \frac{F}{\sqrt{2}} \cdot \frac{\mathrm{Re}\, A_2}{A_0} \quad \epsilon' = i \frac{F}{\sqrt{2}} \frac{\mathrm{Im}\, A_2}{A_0}$

With very small direct CPV and $|\Delta| \ll 1$ and $|\epsilon'| \ll 1$ one finds for the double ratio:

$$\left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 = \frac{\Gamma(K_L \to \pi^0\pi^0)\, \Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^0\pi^0)\, \Gamma(K_L \to \pi^0\pi^0)}$$

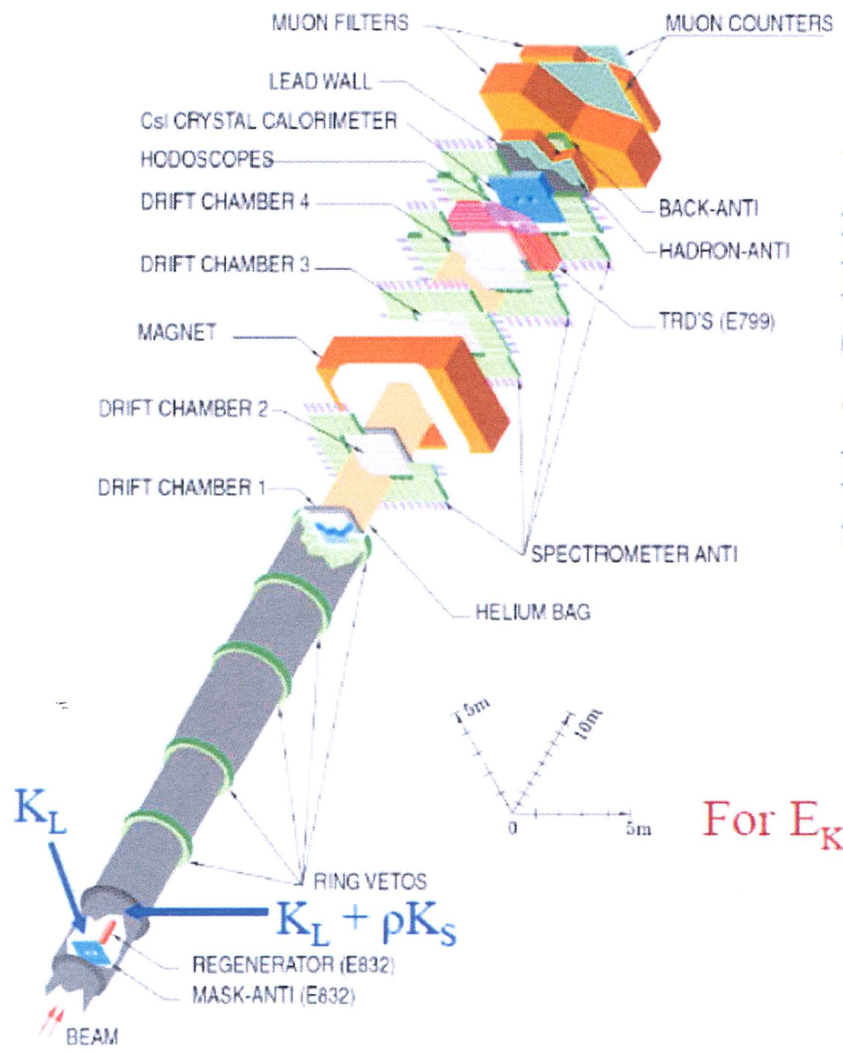
$$= 1 - 6\,\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right)$$

To experiments KTeV (Fermilab) and $\left\{ \begin{array}{l} \text{NA31} \\ \text{NA48 (CERN)} \end{array} \right.$ have precisely measured the above double ratio. To limit the syst. uncertainties as much as possible the 4 decays were measured simultaneously using at the same time a K_L and a K_S beam, and measuring $\pi^0\pi^0$ as well as $\pi^+\pi^-$.

$$\mathrm{Re}(\epsilon'/\epsilon) = (1{,}67 \pm 0{,}23)\, 10^{-3}$$

i.e. direct CPV in Kaon decay is a very tiny (10^{-6}) effect. In B-mesons direct CP is of $O(20\%)$!

The KTeV Detector



- Charged particle momentum resolution < 1% for p>8 GeV/c; Momentum scale known to 0.01% from $K \to \pi^+\pi^-$
- CsI energy resolution < 1% for $E_\gamma > 3$ GeV; energy scale known to 0.05% from $K \to \pi e \nu$.

For $E_K \sim 70$ GeV, K_S: $\gamma\beta c\tau \sim 3.5$m
K_L: $\gamma\beta c\tau \sim 2.2$ km

Re(ε′/ε) Measurements

$$R = \frac{\Gamma\left(K_L \to \pi^0\pi^0\right)}{\Gamma\left(K_S \to \pi^0\pi^0\right)} \Big/ \frac{\Gamma\left(K_L \to \pi^+\pi^-\right)}{\Gamma\left(K_S \to \pi^+\pi^-\right)} \approx 1- \quad 6 \quad \mathrm{Re}\left(\varepsilon' / \varepsilon\right)$$