3.4. Neutral meson mixing and CP violation

In the following neutral flavored meson systems (K^0, D^0, B^0_d, B^0_s) in which particle and anti-particle are distinguished by the flavor quantum number will be discussed in a general way: P^0 and $\overline{P^0}$.

A.) Flavor and CP eigenstates

A generic flavored meson P^0 and $\overline{P^0}$ is described by:

$$F|P^0\rangle = +|P^0\rangle \qquad F|\overline{P^0}\rangle = -|\overline{P^0}\rangle$$

CP conjugation transforms particle into anti-particle but introduces in general a phase η_{CP}:

$$CP|P^0\rangle = \eta_{CP}|\overline{P^0}\rangle \qquad CP|\overline{P^0}\rangle = \eta^*_{CP}|P^0\rangle$$

Besides the flavor states there exists the physical states which are mixtures of the flavor states and which, in case CP is conserved, could be chosen to be CP eigenstates:

$$P_{\underset{+}{1}} = \frac{1}{\sqrt{2}}\left(|P^0\rangle + |\overline{P^0}\rangle\right) \qquad P_{\underset{-}{2}} = \frac{1}{\sqrt{2}}\left(|P^0\rangle - |\overline{P^0}\rangle\right)$$

with $CP|P_1\rangle = +|P_1\rangle$ " $CP|P_2\rangle = -|P_2\rangle$

in case the phase η_{CP} is chosen to be $+1$.
(convention)

B) Effective Lagrangian and the physical states

In the most general case the time dependence of a physical state can be described by:

$$\psi(t) = a(t)\,|P^0\rangle + b(t)\,|\bar{P}^0\rangle$$

$\psi(t)$ should fulfill the Schrödinger-Eq with the non-hermitian effective Hamiltonian $\mathcal{H}$ (non-hermitian, because ψ decays outside the $(P^0, \bar{P}^0)$ subspace):

$$i\hbar \frac{d}{dt}\psi(t) = \mathcal{H}\,\psi(t)$$

One usually splits $\mathcal{H}$, which consists of a flavor conserving part $\mathcal{H}_0$ and a weak flavor-violating part $\mathcal{H}_W$ ($\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_W$), into an hermitian and an anti-hermitian part:

$$\mathcal{H} = M - \frac{i}{2}\Gamma$$

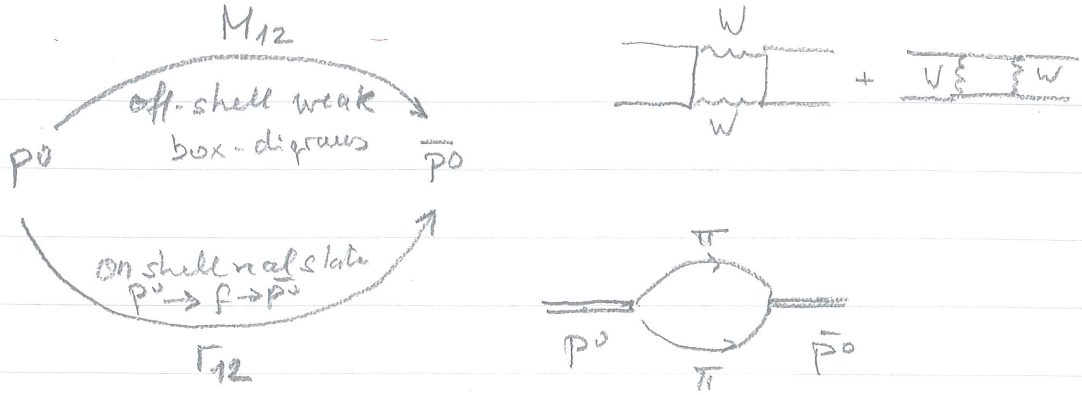
where both M (mass matrix) and Γ (decay matrix) are hermitian.

With the representation $|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$ one can write $\mathcal{H}$ a 2-dim matrix

$$\mathcal{H} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

Hermiticity of M and Γ: $M_{12} = M_{21}^*$, $\Gamma_{12} = \Gamma_{21}^*$ CPT: $M_{11} = M_{22}$ $\Gamma_{11} = \Gamma_{22}$

- The states P^0 and $\bar{P}^0$ are eigenstates to "$\mathcal{H}_0$" $\rightarrow$ no mixing, no decay
  $\mathcal{H}_0$ therefore contributes only to diag(M)
- M_{12} dispersive part of the $P^0 \leftrightarrow \bar{P}^0$ transition describes mixing via virtual intermediate states
- Γ_{12} absorptive part of $P^0 \leftrightarrow \bar{P}^0$ transition describe the mixing via real intermediate states
- Γ_{11}, Γ_{22} describe the decay to real final states



