II. Quark sector

1. Quarks and hadrons

Fundamental particles under the interaction of the SM are quarks:

$$\begin{pmatrix} u \sim 2.5\,\text{MeV} \\ d \sim 5\,\text{MeV} \end{pmatrix} \quad \begin{pmatrix} c \sim 1.27\,\text{GeV} \\ s \sim 95\,\text{MeV} \end{pmatrix} \quad \begin{pmatrix} t \;\; 173\,\text{GeV} \\ b \sim 4.2\,\text{GeV} \end{pmatrix}$$

However, quarks do not exist as free particles but are confined in hadrons:

$q\bar{q}$ = Mesons = Bosons $\quad$ qqq = Baryons = Fermions

(Remark: Observation/confirmation of $Z^+(4430)$ Resonanz $|c\bar{c}d\bar{u}\rangle$ with $J^P = 1^+$)

Exceptions is the Top-quark which decays before it can hadronize:
With a mass larger than the Wb-Threshold the top is decaying nearly entirely to Wb: $t \to Wb$, $|V_{tb}| \approx 1$.

$$\to \Gamma_{top} = \frac{G_F^2 m_t^3}{8\pi\sqrt{2}} \cdot \left(1 - \frac{M_W^2}{m_t^2}\right)\left(1 + 2\frac{M_W^2}{m_t^2}\right)\left[1 + f(\alpha_s)\right]$$

$$\to \Gamma_{top} \approx 1.3\,\text{GeV} \quad \rightsquigarrow \quad \tau \approx 0.5 \cdot 10^{-24}\,\text{s}$$

$\to$ lifetime ($5 \cdot 10^{-25}\,\text{s}$) below the typical hadronization time scale ($\sim \frac{0.5\,\text{fm}}{c} \sim 0.5 \cdot 10^{-23}\,\text{s}$)
no top-flavored hadrons.

The long list of hadrons in the PDG is a bit confusing - not easy to remember.
In the following divide hadrons into

- (pseudo) stable hadrons: „detectable flight length"
  $\to$ lifetime can be measured from displaced vertices.
- unstable particles, also called resonances
  $\to$ lifetime can only be measured via decay width

(Pseudo) stable hadron does not decay through QCD but through weak IA
→ π, K, D, B mesons

<u>Definition</u> A „stable' particle is an eigenstate of the Hamiltonian in the limit that $g_W \to 0$ (i.e. no weak decay)
(Resonances are particles which are not stable)

<u>Example</u>: i) ρ-Meson – Vector-resonance (Spin = 1) of u and d
What is the approximate mass and width?

Since ρ is composed of light quarks, dominant mass contribution comes from QCD binding energy.
⇝ Mass must be in the order $\mathcal{O}(\Lambda_{QCD}) \sim$ few 100 MeV.
⇝ Strong decay, with coupling $\sim \mathcal{O}(1)$: $\Gamma \sim \mathcal{O}(\Lambda_{QCD})$
PDG: $\rho(770)$: $m_\rho = 775$ MeV $\Gamma_\rho = 150$ MeV

ii) B-meson – contains heavy b-quark
→ mass dominated by $m_b = 4.2$ GeV $m_{B^0} = 5280$ MeV
+ QCD binding corrections.
→ B decays weakly, typ. lifetime (b-quark) $\simeq 1.5 \cdot 10^{-12}$ s
with the conversion $\boxed{1\,\text{GeV} = 1.5 \cdot 10^{-24}\,\text{s}^{-1}}$ *)
↳ $\Gamma_B \simeq 4 \cdot 10^{-4}$ eV

<u>Zerfallsbreiten</u> $\Gamma_B / \Gamma_\rho = \dfrac{10^{-4}\,\text{eV}}{10^{8}\,\text{eV}} \simeq 10^{-12}$

d.h. $\tau_B \sim 10^{12} \tau_\rho$ ⟹ B stable compared to ρ

*) $1 = \hbar c = 197$ MeV fm → $1/\text{fm} = 0{,}2\,\text{GeV} = \dfrac{3 \cdot 10^{8}\,\text{m/s}}{10^{-15}\,\text{m}} = 3 \cdot 10^{23}/\text{s}$
⟶ $1.5 \cdot 10^{-24}\,\text{s}^{-1} = 1\,\text{GeV}$

Λ_{QCD}

Running of α_s: $\alpha_s(q^2) \approx \dfrac{1}{\beta_0 \ln(q^2/\Lambda^2)}$

Λ is the QCD scale = scale at which QCD becomes non perturbative.

