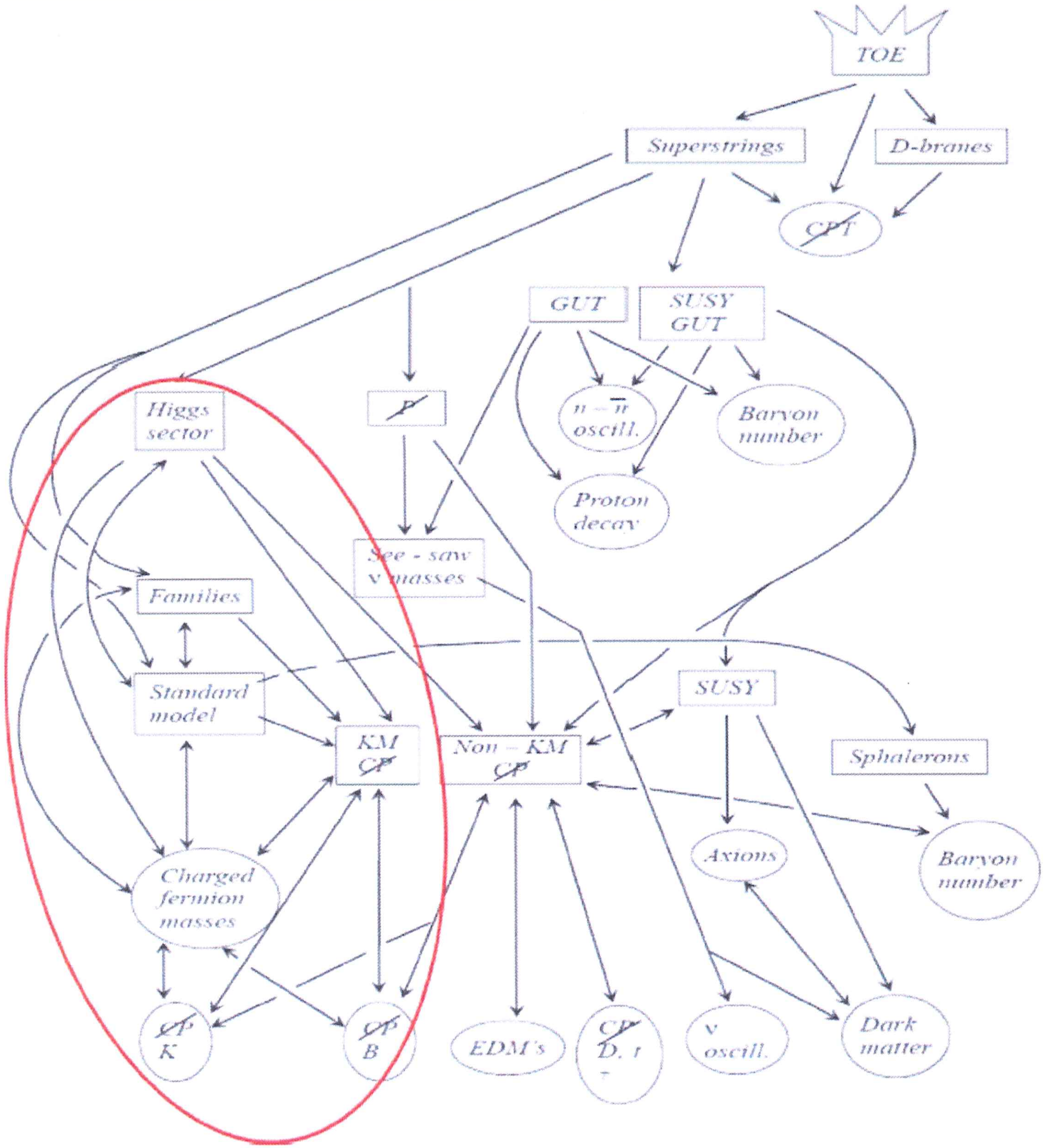
Flavor Physics



I.I. Bigi and A.I. Sanda. *CP Violation.* Cambridge University Press, Cambridge, (England), 2000.

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   1. CKM Matrix
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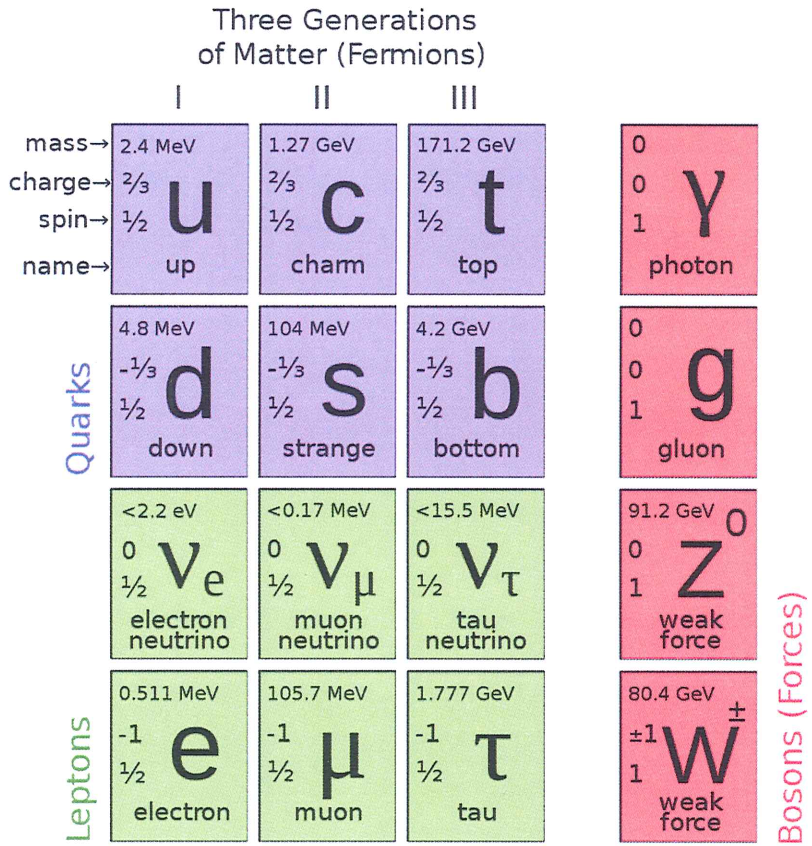
III. Physics with leptons
   1. Muon and Tau-Physics
   2. Charged lepton flavor violation
   3. Neutrinos physics