The physical states $|P_a\rangle$ and $|P_b\rangle$ are obtained by diagonalizing the matrix $\rightarrow$ eigenvalues $\lambda_{a,b}$ & eigenstates $P_{a,b}$:

$$\mathcal{H}|P_{a,b}\rangle = \lambda_{a,b}|P_{a,b}\rangle$$

with $\lambda_{a,b} = m_{a,b} - \frac{i}{2}\Gamma_{a,b}$

where $m_{a,b}$ are the masses and $\Gamma_{a,b}$ are the decay widths:

$$|P_{a,b}(t)\rangle = e^{-im_{a,b}\cdot t}\, e^{-\Gamma_{a,b}\cdot t/2}\, |P_{a,b}(t=0)\rangle$$

(a,b)

The physical states are usually labelled by the properties which distinguish them the best:

lifetime for kaons: K_S, K_L

masses for B mesons: B_H, B_L

CP values for D mesons: D_1, D_2 (assuming no direct CPV)

The physical states can be written as

$$|P_a\rangle = p|P^0\rangle + q|\bar{P}^0\rangle$$
$$|P_b\rangle = p|P^0\rangle - q|\bar{P}^0\rangle$$

with $|p|^2 + |q|^2 = 1$

(Remark: we used CPT invariance: $q_a/p_a \overset{!}{=} q_b/p_b =: q/p$)

One further defines the following quantities:

$$\Delta m = m_b - m_a \qquad \Delta\Gamma = \Gamma_b - \Gamma_a$$

$$\Gamma = \frac{1}{2}(\Gamma_a + \Gamma_b) \qquad m = \frac{1}{2}(m_a + m_b)$$

For q/p one finds:

$$q/p = \pm\sqrt{\frac{\mathcal{H}_{21}}{\mathcal{H}_{12}}} = \pm\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

The sign $\pm$ of q/p determines whether m_a or m_b is heavier:
The usual choice is $\Delta m > 0$: $q/p > \Leftrightarrow$ „+" sign.
Attention: This choice <u>is not</u> fixing the sign of $\Delta\Gamma$
(experiment has to tell whether $\Delta\Gamma \gtrless 0$; i.e. CP even/odd lives longer)

<u>Remark:</u> While P^0 and $\overline{P^0}$ as well as P_1 and P_2 are orthogonal P_a and P_b are in general <u>not</u> orthogonal:

$$\xi = \langle P_a | P_b \rangle = |p|^2 - |q|^2 \neq 0$$

If CP symmetry is conserved and P_a, P_b are CP eigenstates: $|q/p| = 1 \rightarrow \xi = 0$

c) Time evolution of flavor states:

Physical states: $|P_a\rangle = p\,|P^0\rangle + q\,|\bar{P}^0\rangle$
$|P_b\rangle = p\,|\bar{P}^0\rangle - q\,|\bar{P}^0\rangle$

[Depending on the properties which distinguish the states best one could label them (P_H, P_L), (P_S, P_L) or (P_1, P_2)]

For the flavor states one obtains correspondingly:

$$|P^0\rangle = \frac{1}{2p}\left[\,|P_a\rangle + |P_b\rangle\,\right]$$
$$|\bar{P}^0\rangle = \frac{1}{2q}\left[\,|P_a\rangle - |P_b\rangle\,\right]$$

Using the time dependence of $|P_a\rangle$ and $|P_b\rangle$ as above:

$$|P^0(t)\rangle = \frac{1}{2p}\left\{ e^{-im_a t}\,e^{-\Gamma_a t/2}\,\underbrace{|P_a(0)\rangle}_{=p|P^0\rangle + q|\bar{P}^0\rangle} + e^{-im_b t}\,e^{-\Gamma_b t/2}\,\underbrace{|P_b(0)\rangle}_{=p|P^0\rangle - q|\bar{P}^0\rangle} \right\}$$

$$= g_+(t)\,|P^0\rangle + \left(\frac{q}{p}\right) g_-(t)\,|\bar{P}^0\rangle$$

$$|\bar{P}^0(t)\rangle = g_+(t)\,|\bar{P}\rangle + \left(\frac{p}{q}\right) g_-(t)\,|P^0\rangle$$

with: $g_{\pm}(t) = \frac{1}{2}\left(e^{-im_a t - \Gamma_a t/2} \pm e^{-im_b t - \Gamma_b t/2}\right)$

$$= \frac{1}{2}\,e^{-imt}\left(e^{-\frac{i}{2}\Delta m t - \Gamma_a t/2} \pm e^{+\frac{i}{2}\Delta m t - \Gamma_b t/2}\right)$$

$g_-(t) = \quad$ " $\quad($ " $\quad - \quad$ " $\quad)$

with $m = \frac{1}{2}(m_a + m_b)$ as above.