$\Lambda_{\overline{MS}} \simeq 220$ MeV

Quantum numbers

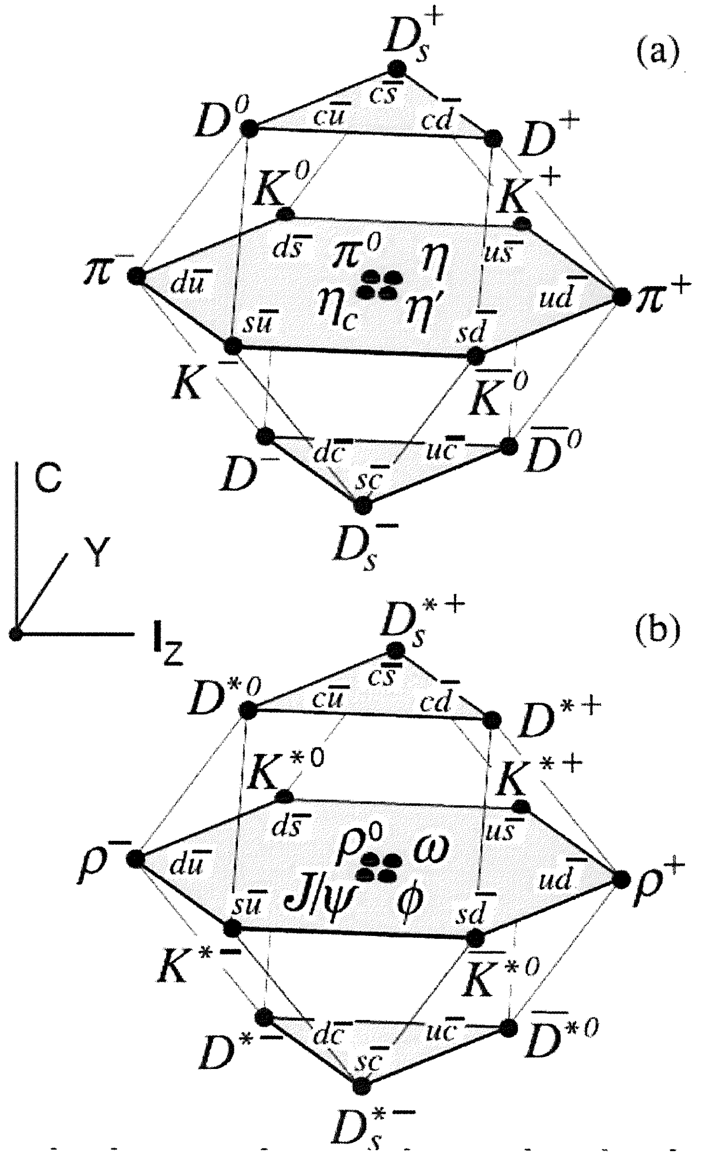
	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	-1	0	0	0
C – charm	0	0	0	$+1$	0	0
B – bottomness	0	0	0	0	-1	0
T – topness	0	0	0	0	0	$+1$

Generalized Gell-Mann-Nishijima formula

$$\mathsf{Q} = \mathsf{I}_z + \frac{\mathcal{B} + \mathsf{S} + \mathsf{C} + \mathsf{B} + \mathsf{T}}{2}$$

Hypercharge

$$\mathsf{Y} = \mathcal{B} + \mathsf{S} - \frac{\mathsf{C} - \mathsf{B} + \mathsf{T}}{3}$$