Lecture not fully planned through – let´s see how far we get!

I. Standard Model

Fig. I.1



http://de.wikipedia.org/wiki/Standardmodell

Ordinary matter is made of particles of the 1st generation

Particles of the 3 generations carry the same charges under the Standard Model gauge group: SU(3) × SU(2) × U(1)

Flavor sector: Masses and Mixing

Fig. I.2

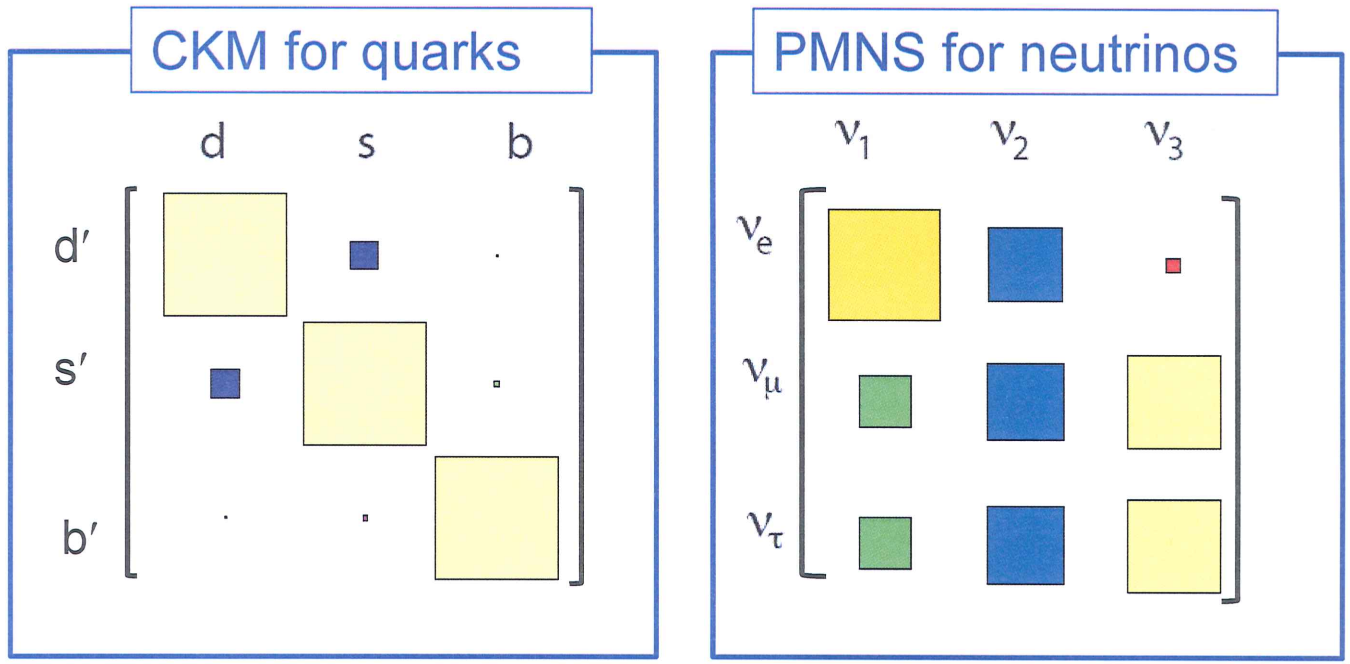
Flavor parameters:

- 6 quark masses
- 3 quark mixing angles + 1 phase: CKM matrix
- 3 + 3 lepton masses
- 3 lepton mixing angles + 1phase : PMNS matrix

} 20 parameters

Mixing:

Area ~V^2



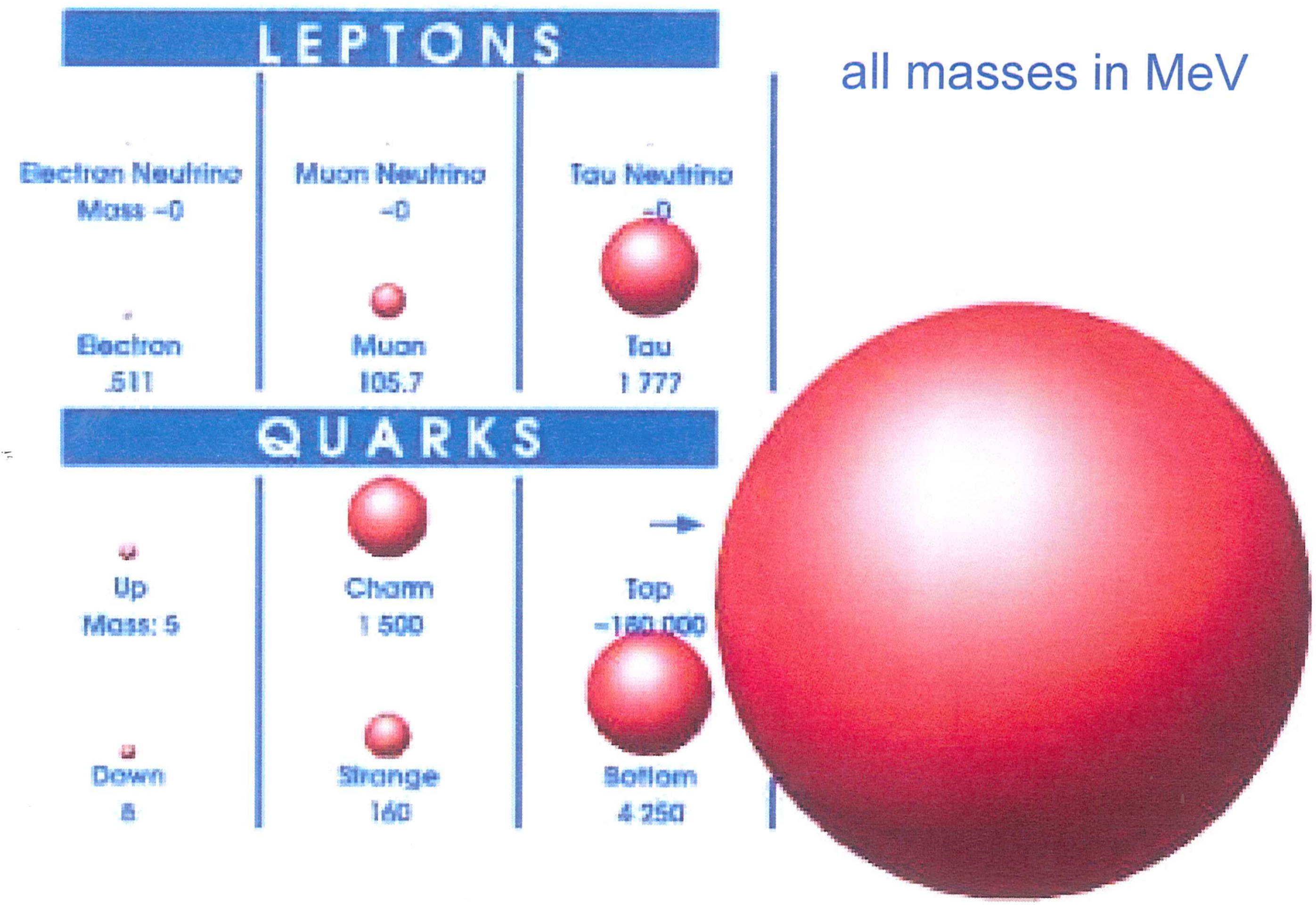
Pontecorvo–Maki–Nakagawa–Sakata

Why these values? Are the two related? Are they related to masses?

Fermion Masses

Fig. I.3

http://www.physics.mcmaster.ca/ElementaryParticle/



all masses in MeV

I. Flavor in the Standard Model

1. Fundamental particles

3 Generations of fundamental particles

	I	II	III	Q [e]
Quarks	u	c	t	$+2/3$
	d	s	b	$-1/3$
Leptons	ν_e	ν_μ	ν_τ	0
	e	μ	τ	-1

The 3 generations carry the same charges under the SM gauge group $SU(3)_C \times SU(2)_L \times U_Y(1)$.

Flavor physics: Difference between the 3 Generations and the phenomenology related to it.

Remark: Chirality

Exp. observation: Only left-handed fermion components ψ_L participate in the weak charged current interactions.
→ W bosons only couple to LH fermion components (RH antifermions)

$\boxed{\text{Parity is violated maximally!}}$

Fermion components: $\psi_L = P_L \psi = \frac{1}{2}(1-\gamma^5)\psi$
$\psi_R = P_R \psi = \frac{1}{2}(1+\gamma^5)\psi$

with projectors P_L and P_R: $P_L^2 = P_L$ $\quad P_L P_R = P_R P_L = 0$
$P_R^2 = P_R$ $\quad P_L + P_R = 1$

One finds the following decomposition:

$$(*) \quad \bar{\psi}\gamma^\mu\psi = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R \qquad \text{fermion currents}$$

$$\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L \qquad \text{mass terms}$$

with $\bar{\psi} = \psi^\dagger\gamma^0$

Excercise: Show $(*)$

[For QCD with n massless quarks:
Global chiral symmetry: $U(n)_L \times U(n)_R$ } broken by quark masses

In order to accomodate observed parity violation, left and right-handed quarks are assigned to different representations of $SU(2)_L$: LH fermions are doublets with weak isospin $\frac{1}{2} = T$
RH fermions are singlets; no weak charge

[Excercise - Solution

1) $\bar{\psi}_L = \psi_L^\dagger\gamma^0 = \left(\psi^\dagger \cdot \frac{1}{2}(1-\gamma^5)\right)\gamma^0 = \psi^\dagger\gamma^0\frac{1}{2}(1+\gamma^5) = \bar{\psi}\,\frac{1}{2}(1+\gamma^5)$