Starting from a pure sample of P^0 mesons (e.g. produced in strong interaction) the probability to measure the flavor state $\overline{P^0}$ at time t is given by:

$$\left|\langle \overline{P^0} \,|\, P^0(t)\rangle\right|^2 = |g_-(t)|^2 \left|\frac{q}{p}\right|^2$$

and correspondingly for a pure $\overline{P^0}$ sample at $t=0$:

$$\left|\langle P^0 \,|\, \overline{P^0}(t)\rangle\right|^2 = |g_-(t)|^2 \left|\frac{p}{q}\right|^2$$

Possibility for CPV in mixing

With the above expressions for $g_\pm(t)$ one finds:

$$|g_\pm(t)|^2 = \frac{1}{4}\left(e^{-\Gamma_a t} + e^{-\Gamma_b t} \pm e^{-\Gamma t}\left(e^{-i\Delta m t} + e^{+i\Delta m t}\right)\right)$$

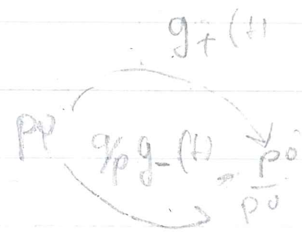
$$= \frac{e^{-\Gamma t}}{2}\left(\cosh\left(\tfrac{1}{2}\Delta\Gamma \cdot t\right) \pm \cos(\Delta m t)\right)$$

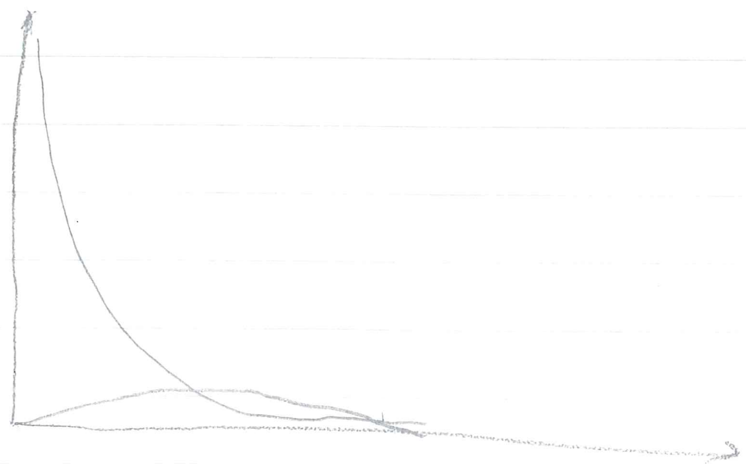
See Chap. 3.1 K^0-$\overline{K^0}$ Mixing

For $\Delta\Gamma \approx 0$ (in B-mesons) and $|p/q| \approx 1$ (fulfilled for all P^0):

$$\mathrm{Prob}(P^0 \to P^0)(t) = \frac{e^{-\Gamma t}}{2}\left(1 + \cos(\Delta m t)\right)$$

$$\mathrm{Prob}(P^0 \to \overline{P^0})(t) = \frac{e^{-\Gamma t}}{2}\left(1 - \cos(\Delta m t)\right)$$

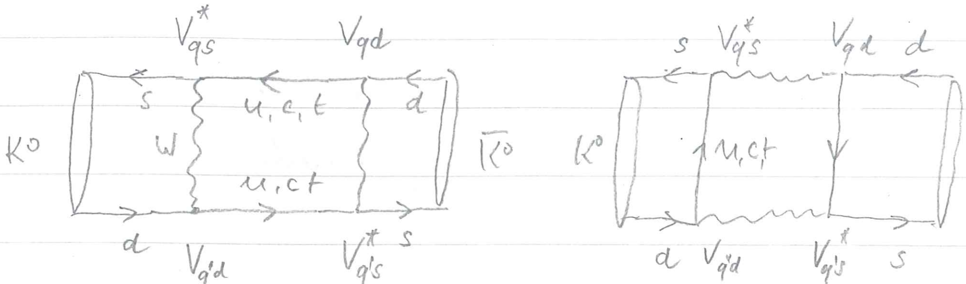




Fig

D. Standard model prediction for mixing

The short distance contribution to the $P^0 \leftrightarrow \bar{P}^0$ transition is described by M_{12} respectively Δm and can be calculated from the contributing box diagrams. For Kaons one obtains



Propagator q, q'

$$\mathcal{M} \sim \sum_{q,q'} V^*_{qs} V_{qd} \Pi_q \cdot V_{q'd} V^*_{q's} \Pi_{q'} \quad \text{with } qq' = \begin{matrix} uu & uc & ut \\ cu & cc & ct \\ tu & tc & tt \end{matrix}$$

$$\sim \int d^4k \, k_\mu k_\nu \left(\frac{V^*_{us} V_{ud}}{k^2 - m_u^2} + \frac{V^*_{cs} V_{cd}}{k^2 - m_c^2} + \frac{V^*_{ts} V_{td}}{k^2 - m_t^2} \right)^2$$

$$\sim \int d^4k \, k_\mu k_\nu \left(V^*_{cs} V_{cd} \left[\frac{1}{k^2 - m_c^2} - \frac{1}{k^2 - m_u^2} \right] + V^*_{ts} V_{td} \left[\frac{1}{k^2 - m_t^2} - \frac{1}{k^2 - m_u^2} \right] \right)^2$$

↑ Unitarity relation: $V^*_{us} V_{ud} + V^*_{cs} V_{cd} + V^*_{ts} V_{td} = 0$

Neglecting in the further calculations the u-quark mass the loop integrals results to 3 terms described by the Inami-Lim-Fkt.