(a)
D_s^+
D^0
D^+
$c\bar{u}$
$c\bar{s}$
$c\bar{d}$
K^0
K^+
π^-
π^+
$d\bar{s}$
$u\bar{s}$
$d\bar{u}$
$u\bar{d}$
π^0
η
η_c
η'
$s\bar{u}$
$s\bar{d}$
K^-
$\bar{K}^0$
D^-
$\bar{D}^0$
$d\bar{c}$
$u\bar{c}$
$s\bar{c}$
D_s^-
C
Y
I_Z
(b)
D_s^{*+}
D^{*0}
D^{*+}
$c\bar{u}$
$c\bar{s}$
$c\bar{d}$
K^{*0}
K^{*+}
ρ^-
ρ^+
$d\bar{s}$
$u\bar{s}$
$d\bar{u}$
$u\bar{d}$
ρ^0
ω
J/ψ
ϕ
$s\bar{u}$
$s\bar{d}$
K^{*-}
$\bar{K}^{*0}$
D^{*-}
$\bar{D}^{*0}$
$d\bar{c}$
$u\bar{c}$
$s\bar{c}$
D_s^{*-}

1.1 Hadronic quantum numbers

To describe hadronic states 3 types of quantum number (QN) are used:

(i) exact QN:

There are only 2 exact QN: electr. charge Q and spin J *)
Mixing between two hadronic states is only possible if they exhibit the same exact quantum numbers.

*) Remark: Since baryons always have half-integer spin ($2J+1$ = even) and mesons have integer spin ($2J+1$ = odd) the baryon number ~~is~~ implicitly conserved if the spin is conserved.

(ii) QN exact only under QCD but not under weak interaction

Parity P, charge conjugation C and flavor number

a) P and C: In the PDG mesons are denoted by their spin J parity and C-conjugation: J^{PC} superscript: exact only in QCD

Examples: 1) $J^{PC}(\pi^0) = 0^{-+}$

negative internal parity?

$\rightarrow \pi^0$ is lowest $q\bar{q}$ state: $l=0 \rightarrow \uparrow\downarrow$ to get $J=0$

This leads to a minus sign when P is applied:

$\uparrow\downarrow \longrightarrow \downarrow\uparrow$ (wave function picks up "$-$" sign)

2) $J^P(\pi^+) = 0^-$ — no "C"-value!

π^+ is not an eigenstate under C: $\pi^+ \xrightarrow{C} \pi^-$

For the π^+ (and also for other charged mesons) the PDG lists an additional QN: $I^G = 1^-$, where I is the isospin ($I=1$ $I_3=+1$) and the G parity is defined as:

G-parity: $G = C \cdot \underbrace{e^{i\pi I_2}}$

rotation in iso-spin space around the y-axis: $\pi^+(I_3=+1) \to \pi^-(I_3=-1)$

The rotation in isospin space gives for the π^+ an additional phase π (i.e. a „−" sign): $G(\pi^+) = -\pi^+$

Grup theory $G(\pi^+) = -1$

b) Flavor numbers

- for the light (historically called „flavorless") mesons composed of u and d quarks the isospin is used. (Isospin is in principle even in QCD only approximate, since it neglects the mass diff. betwee u and d quaks $\Delta m_{ud} \ll \Lambda_{QCD}$ ↝ good approximation)

- Strangeness, charm, beaty / bottomness:

1.2 Masses and mixing :

Beside the problem that quarks are bound in strongly-coupled bound-states, there is a second problem associated with QCD: Once a quark of a given flavor is bound in a physical-state meson, the meson does not necessarily preserve that quark's flavor:

[Idea of mixing for systems which live in diff. basis can be seen already for light meson states.]

Example : u-quark which hadronize into a neutral π.

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

This state is 50% $u\bar{u}$ and 50% $d\bar{d}$!