$\gamma^5 = \gamma^{5\dagger}$ $\qquad$ $\gamma^5\gamma^0 = -\gamma^0\gamma^5$

damit hat man: $\bar{\psi}_L\gamma^\mu\psi_R = \frac{1}{4}\bar{\psi}(1+\gamma^5)\gamma^\mu(1+\gamma^5)\psi$

$\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5 \rightarrow = \underbrace{\frac{1}{4}\bar{\psi}\gamma^\mu\underbrace{(1-\gamma^5)(1+\gamma^5)}_{=0}\psi}_{=0}$

damit wird $\bar{\psi}\gamma^\mu\psi = (\bar{\psi}_L + \bar{\psi}_R)\gamma^\mu(\psi_L + \psi_R) = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R$

2. Particle representations and charges under $SU(3)_c \times SU(2)_L \times U_Y(1)$
$Y = Q - T_3$

Each generation consists of the following five fermion multiplets

$Q_{Li}\ (3, 2, +1/6) = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$ LH Quark multiplett
↑ generation index
Triplet under $SU(3)_c$
Doublett under $SU(2)_L$ $T = \frac{1}{2}$
$U(1)$ Hypercharge $Y = +\frac{1}{6}$

$u_{Ri}\ (3, 1, +\frac{2}{3})$ $SU(2)_L$ singlet $T = 0$

$d_{Ri}\ (3, 1, -\frac{1}{3})$

$L_{Li}\ (1, 2, -\frac{1}{2}) = \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix}$ Leptons are $SU(3)_c$ singlets
Isospin doublet and Hypercharge $-\frac{1}{2}$

$l_{Ri}\ (1, 1, -1)$

3) Yukawa coupling to Higgs and origin of mass

SM Lagrangian: $\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$

a) Kinetic term describes dynamics of fermions

$$\mathcal{L}_{Kinetic} = i\bar{\Psi} D^\mu \gamma_\mu \Psi$$

with the kovariant derivative

$$D^\mu = \partial^\mu + i g_s G_a^\mu \lambda_a + i g W_b^\mu \sigma_b + i g' B^\mu Y$$

G_a^μ, W_b^μ and B^μ are the 8 + 3 + 1 gauge fields

Kinetic term for LH quarks and leptons

$$\mathcal{L}^{quark}_{kinetic} = i\,\overline{Q}_{Li}\,\gamma_\mu\Big(\partial^\mu + \frac{i}{2} g_s G^\mu_a \lambda_a + \underbrace{\frac{i}{2} g W^\mu_b \tau_b}_{T} + \underbrace{\frac{i}{6} g' B^\mu}_{Y}\Big) Q_{Li}$$

$$\mathcal{L}_{kinetic} = i\,\overline{L}_{Li}\,\gamma_\mu\Big(\partial^\mu + \frac{i}{2} g W^\mu_b \tau_b + \frac{i}{2} g' B^\mu\Big) L_{Li}$$

(expression for RH Quarks + Leptons less interesting: only NC!)

b) Higgs - Potential describing the scalar self-interactions:

$$\mathcal{L}_{Higgs} = \mu\ \phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2$$

(vacuum stability : $\lambda > 0$; Pattern of spontaneous symmetric breaking: $\mu^2 < 0$)

c) Yukawa interaction with Higgs-Field:

-> Split into leptonic and baryonic (quark) part:

$$-\mathcal{L}^{lept}_{Yukawa} = Y^\ell_{ij} \cdot \overline{L}_{Li}\ \phi \cdot \ell_{Rj} + h.c.$$

After symmetry breaking Higgs acquires" VEV $\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

-> lead to masses for the charged leptons

(Lepton Yukawa matrix diagonal (no mixing) and involve 3 parameters = charged lepton masses. $m_{\ell i} = Y^\ell_{ii} \cdot \frac{v}{\sqrt{2}}$

Note that neutrinos stay massless in SM.

→ For the quarks one obtains:

$$-\mathcal{L}^{quarks}_{Yukawa} = Y^d_{ij}\,\overline{Q}_{Li}\,\phi\,d_{Rj} + Y^u_{ij}\,\overline{Q}_{Li}\,\tilde{\phi}\,u_{Rj} + h.c.$$

This is where the quark masses and the flavor mixing arises: The Yukawa matrices describing the Yukawa interaction are in general complex and non-diagonal (→ flavor structure)

After symmetric breaking and with the doublet $Q_{Li} = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$ one can rewrite the Yukawa interaction as:

$$-\mathcal{L}^{quarks}_{Yukawa} = -\frac{v}{\sqrt{2}}\left(\overline{d}_{Li}\,Y^{d}_{ij}\,d_{Rj} + \overline{u}_{Li}\,Y^{u}_{ij}\,u_{Rj}\right) + h.c.$$

Mass eigenstates $\tilde{u}$ and $\tilde{d}$ of quarks can be obtained by a unitary transformation:

$$\tilde{u}_L = V_{L,u}\,u_L \qquad \tilde{d}_L = V_{L,d}\,d_L$$
$$\tilde{u}_R = V_{R,u}\,u_R \qquad \tilde{d}_R = V_{R,d}\,d_R$$

with all matrices: $V_{A,q} \cdot V^{+}_{Aq} = 1$

The matrices V_{Aq} are chosen such that the matrices

$$M_d = \frac{v}{\sqrt{2}} \cdot V_{Ld} \cdot Y^d \cdot V^{+}_{Rd} \overset{!}{=} \mathrm{diag}(m_d, m_s, m_b)$$

$$M_u = \frac{v}{\sqrt{2}}\, V_{Lu} \cdot Y^u \cdot V_{Ru} = \mathrm{diag}(m_u, m_c, m_t)$$

are diagonal.