Internal quarks:

cc	$S_0(m_c^2/M_W^2)$	$\simeq 3.5 \cdot 10^{-4}$
tt	$S_0(m_t^2/M_W^2)$	$\simeq 2.5$
ct	$S_0(m_c/M_W, m_t/M_W)$	$\approx 3 \cdot 10^{-3}$

For the CKM factors one obtains: "$|V_{cd} V^*_{cs}|^2 \gg |V_{td} V^*_{ts}|^2$

$\sim \lambda^2 \qquad \sim \lambda^{10}$

$\sim 2.7 \cdot 10^{-2} \qquad \sim 1.1 \cdot 10^{-7}$

$$\rightarrow \quad M_{K\bar{K}^0} \sim S_0(m_c^2/M_W^2) \, |V_{cd} V^*_{cs}|^2$$

Taking the hadronic part into account;

$$\langle K^0 | J_\mu J^\mu | \bar{K}^0 \rangle = \sum_X \langle K^0 | J_\mu | X \rangle \langle X | J^\mu | \bar{K}^0 \rangle$$

$$= \underset{\substack{\uparrow \\ \text{bag factor which accounts} \\ \text{for the vacuum insertion.}}}{B_K} \langle K^0 | J_\mu | 0 \rangle \underbrace{\langle 0 | J^\mu | K^0 \rangle}_{\text{decay constant}} = B_K f_K^2 p_\mu p^\mu$$

one finally obtains:

$$\Delta m_K = 2 |M_{12}| = \frac{G_F^2 \cdot M_W^2}{6\pi^2} \cdot \underbrace{\eta_{QCD}}_{\text{perturbative QCD corrections}} B_K \cdot f_K^2 m_K \left[S_0(m_c^2/M_W^2) |V_{cd} V_{cs}^*|^2 \right]$$

For the B system one has $|V_{td} V_{tb}^*|^2 \sim |V_{cd} V_{cb}^*|^2$ (both $\sim A^2 \lambda^6$). because of $m_t^2 \gg m_c^2$ it is now the top-loop which contributes:

$$\Delta m_{B_d} = \frac{G_F^2 m_W^2}{6\pi^2} \cdot \eta_{QCD} \underbrace{B_B \cdot f_B^2}_{\text{changes for } B_s} m_B \left[S_0(m_t^2/M_W^2) \underbrace{|V_{td} V_{tb}^*|^2}_{B_s:\ V_{ts} V_{tb}} \right]^2$$

For D-mesons the mass of the heaviest internal quark (d-type: b-quark m_b) is not large enough to compensate the large CKM suppression $\sim |V_{ub} V_{cb}^*|^2$. As a result the light s-quark dominates the short range mixing:

$$\Delta m_D \sim |V_{us} V_{cs}^*|^2 \cdot \underbrace{S_0(m_s^2/M_W^2)}_{\sim m_s^2/M_W^2} \sim \lambda^2 \underbrace{S_0(m_s^2/M_W^2)}_{\sim m_s^2/M_W^2}$$

→ Mixing parameters are small (very slow mixing): most of the D's decay before they mix.

As mentioned earlier the mixing consists of 2 components M_{12} and $\frac{1}{2}\Gamma_{12}$. The calculation of Γ_{12} (real intermediate states) is difficult and is important for kaons and D mesons. Within the SM Γ_{12} can be approximated by the „on shell" (absorptive) part of the box diagrams
→ quark representation of the final state (very poor approx. for kaons)

To describe the mixing often, in addition to Δm, dimensionless parameters x and y are introduced:

$$x = \frac{\Delta m}{\Gamma} \qquad y = \frac{\Delta\Gamma}{2\Gamma}$$

<u>Summary of mixing parameters for neutral mesons</u>

System	τ	Δm	x	y
K^0	$0{,}26 \cdot 10^{-9}$ s	$5.29\,\mathrm{ns}^{-1}$	0,477	-1
D^0	$0.41 \cdot 10^{-12}$ s	$0.0024\,\mathrm{ps}^{-1}$	0,0097	0,0078
B_d^0	$1{,}53 \cdot 10^{-12}$ s	$0.507\,\mathrm{ps}^{-1}$	0.78	0,0015 *)
B_s	$1{,}47 \cdot 10^{-12}$ s	$17.77\,\mathrm{ps}^{-1}$	26.1	0,06 *)

*) theoretical values.