Mixing : [A given quark pair describes a meson state with a definite flavor in the sense of $q\bar{q}$. As seen for the π^0 these mesons (flavor states) can mix and the physical mesons are linear combinations of diff. flavor states.]

Question: Why exhibits the $|\pi^0\rangle = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$ maximum mixing while the ψ is a definite flavor state ? (no contributions from light $u\bar{u}$, $d\bar{d}$ or heavy quarks)

The answer is related to the origin of the meson masses:
We saw for the ρ and B that masses are either predominately the effect of QCD binding ($\mathcal{O}(\Lambda_{QCD})$) or the effect of valence quarks masses.

For the light mesons $q\bar{q}$, $q = u, d, s$ isospin is a good symmetry and the masses come from the QCD binding!
(+ additional internal degrees of freedom, like Spin)

u, d states: Each quark has isospin $\frac{1}{2}$. $q\bar{q}$ bound state has $I = 0$ or $I = 1$:

$$q\bar{q} : \quad 2 \otimes \bar{2} = \underset{\substack{\uparrow \\ \text{isospin singlet}}}{1} \oplus \underset{\substack{\uparrow \\ \text{triplet}}}{3}$$

$$\pi : \begin{cases} u\bar{d} \\ \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \\ \bar{u}d \end{cases}$$

$$\eta : \quad \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

- Within the triplet masses are nearly degenerate : m_3
- Singlet state has a different mass : m_1

→ QCD potential gives mass according to the isospin assignment

$$\mathcal{L}_{mass,iso} \sim \underbrace{(\langle 1|, \langle 3|)}_{\text{isospin basis}} \begin{pmatrix} m_1 & 0 \\ 0 & m_3 \end{pmatrix} \begin{pmatrix} \langle 1| \\ \langle 3| \end{pmatrix}$$

heavy states e.g. charmonia $c\bar{c}$, bottomonia $b\bar{b}$

The masses are defined through the flavor content

$$\mathcal{L}_{mass,flavor} \sim \underbrace{(\langle q| \; \langle q'|)}_{\text{flavor basis}} \begin{pmatrix} m_q & 0 \\ 0 & m_{q'} \end{pmatrix} \Big(\quad \Big)$$

(mixing: $\frac{1}{\sqrt{2}}(c\bar{c} + b\bar{b})$)

For real mesons we need to take both effects into account

The real mass matrix lies in general between the two bases: Two-state system with oscillations.

if iso-spin splitting is more important:

iso-spin basis is the natural basis and flavor effect acts as perturbation
→ flavor effect leads to off-diagonal element and thus to mixing between the isospin states :

if flavor-splitting is dominant:

flavor basis is the natural basis and iso-spin splitting acts as perturbation
→ leads to mixing

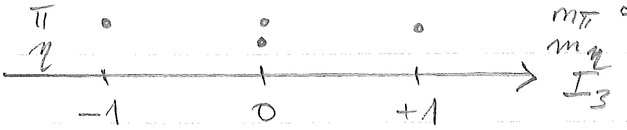
e.g.: Hypothetical $\cos\theta\,|b\bar{b}\rangle + \sin\theta\,|c\bar{c}\rangle$ state

$$\sin\theta \sim \frac{\Lambda_{QCD}}{m_b^2 - m_c^2} \ll 1$$

→ reason why this kind of mixing does not appear.

a) Pseudo-scalar mesons

u, d quarks:

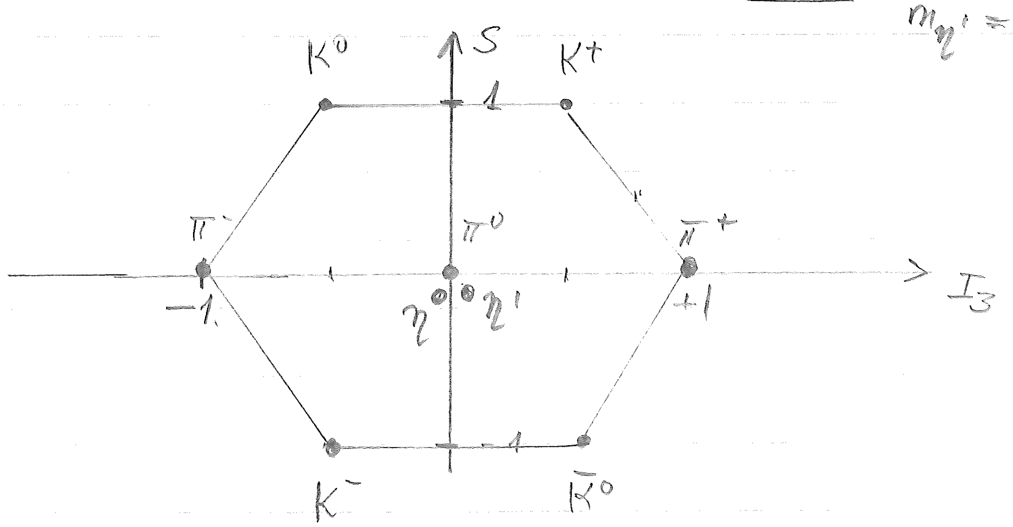


$m_\pi = 135$ MeV
$m_\eta = 550$ MeV

+ s quark : $SU(3)_{\text{iso + strangeness}}$ (often „flavor SU(3)")

$$3 \otimes \bar{3} = 1 \oplus 8$$

Singlet: η'
$m_{\eta'} = 960$ MeV



SU(3) group Theory:

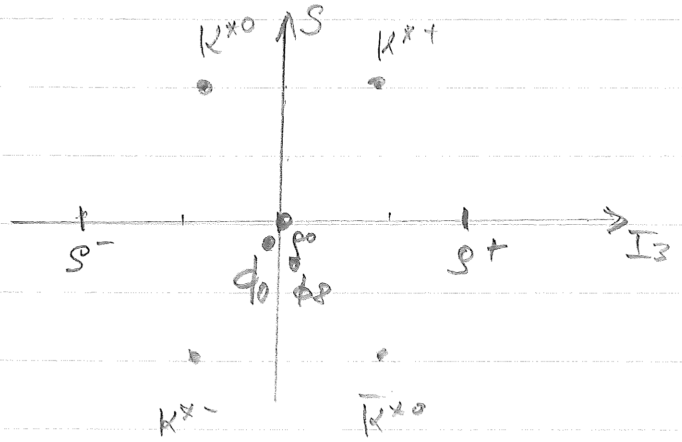
$$\text{Oktet}\begin{cases}\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) & I=1 \quad I_3=0 \\ \eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) & I=0 \quad I_3=0\end{cases}$$

$$\text{Singlet} \quad \eta_0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} - s\bar{s}) \qquad I=0 \quad I_3 0$$

→ Isospin basis, but the mass diff $|m_s - m_{u,d}|$ leads to mixing between the two state → 10% correction. (physical states are mixture of the isospin states)
$\eta_8, \eta_0 \rightarrow$ physical states η', η'

b) Vector mesons $J^P = 1^-$ (spin alignment different w/r to pseudo-scalars)

SU(3) group theory:
$1 \oplus 8$



In principle the same as for pseudo-scalar mesons but the mixing is large:
Physical states ϕ, ω differ significantly from isospin state ϕ_0 ϕ_8:

$$\phi = \cos\theta \, \phi_8 - \sin\theta \, \phi_0 \qquad \text{with } \theta = 37^0$$
$$\omega = \sin\theta \, \phi_8 + \cos\theta \, \phi_0$$

Ideal mixing $\tan\theta = 2\sqrt{2} \approx 35.5^0$;

$$\left.\begin{aligned}\phi' &= s\bar{s} \\ \omega' &= \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\end{aligned}\right\}$$

Real ϕ is nearly a pure $s\bar{s}$ state!