After this transformation quark mass terms appear as usual Dirac terms:

$$-\mathcal{L}^{quark}_{Yukawa} = \bar{\tilde{d}}_{Li} (M_d)_{ii} \tilde{d}_{Ri} + \bar{\tilde{u}}_{Li} (M_U)_{ii} U_{Ri}$$

→ leads to 6 quark masses: $m_u, \dots m_t$

However there is a non-vanishing effect on the charged currents.
→ u, d quarks appear together and are transformed by diff. $V_{L,q}$

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \left(\bar{u}_{Li} \gamma^\mu W^+_\mu d_{Lj} + \bar{d}_{Li} \gamma^\mu W^-_\mu u_{Li} \right)$$

$$\downarrow \quad u_L \rightsquigarrow \tilde{u}_{Li} = (V_{L,U})_{ij} u_j \quad \text{und} \quad d_L \rightsquigarrow \tilde{d}_L = V_{L,d}\, d$$

$$u_L = (V_{L,U})^+ \tilde{u}_L$$

$$\mathcal{L}_{CC} = \frac{-g}{\sqrt{2}} \Big(\bar{\tilde{u}}_{Li} \gamma^\mu W^+_\mu \underbrace{(V_{L,U} \cdot V^+_{L,d})_{ij}}_{V_{CKM}} \tilde{d}_{Lj} + \bar{\tilde{d}}_{Li} \gamma^\mu W^-_\mu \underbrace{(V_{L,d} V^+_{L,U})_{ij}}_{V^+_{CKM}} \tilde{u}_{Lj} \Big)$$

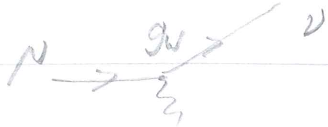
There is no net effect on the neutral currents.

With $(u_1, u_2, u_3) = (u, c, t)$ und $(d_1, d_2, d_3) = (d, s, b)$
This choice: $d_i = (V_{CKM})_{ij} \tilde{d}_j$ → weak state are mixtures of mass; $u_i = \tilde{u}_i$

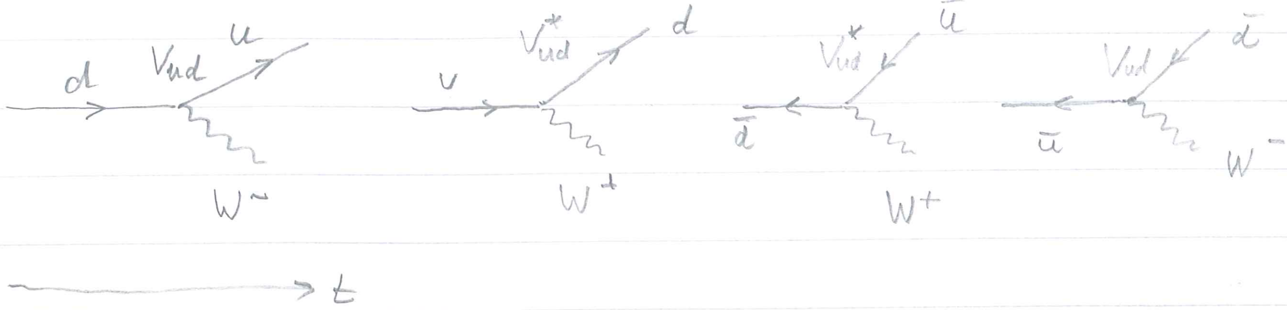
$$\underset{\text{interaction eigenstates}}{\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} \tilde{d}_L \\ \tilde{s}_L \\ \tilde{b}_L \end{pmatrix} \leftarrow \text{Mass eigenstates}$$

Since V_{CKM} is product of unitary matrices V_{CKM} is unitary: $V_{CKM} V^+_{CKM} = 1$

Bem.: For the rest of the lecture I will use d and u for the mass (flavor) eigenstates and d' for the weak eigenstate.



Pictorial explanation of CKM elements:



4) Properties of V_{CKM}

V_{CKM} is a complex and unitary matrix: $V_{CKM} V_{CKM}^+ = 1$

# parameters of compl. 3x3	:	18 = 9 real + 9 phases
after unitarity condition (→ 9 parameters removed)	:	9 = 3 real + 6 phases
5 phases are absolute quark phases and are not observable		4 = 3 real + 1 phase

Unobservable Quark phases:

Rephasing of quark fields possible: $q_L = e^{i\phi_q} \cdot q_L$

for $q = u, d, c, s, t, b$

Under this phase transformation:

$$V_{CKM} \mapsto \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{i\phi_t} \end{pmatrix} V_{CKM} \begin{pmatrix} e^{i\phi_d} & 0 & 0 \\ 0 & e^{i\phi_s} & 0 \\ 0 & 0 & e^{i\phi_b} \end{pmatrix}$$

or

$$V_{\alpha\beta} \to \exp\left(i[\phi_\beta - \phi_\alpha]\right) V_{\alpha\beta}$$

this allows to rotate away 5 phases related to phase diff.