Figure with the mixing diagrams for the 4 neutral mesons,
→ Niels Tuning paper

E) Neutral meson decay

In addition to the oscillation we consider in the following also the subsequent decay of neutral mesons.
We consider 4 different decay amplitudes:

$$\mathcal{A}_f = \mathcal{A}(P^0 \to f) \qquad \bar{\mathcal{A}}_f = \mathcal{A}(\bar{P}^0 \to f)$$
$$\mathcal{A}_{\bar{f}} = \mathcal{A}(P^0 \to \bar{f}) \qquad \bar{\mathcal{A}}_{\bar{f}} = \mathcal{A}(\bar{P}^0 \to \bar{f})$$

and we define the complex parameter λ (observable): Modulus & phase of λ.

$$\lambda_f = \frac{q}{p}\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} \qquad \bar{\lambda}_f = \frac{1}{\lambda_f} \qquad \lambda_{\bar{f}} = \frac{q}{p}\frac{\bar{\mathcal{A}}_{\bar{f}}}{\mathcal{A}_{\bar{f}}} \qquad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}}$$

The time-dependent decay rate $\Gamma(P^0 \to f)(t) = |\mathcal{A}(P^0 \to f)|^2$ gives the probability that an initial P^0 decay at time t into the final state f:

$$P^0 \xrightarrow{g_+ \cdot \mathcal{A}_f} f \qquad P^0 \xrightarrow{\frac{q}{p} g_-} \bar{P}^0 \xrightarrow{\bar{\mathcal{A}}_f} f$$

$$|\mathcal{A}(P^0 \to f)(t)|^2 = |\mathcal{A}(P^0 \to f)(t) + \mathcal{A}(P^0 \to \bar{P}^0 \to f)|^2$$

$$\to \quad \Gamma(P^0 \to f)(t) = |\mathcal{A}_f|^2 \left[|g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\,\mathrm{Re}\{\lambda_f\, g_+^*(t)\, g_-(t)\} \right]$$

analog
$$\Gamma(\bar{P}^0 \to f)(t) = |\mathcal{A}_f|^2 \left|\frac{p}{q}\right|^2 \left[|g_-(t)|^2 + |\lambda_f|^2 |g_+(t)|^2 + 2\,\mathrm{Re}\{\lambda_f\, g_+(t)\, g_-^*(t)\} \right]$$

with
$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left(\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) \pm \cos(\Delta m t)\right)$$

$$g_\mp^*(t)\, g_\pm(t) = \frac{e^{-\Gamma t}}{2}\left(\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) \pm i \sin(\Delta m t)\right)$$

In this way one obtains the „Master Equations" for time-dependent neutral meson decays:

$$\Gamma(P^0 \to f)(t) = |A_f|^2 \frac{e^{-\Gamma t}}{2} \cdot$$

$$\left((1+|\lambda_f|^2)\cosh\left(\tfrac{\Delta\Gamma}{2}t\right) + 2\,\mathrm{Re}\,\lambda_f \sinh\left(\tfrac{\Delta\Gamma}{2}t\right) + (1-|\lambda_f|^2)\cos(\Delta m t) - 2\,\mathrm{Im}\,\lambda_f \sin(\Delta m t) \right)$$

$$= |A_f|^2 (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\left(\cosh\left(\tfrac{\Delta\Gamma}{2}t\right) + D_f \sinh\left(\tfrac{\Delta\Gamma}{2}t\right) + \underline{\underline{C_f \cos\Delta m t}} - S_f \cdot \sin\Delta m t\right)$$

with $D_f = \frac{2\,\mathrm{Re}\,\lambda_f}{1+|\lambda_f|^2}$ $\quad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}$ $\quad S_f = \frac{2\,\mathrm{Im}\,\lambda_f}{1+|\lambda_f|^2}$

analog:

$$\Gamma(\bar{P}^0 \to f)(t) =$$

$$|A_f|^2 \cdot \left|\frac{p}{q}\right|^2 (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\left(\cosh\left(\tfrac{\Delta\Gamma t}{2}\right) + D_f \sinh\left(\tfrac{\Delta\Gamma t}{2}\right) - C_f \cos\Delta m t + S_f \sin\Delta m t\right)$$

i.e. $\not{CP}$ differences in the time-dependent decay rate (CPV):

$$A_{CP}(t) = \frac{\Gamma(P^0\to f)(t) - \Gamma(\bar{P}^0 \to f)}{\Gamma + \Gamma}$$

$$= \frac{2C_f\cos(\Delta m t) - 2S_f \sin(\Delta m t)}{2\cosh\left(\frac{\Delta\Gamma t}{2}\right) + 2D_f \sinh\left(\frac{\Delta\Gamma t}{2}\right)} \neq 0$$

if $\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f} \neq 1$

= CP violation in the interference between mixing and decay.

$\Rightarrow$ Master Eq for time dependent CP V in neutral meson decays!

F) Classification of CP violation

Usually the observed CP violating effects in meson decays are classified in the following way:

(i) CPV in decay:

$$\Gamma(P^0 \to f) \neq \Gamma(\bar{P}^0 \to \bar{f})$$

implies $\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| \neq 1$ if $\bar{f} = \eta_{CP} f$: $\left|\frac{\bar{A}_f}{A_f}\right| \neq 1$

e.g. $\Gamma(B^0 \to K^+\pi^-) \neq \Gamma(\bar{B}^0 \to K^-\pi^+)$

In charged mesons where no mixing is possible this is the only type of CP V which can occure.