1.3 Names of hadrons

Historically mesons of light quarks are called „unflavored"
in contrast to „flavored" mesons of heavy quarks:

Kaons: net-strangeness (s quark $\Rightarrow S=-1$)
D-mesons: net-charm (c-quark $\Rightarrow C=+1$)
B-mesons: net-beauty (b-quarks $\Rightarrow B=-1$)

- Pseudoscalar mesons: $J^{PC}=0^{-+}$: $\pi, \eta, \eta', K, D, B$

- Vector mesons: $J^{P}=1^{--}$: $\rho, \omega, \phi, K^*, D^*, B^*$

- other mesons: $J^{PC}=0^{++}$ mesons: $a_0, a_1, a_2, \ldots$
  ($\sim$ relative angular momentum $\ell=1$, Spin $S=1 \rightarrow J=0$)
  $J^{PC}=1^{+-}$ mesons: b_0, b_1, b_2
  ($\ell=1$, $S=0 \rightarrow J=1$)

- Baryons (half-integer spin):
  - Baryons with 3 light u, d quarks
    $I=\frac{1}{2}$: nucleons N
    $I=\frac{3}{2}$: Δ's
  - Baryons with 2 light (u,d) quarks $+ s, c, b$:
    $I=0$: Λ (Λ_c, Λ_b)
    $I=1$: Σ (Σ_c, Σ_b)
  - Baryons with 1 light quark"
    $I=\frac{1}{2}$: $\underset{ss}{\Xi}$ $(\underset{sc}{\Xi_c}, \underset{sb}{\Xi_b})$
  - Baryons with no light quark
    $\Omega\,(sss)$ $\Omega_c\,(ssc)$ $\Omega_b\,(ssb)$

Hyperons

1.4 Weak-decays of hadrons

Theoretically quarks are the fundamental particles participating in IA.
Experimentally however one observes / probes hadrons as asymptotic states

↳ Introduce a clever parametrization to treat the problem.

- Factorize the different physical effects in the transition amplitude. Factorization exploits the idea that one can separate diff. kinds of physics in a factorizable way: e.g. physics at diff. scales decouple.

- Form factors: describes shape corrections to the approximation that the scattering object is not point like (→ familiar concept in non-relativistic Rutherford scattering)

- Decay constant: Absorbes the non-perturbative properties of meson decays.

Example: $\pi \to \ell \nu$: $\mathcal{A} = \langle \ell\nu | \mathcal{O} | \pi \rangle$

asymptotic states

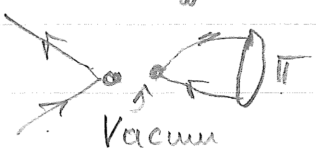
As the decay is a low energy process $m_\ell, m_\pi < M_W$ one can integrate-out the W-boson → effective 4 fermion IA

↳ Effective operator $\mathcal{O}$: $\mathcal{O} \sim \mathcal{O}_\ell \frac{1}{M_W^2} \mathcal{O}_\pi$

"symbolically" $= \left("\bar{\ell} \gamma^5 \gamma^\mu \nu" \right) \frac{1}{M_W^2} \left("\bar{u} \gamma^5 \gamma_\mu d" \right)$

The matrix element should factorize the same way:

$$\langle l\nu | O | \pi \rangle = \langle l\nu | O_e | 0 \rangle \frac{1}{M_W^2} \langle 0 | O_H | \pi \rangle$$



As we don't know how to calculate $\langle 0 | O_H | \pi \rangle$ one treats it as a parameter of the theory:

$$\pi\text{- decay const} \quad f_\pi \sim \langle 0 | O_H | \pi \rangle$$

The decay const. absorbes all the non-perturbative „brown mud" that keep the π together:
The operator O_H should annihilate the 2 quarks in the pion and must therefore be of the form:

$$\bar{u}\Gamma d \quad \text{with coupling structure } \Gamma = S, P, A, V, T$$

Since the parity of the vacuum is „even" and the parity of π is „odd" the main contribution comes from the axial-vector operator:

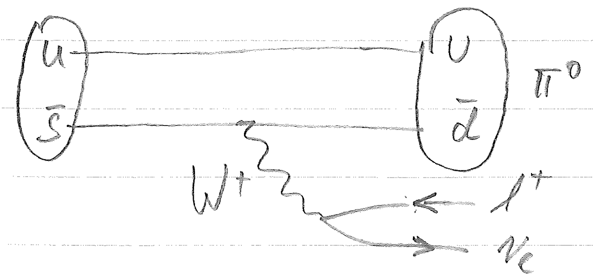
$$\langle 0 | O_H | \pi \rangle = \underbrace{\langle 0 | \bar{u} \gamma^5 \gamma^\mu d | \pi \rangle}_{} = \langle 0 | A^\mu | \pi \rangle \overset{!}{=} -i f_\pi q^\mu$$

we don't how to calculate
→ we absorb our ignorance in the decay constant (must have mass dimension)
→ As the Lorentz structure should be an (axial) vector and the only vector involved in the decay of the scalar π is the momentum transfer: $\langle 0 | O_H | \pi \rangle \sim q^\mu$

The decay constant f_π can be measured (π decay): $f_\pi \simeq 131\,\text{MeV}$
One obtains the matrix element for the decay:

$$M(\pi^- \to \mu^- \bar{\nu}) = \frac{-g^2}{4M_W^2} \cdot f_\pi \cdot (V_{ud}) \cdot m_\mu (\bar{\mu}_R \nu_L)$$

Example: $K^+ \to \pi^0 l^+ \nu$



$$\mathcal{A} = \langle \pi^0 l\nu | \mathcal{O} | K^+ \rangle$$

$$= \underbrace{\langle l\nu | \mathcal{O}_e | 0 \rangle}_{\text{easy - see above}} \cdot \frac{1}{M_W^2} \cdot \underbrace{\langle \pi | \mathcal{O}_H | K^+ \rangle}_{\substack{\text{QCD binding of quarks} \\ \hookrightarrow \text{difficult}}}$$

→ Absorb the non-perturbative effects into a form-factor

$$F \sim \langle H_2 | \mathcal{O} | H_1 \rangle$$

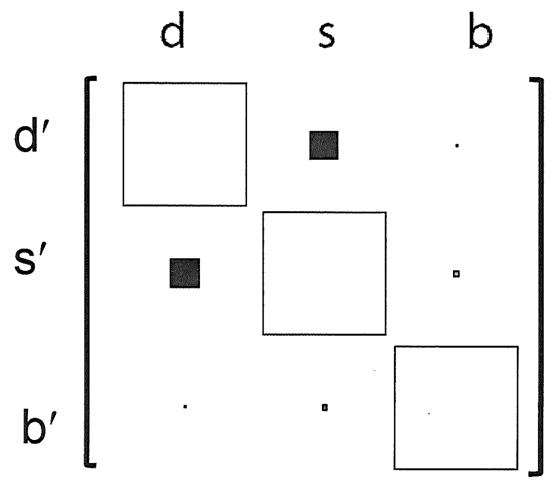
Form factors can be determined (measured) in semi-leptonic decays.

