Flavor Physics – Exercise Sheet 7 – SomSem 2014

Discussion: 13/06 during the tutorial

Exercise 1: Meson oscillation

The effective Hamiltonian $\mathcal{H}$ describing the time-development of a system of neutral mesons $(P^0, \bar{P^0})$ can be written as

$$\mathcal{H} = \begin{pmatrix} H & H_{12} \\ H_{21} & H \end{pmatrix} = \begin{pmatrix} m - \frac{i}{2}\Gamma & m_{12} - \frac{i}{2}\Gamma_{12} \\ m_{12}^* - \frac{i}{2}\Gamma_{12}^* & m - \frac{i}{2}\Gamma \end{pmatrix}$$

The eigenvalues are given by solving the eigenvalue problem,

$$det\left(\mathcal{H} - \lambda \cdot \mathbf{1}\right) = \left(H - \lambda\right)^2 - H_{12}H_{21} = 0$$

a) Give the eigenvalues $\lambda_{H,L}$ as functions of m, Γ, m_{12} and Γ_{12}.

b) Show that

$$\begin{aligned} \frac{\Delta m}{2} &= \Re\sqrt{H_{12}H_{21}} \\ \frac{\Delta \Gamma}{2} &= -\Im\sqrt{H_{12}H_{21}} \\ \frac{\Delta \lambda}{2} &= \lambda_H - \lambda_L \\ &= \sqrt{\left|m_{12}\right|^2 - \tfrac{1}{4}\left|\Gamma_{12}\right|^2 - i\Re\left(m_{12}\Gamma_{12}^*\right)} \end{aligned}$$

c) Show that $\Delta m \cdot \Delta\Gamma = 4\Re\left(m_{12}\Gamma_{12}^*\right)$.

d) In the limit $\Gamma_{12} \ll m_{12}$ show that $\Delta m = 2\left|m_{12}\right|$.