(ii) CPV in mixing

$$P(P^0 \to \bar{P}^0) \neq P(\bar{P}^0 \to P^0)$$

this implies $\left|\frac{q}{p}\right| \neq 1$ (see mixing equations)

while for B-mesons $\left|\frac{q}{p}\right| = 1 + \mathcal{O}(10^{-4} .. 10^{-5}) \simeq 1$

this is the dominating effect for kaons. ($\mathcal{O}(10^{-3})$)

(iii) CPV in interference between a decay w/ and w/o mixing:

→ time-dependent effect (see above)

no effect in time-integrated measurements

can only occure if $\mathrm{Im}(\lambda_f) = \mathrm{Im}\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$

i.e. if either q/p or $\bar{A}_f/A_f$ has a non trivial phase.

An alternative classification distinguishes between direct and indirect CPV.

Direct CPV : $A(P^0 \to f) \neq A(\bar{P^0} \to \bar{f})$

Indirect CPV : CPV that involves the mixing phenomenum in any way

Final remarks

CP violating effects all depend on J_{CP} and should be of the same order in the SM.

The observable asymmetries = ratios of CP violating to CP conserving quantities are enhanced for suppressed quantities: Observable asymmetries larger in B decays than in kaons → Bs have smaller CKM couplings and larger lifetimes (suppressed w/r to kaons)

To exhibit a CP violating phase the process must involve at least 4 CKM mixing elements (definition of J_{CP})

→ Below the charm threshold on-shell processes cannot violate CP as only V_{ud} and V_{us} are involved.

→ CPV in kaon sector only through virtual processes to which also heavier quarks can contribute: $K^0 - \bar{K}^0$ mixing or penguin decays.

→ Above the charm-threshold CPV also in on-shell processes possible!

Also semileptonic kaon decays allow to show that the major effect comes from CPV in mixing: $K^0 \to \pi^- l^+ \nu_e$

1) with a single hadron in the final state CPT enforces the equality of $K^0 \to f$ and $\bar{K}^0 \to \bar{f}$

$\Rightarrow$ no direct CPV.

2. There can be no interference effects of $K^0 \to f$ and $\bar{K}^0 \to f$ through mixing.

$$K^0 \to f \; + \quad \searrow \bar{K}^0 \to \bar{f}$$

$\Rightarrow$ CPV can only appear through mixing.

Time dependent mixing

$$
\begin{aligned}
g_+(t) &= \frac{1}{2}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t}+e^{-im_L t-\frac{1}{2}\Gamma_L t}\right)=\frac{1}{2}e^{-iMt}\left(e^{-i\frac{1}{2}\Delta mt-\frac{1}{2}\Gamma_H t}+e^{+i\frac{1}{2}\Delta mt-\frac{1}{2}\Gamma_L t}\right)\\
g_-(t) &= \frac{1}{2}\left(e^{-im_H t-\frac{1}{2}\Gamma_H t}-e^{-im_L t-\frac{1}{2}\Gamma_L t}\right)=\frac{1}{2}e^{-iMt}\left(e^{-i\frac{1}{2}\Delta mt-\frac{1}{2}\Gamma_H t}-e^{+i\frac{1}{2}\Delta mt-\frac{1}{2}\Gamma_L t}\right)
\end{aligned}
$$

$$
\begin{aligned}
|g_\pm(t)|^2 &= \frac{1}{4}\left(e^{-\Gamma_H t}+e^{-\Gamma_L t}\pm e^{-\Gamma t}(e^{-i\Delta mt}+e^{+i\Delta mt})\right)\\
&= \frac{1}{4}\left(e^{-\Gamma_H t}+e^{-\Gamma_L t}\pm 2e^{-\Gamma t}\cos\Delta mt\right)\\
&= \frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t\pm\cos\Delta mt\right)
\end{aligned}
$$