CKM Matrix - PDG Parameterization

$$V_{\mathrm{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

Hierarchical structure



Wolfenstein Parameterization

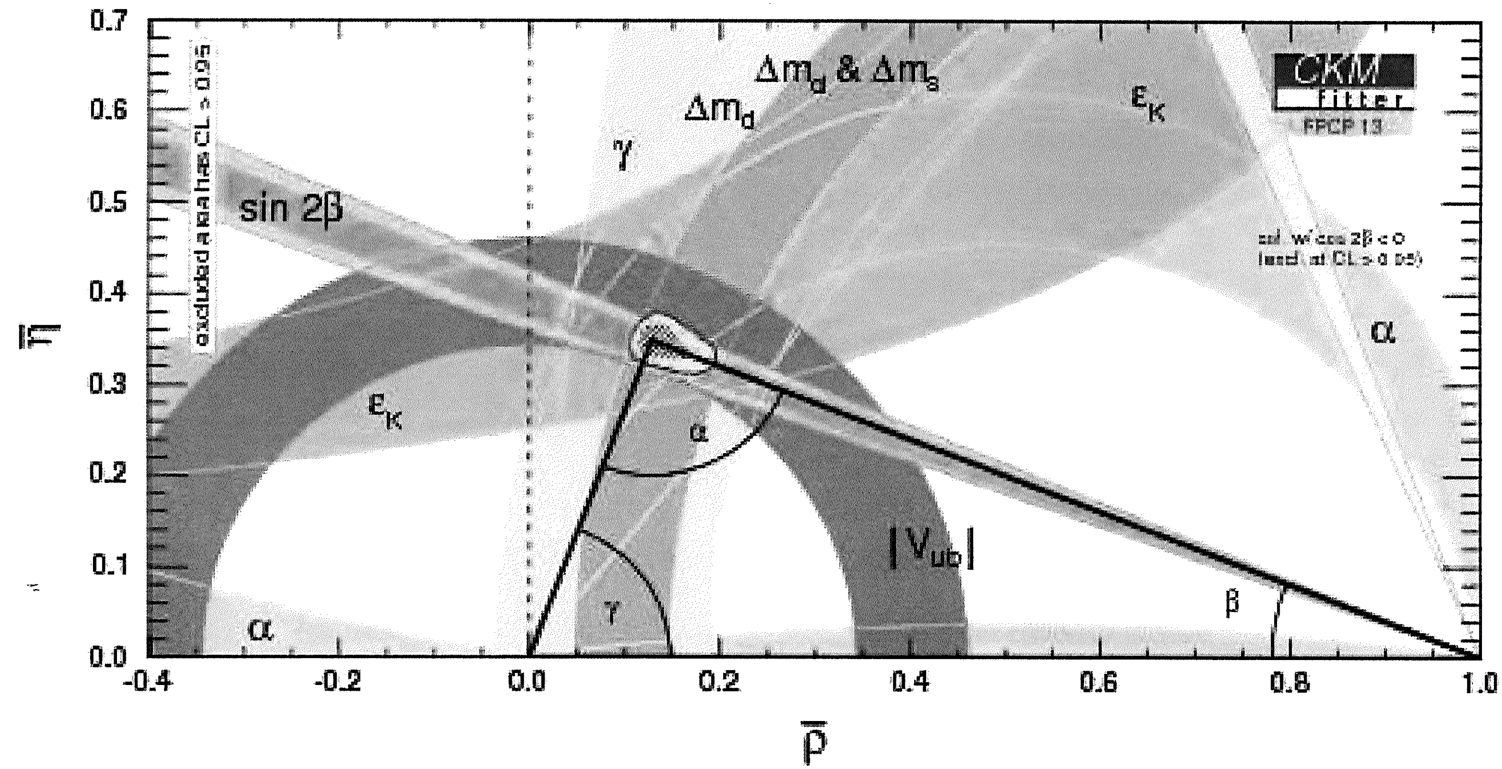
$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \delta V$$

$$\delta V = \begin{pmatrix} -\frac{1}{8}\lambda^4 & 0 & 0 \\ \frac{1}{2}A^2\lambda^5(1-2(\rho+i\eta)) & -\frac{1}{8}\lambda^4(1+4A^2) & 0 \\ \frac{1}{2}A\lambda^5(\rho+i\eta) & \frac{1}{2}A\lambda^4(1-2(\rho+i\eta)) & -\frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

Unitarity Triangle

http://ckmfitter.in2p3.fr/



$A = 0.823^{+0.012}_{-0.033}$ $\lambda = 0.22457^{+0.00186}_{-0.00014}$

$\bar{\rho} = 0.1289^{+0.0176}_{-0.0094}$ $\bar{\eta} = 0.348 \pm 0.012$