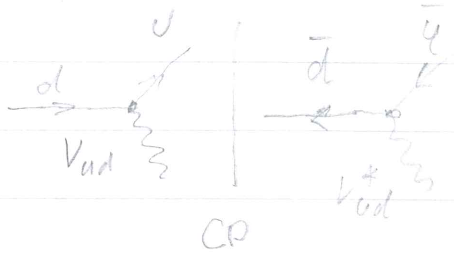
In general for N quark generations: $N \times N$ mixing matrix

$\frac{1}{2} N(N-1)$ real parameters

$\frac{1}{2}(N-1)(N-2)$ phases

N	# real param	# phases
2	1	0
3	3	1
4	6	3

- Phase in CKM matrix is the source of CPV:
- True also for neutrinos!



5) Summary of Flavor sector in SM

Quark sector
- 6 masses
- 4 CKM parameter

Lepton sector
- 3 charged Lepton masses

+ extension: massive neutrinos
+ 3 neutrino masses
+ 4 mixing parameter
= "PMNS matrix.

= Pontecorvo - Maki - Nakagawa - Sakata

So far masses and mixing parameters are exp. inputs to theory. There is no deeper understanding of the mass hierarchy and the mixing parameters $\longrightarrow$ Is there a relation between Masses & mixing and leptons & quarks?

Light neutrino generations

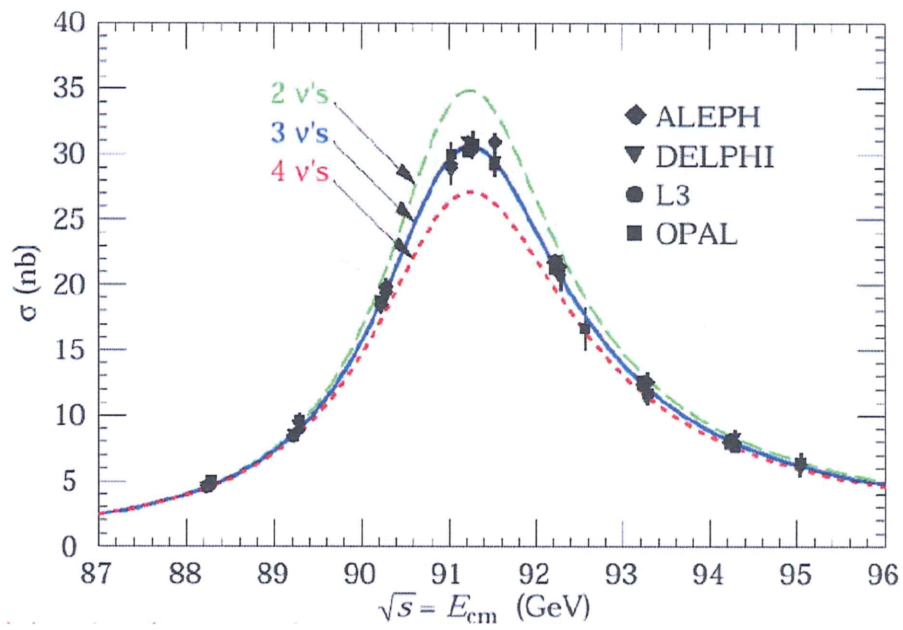
Fig. I.4

$$\Gamma_Z = \Gamma_{had} + 3\cdot\Gamma_\ell + \underbrace{N_\nu \cdot \Gamma_\nu}_{\text{invisible}:\,\Gamma_{inv}} \rightarrow \left\{ \begin{array}{l} e^+e^- \to Z \to \nu_e\bar{\nu}_e \\ e^+e^- \to Z \to \nu_\mu\bar{\nu}_\mu \\ e^+e^- \to Z \to \nu_\tau\bar{\nu}_\tau \end{array} \right.$$

$$\boxed{\Gamma_{inv} = 0.4990 \pm 0.0015\ \text{GeV}}$$

$$\boxed{N_\nu = \frac{\Gamma_{inv}}{\Gamma_{\nu,SM}}} = \underbrace{\left(\frac{\Gamma_{inv}}{\Gamma_\ell}\right)_{exp}}_{5.9431\pm0.0163} \cdot \underbrace{\left(\frac{\Gamma_\ell}{\Gamma_\nu}\right)_{SM}}_{=1/1.991\pm0.001}$$

(small theo. uncertainties from m_{top} M_H)



$$\boxed{N_\nu = 2.9840 \pm 0.0082}$$

6) Limits on possible „sequential" fourth generation

6.1 LEP bounds: additional heavy neutrinos

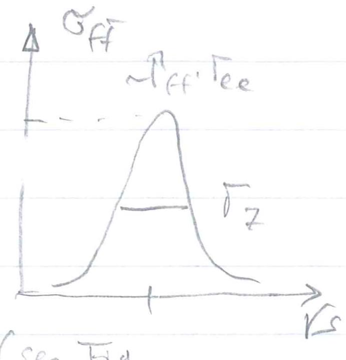
LEP studied the properties of the Z-Boson via resonant Z Boson production in e^+e^- annihilation:

$$e^+e^- \to Z \to f\bar{f}$$

$f\bar{f} = q\bar{q} \to$ Hadrons
$= ee, \mu\mu, \tau\tau$: Leptons
$= \nu_x\bar{\nu}_x$, $x = e, \mu, \tau$: invisible

total Z decay width: $\Gamma_Z = \Gamma_{had} + 3\Gamma_{Lep} + \Gamma_{inv}$

↳ number of light neutrinos $\left(m_\nu < \frac{m_Z}{2}\right)$:



(see Fig

$$N_\nu = \frac{\Gamma_{inv}^{meas}}{\Gamma_\nu^{SM}} = \left(\frac{\Gamma_{inv}}{\Gamma_{lep}}\right)_{meas} \cdot \left(\frac{\Gamma_{lep}}{\Gamma_\nu}\right)_{SM}$$

$$N_\nu = 2.984 \pm 0.008$$

i.e. perfect agreement with SM expectation 3
→ puts limit on $m_{\nu_4} > \frac{m_Z}{2}$!

Direct searches at LEP200 put constraints on heavy quarks & leptons:

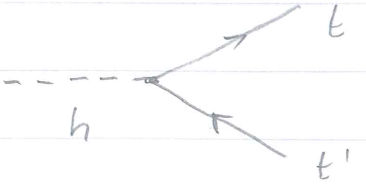
$$m_{b'}, m_{t'}, m_{e_4} > 100 \text{ GeV}$$

6.2 Direct searches at Tevatron and LHC

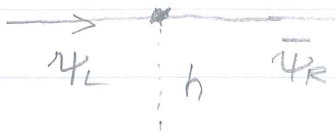
Tevatron & LHC $m_{t'}, m_{b'} \gtrsim 400$ GeV

experimental signatures
$b'\bar{b}' \to tWtW$
↳ $\underbrace{WbW \;\; WbW}_{\text{leptons + jets}}$
$t'\bar{t}' \to bWbW$ } b-jets + jets + lept

6.3 Constraints from Higgs production

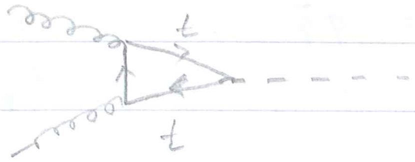


coupling to Higgs ~ $m_{t'}, m_{b'}, m_{\ell'} \dots$



~ $m_\psi \bar{\psi}_R \psi_L h$

Main Higgs production channel at LHC is Gluon-Fusion:



in SM main contribution from top

the presence of additional heavy quarks will increase the effective ggH coupling by roughly a factor 3 i.e. we would expect an enhanced Higgs production rate w.r. to the SM expectation and thus more detectable Higgs decays:

	$\sigma(gg \to H) \cdot BR(H \to \text{finals})$ / SM-expect.	
$gg \to H \to ZZ$	$\sim \times 5 \dots 8$	in contradiction with observation: no enhancement in $H \to ZZ$ and $H \to \tau\tau$ seen!
$gg \to H \to f\bar{f}$	$\sim \times 5$	
$gg \to H \to \gamma\gamma$	~ 1 (*)	

*) Remark: $H \to \gamma\gamma$

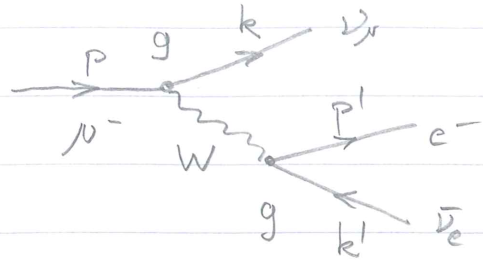
→ gives a reduction of $BR(H \to \gamma\gamma)$ for models with 4th generation and therefore compensates enhanced cross section

→ $\boxed{\text{Observed Higgs} \to ZZ, \tau\tau \text{ Rates ruled out the existence of a 4th generation}}$ 5.6σ

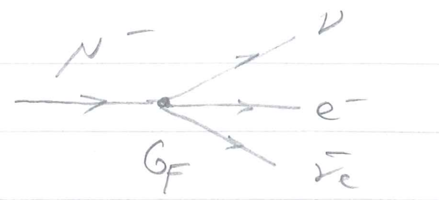
7. Weak decays:

7.1 Muon decay

effektive local IA



$m_\mu \ll m_W \Longrightarrow$



$$\mathcal{M} \sim g\, J^{\text{Muon},\alpha} \left(\frac{g_{\alpha\beta}}{M_W^2 - q^2} \right) \cdot g \cdot J^{\text{electr},\beta} \rightarrow \mathcal{M} \sim \frac{G_F}{\sqrt{2}} J^{\text{Muon}\,\alpha} J_\alpha^{\text{Electr}}$$

$$\text{with } \boxed{\frac{G}{\sqrt{2}} = \frac{g^2}{8 M_W^2}}$$

If one writes out the fermion currents one obtains:

$$\mathcal{M} = \frac{4 G_F}{\sqrt{2}} \left(\bar{\nu}_{\mu,L}\, \gamma^\alpha\, \mu_L \right) \left(\bar{e}_L\, \gamma_\alpha\, \nu_{e,L} \right)$$