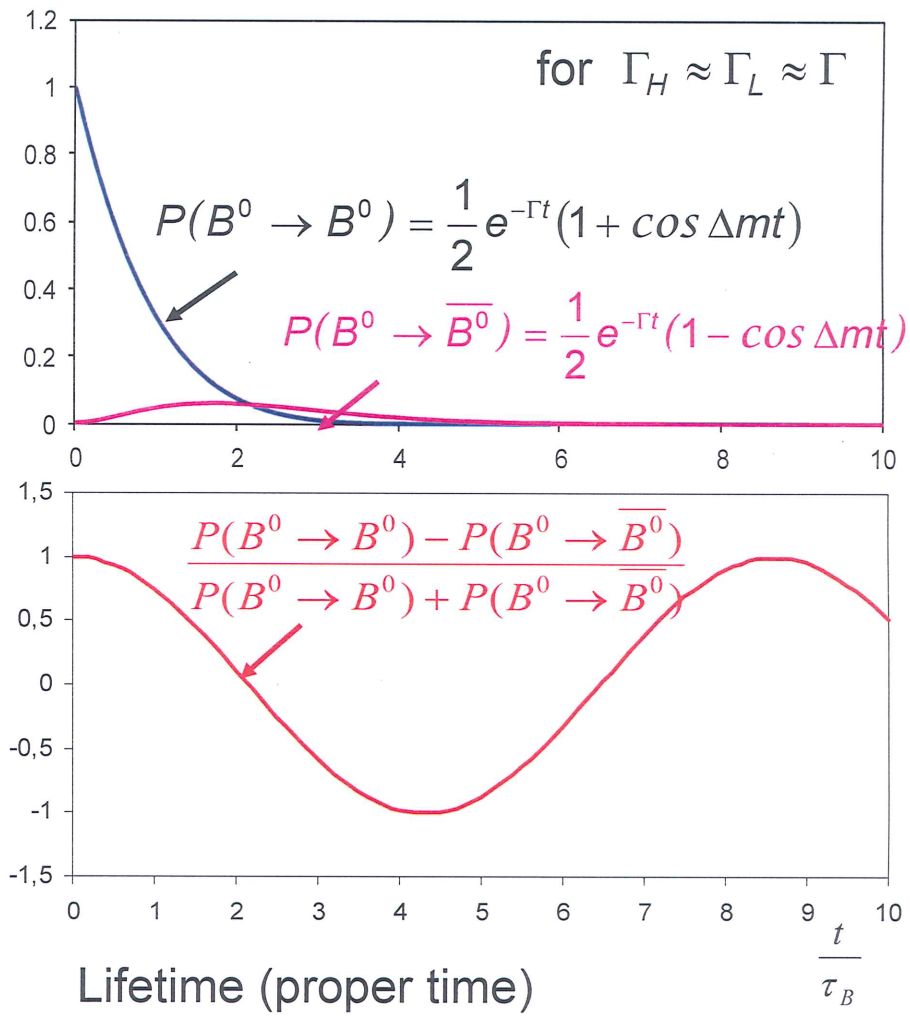
Time-dependent mixing $(\Delta\Gamma\approx 0)$

Mixing probability:

$$B_d \rightarrow f = X\,\mu^+ \quad \text{unmixed}$$

$$B_d \rightarrow \overline{B}_d \rightarrow \overline{f} = \overline{X}\,\mu^- \quad \text{mixed}$$

$$A(t) = \frac{unmixed(t) - mixed(t)}{unmixed(t) + mixed(t)} = \cos(\Delta m t)$$



Direct CP Violation in $K^0 \to \pi\pi$

$$\left.\begin{matrix}(\pi^+\pi^-)\\(\pi^0\pi^0)\end{matrix}\right\} \mathrm{CP} = +1$$

Subtle differences between K_L and K_S decays to 2π due to different contributions from isospin I=0 and 2.

INDIRECT CPV

DIRECT CPV

$$\frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \varepsilon - 2\varepsilon'$$

$$\frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \varepsilon + \varepsilon'$$

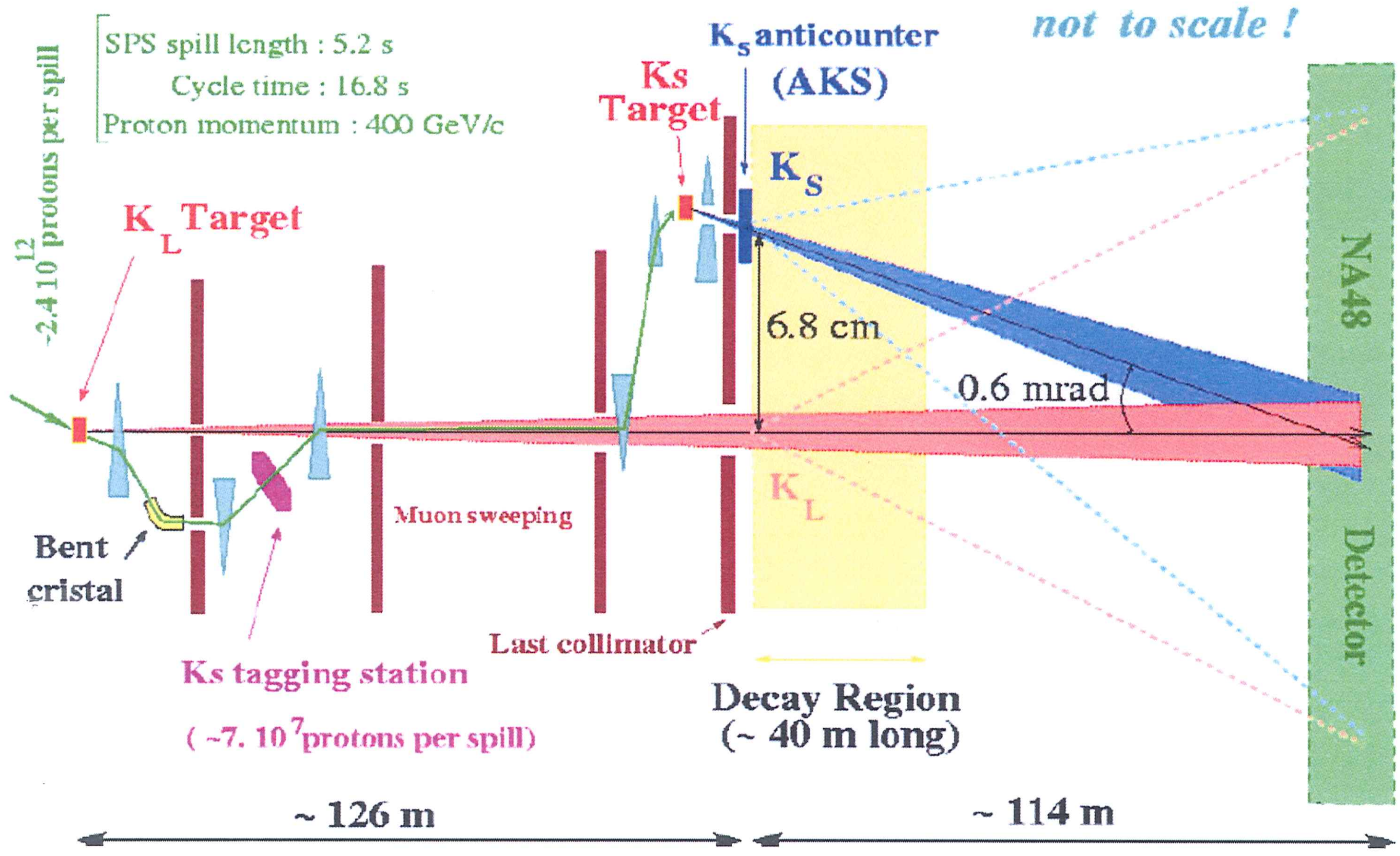
$$\langle 2\pi | K_L \rangle = \varepsilon \langle 2\pi | K_1 \rangle + \langle 2\pi | K_2 \rangle$$

In consequence...

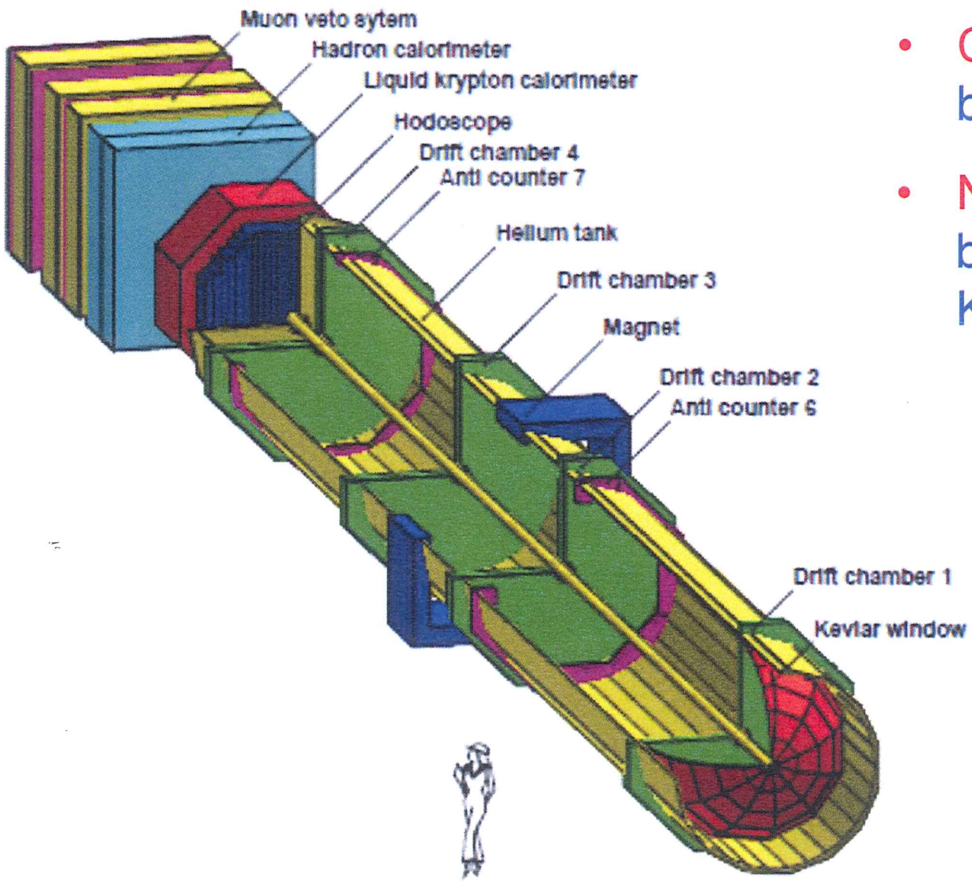
If ε' ≠0 particle and antiparticle decay rates differ:

$$\frac{\Gamma(K^0 \to \pi^+\pi^-) - \Gamma(\overline{K^0} \to \pi^+\pi^-)}{\Gamma(K^0 \to \pi^+\pi^-) + \Gamma(\overline{K^0} \to \pi^+\pi^-)} = 2\,\mathrm{Re}(\varepsilon')$$

NA48 Simultaneous K_L and K_S beams



NA48 Detector



- Charged particles reconstructed by magnetic spectrometer
- Neutral particles reconstructed by quasi-homogenous Liquid Krypton e.m. calorimeter