After summing over all spin configs in the final state and averaging over the spins of the initial state one obtains

$$\overline{|\mathcal{M}|^2} = 64\, G_F^2 \cdot (k \cdot p')(k' \cdot p)$$

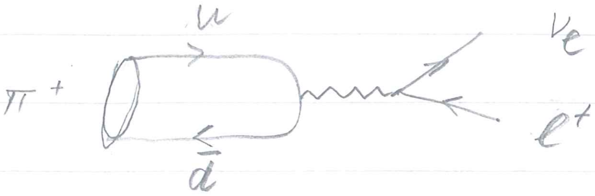
Taking phase space into account one finally obtains for the decay width Γ ($1/\tau$) of the muon:

$$\Gamma = \frac{1}{\tau} = \frac{G_F^2 \cdot m_\mu^5}{192\, \pi^3}$$

From the muon lifetime (2,2 μs) and it's mass one obtains $G_F = 1{,}16 \cdot 10^{-5}\,\text{GeV}^{-2}$

7.2 Pion decay

$\pi^+ \to \mu^+ \nu_\mu$
$\pi^+ \to e^+ \nu_e$



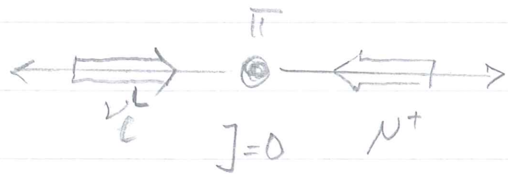
Text-book example of helicity suppression:

From phase space one would expect $BR(\pi^+ \to e^+\nu) \gg BR(\pi^+ \to \mu^+\nu)$
however one measures: $\frac{BR(\pi^+ \to e^+\nu)}{BR(\pi^+ \to \mu^+\nu)} \simeq (1.230 \pm 0.004) (\;)^{-4}$

(The Pion decays essentially only into μ, e decay suppressed !!)

Can be understood using angular momentum conservation

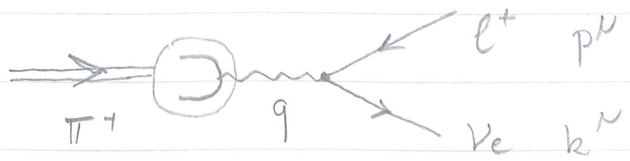
Spin $J(\pi) = 0$:



Due to angular momentum conservation and the LH ν
the μ^+ must have negative helicity (observable).
In the massless limit this is not possible.
However for massive leptons chirality flip is possible
and inserting a mass term would allow coupling to wrong chirality slight

Reminder: [Helicity $H = \frac{\vec{S}\cdot\vec{p}}{|\vec{p}|}$ $\rightarrow$ "$H = \pm\frac{1}{2}$"
Helicity of particle produced in weak IA: $-\frac{1}{2}\frac{v}{c}$]

Problem: Although quarks are the fundamental particles participating in the weak interaction it is the pion which is the asymptotic object.



Matrix element $\mathcal{M} \sim \frac{G_F}{\sqrt{2}} \cdot (\pi)_\mu \, (\text{Lepton})^\mu$

Pion current: With $J(\pi) = 0$ the only relevant 4-Vector is the momentum transfer $q^\mu = p^\mu + k^\mu$ to the lepton system

$$(\pi)_\mu = q_\mu \cdot f_\pi$$

Normalisation is called pion decay constant (see below).

Matrix element considering the chirality flip:

$$\mathcal{M}(\pi \to \mu\nu) = -\sqrt{2}\, G_F \cdot f_\pi \cdot V_{ud} \; m_\mu \cdot (\bar{\mu}_R \nu_L)$$

Including the phase space one obtains for $\Gamma = \frac{1}{\tau}$:

$$\Gamma(\pi^+ \to \ell^+ \nu_e) = \frac{G_F^2}{8\pi} \cdot |V_{ud}|^2 f_\pi^2 \cdot m_\ell^2 \cdot m_\pi \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2$$

$$\to \frac{\Gamma(\pi^+ \to e^+\nu)}{\Gamma(\pi^+ \to \mu^+\nu)} = \left(\frac{m_e^2}{m_\mu^2}\right) \cdot \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.275 \cdot 10^{-4}$$

A theoretical calculation including radiative corrections leads to $(1.2354 \pm 0.0002) 10^{-4}$ in perfect agreement with measurement.

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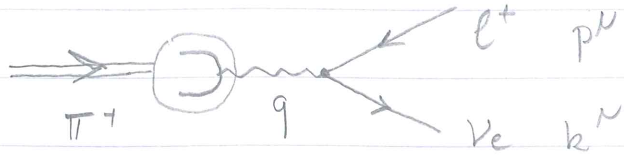

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Normalisation is called pion decay constant (see below).

Matrix element considering the chirality flip:

$$\mathcal{M}(\pi \to \mu\nu) = -\sqrt{2}\, G_F \cdot f_\pi \cdot V_{ud}\, m_\mu \cdot (\bar{\mu}_R \nu_L)$$

Including the phase space one obtains for $\Gamma = \frac{1}{\tau}$:

$$\Gamma(\pi^+ \to \ell^+ \nu_e) = \frac{G_F^2}{8\pi} \cdot |V_{ud}|^2 f_\pi^2 \cdot m_\ell^2 \cdot m_\pi \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2$$

$$\to \frac{\Gamma(\pi^+ \to e^+\nu)}{\Gamma(\pi^+ \to \mu^+\nu)} = \left(\frac{m_e^2}{m_\mu^2}\right) \cdot \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.275 \cdot 10^{-4}$$

A theoretical calculation including radiative corrections leads to $(1.2354 \pm 0.0002)\,10^{-4}$ in perfect agreement with measurement.