Flavor Physics – Exercise Sheet 5 – SomSem 2014

Discussion: 30/05 during the tutorial

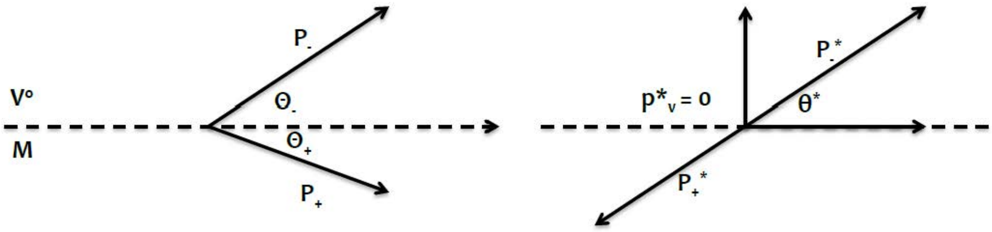
Exercise 1: Armenteros-Podolanski plot for V^0 decays

A neutral V^0-meson decays into a positive and a negative particle of mass m^+ and m^-. Assume that the decay angular distribution in the V^0 rest frame is isotropic. Let p_L^+ (p_L^-) be the positive (negative) particle's momentum component in the lab frame along the V^0 flight direction. The quantity α is defined as:

$$\alpha = \frac{p_L^+ - p_L^-}{p_L^+ + p_L^-}$$

Show that with the transverse momentum component $p_T = p_T^+ = p_T^-$ of the decay particles with respect to V^0 flight direction, the points (α, p_T) describe an ellipse. Discuss how this could be used to separate $\Lambda \to p\pi^-$ decays from $K^0 \to \pi^+\pi^-$ decays.

Hint: Use the longitudinal momenta $p_L^{\pm,*}$ in the V^0 rest frame and perform a boost into the lab frame to calculate $p_L^\pm$. Express α as a function of θ^*, the decay angle in the V^0 rest frame. With the equation $\sin^2\theta^* + \cos^2\theta^* = 1$ one can derive the ellipse equation for α and p_T.



Exercise 2: θ/τ puzzle

Please read the attached article on the spin and the parity of the τ^+($i.e.K^+$) decaying into 3 pions (taken from Cahn and Goldhaber: Experimental Foundations of Particle Physics, *http://archive.org/details/CahnGoldhaberExperimentalFoundationsOfParticlePhysics*). What are the main arguments for the τ^+ being a $J^P = 0^-$ state. For further information you can also consult the original article by R.H Dalitz (Phys. Rev. 94 (1954) 1046; accessible online from inside the university network; use VPN from outside).

Figure 3.1. A κ (K) meson stops at P, decaying into a muon and neutrals. The muon decays at Q to an electron and neutrals. The muon track is shown in two long sections. Note the lighter ionization produced by the electron, contrasted with the heavy ionization produced by the muon near the end of its range. The mass of the κ was measured by scattering and grain density to be 562 ± 70 MeV (Ref. 3.4).

theta mesons had just about the same mass and set the stage for the famous puzzle about the parities of these particles.

The year 1953 marked a turning point in the investigation of the new V-particles. The great achievements of cosmic-ray physics in exploring the new particles was summarized

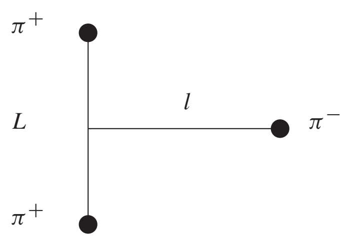


Figure 3.2. Diagram for the angular momentum in τ^+ meson decay. The $\pi^+\pi^+$ angular momentum, L, must be even. The orbital angular momentum, l, of the π^- must be added to L to obtain the total angular momentum (that is, the spin) of the tau.

in a meeting at Bagnères-de-Bigorre in France. The V_1^0 was well established, as was the tau. There were indications of both positive and negative hyperons (particles heavier than a proton). The negative hyperon was observed in a cascade that produced a neutral hyperon that itself decayed (Refs. 3.6, 3.7) There was a κ, which decayed into a muon plus neutrals, and a χ, which decayed into a charged pion plus neutrals. The $\theta \to \pi^+\pi^-$ was established, too.

At the Bagnères Conference, Richard Dalitz presented his analysis of the tau that was designed to determine its spin and parity through its decay into three pions. Some immediate observations about the spin and parity of the tau are possible. If there is no orbital angular momentum in the decay, the spin is zero and the parity is $(-1)^3$ because the parity of each pion is -1, and thus $J^P = 0^-$. The system of $\pi^+\pi^+$ can have only even angular momentum because of Bose statistics. Dalitz indicated this angular momentum by L and the orbital angular momentum of the system consisting of the π^- and the $(\pi^+\pi^+)$ by l. See Figure 3.2. Then the total angular momentum, J, was the vector sum of L and l. If $L = 0$, then $J = l$, and $P = (-1)^{J+1}$. For $L = 2$, other combinations were possible. Dalitz noted that since the sum of the pion energies was a constant, $E_1 + E_2 + E_3 = Q$, each event could be specified by two energies and indicated on a two-dimensional plot. (Here we are using kinetic energies, that is relativistic energies less rest masses.) If E_1 corresponds to the more energetic π^+ and E_2 to the less energetic π^+, all the points fall on one half of the plot. See Figure 3.3. If the decay involves no angular momentum and there are no effects from interactions between the produced pions, the points will be evenly distributed on the plot. Deviations from such a distribution give indications of the spin and parity. For example, as $E_3 \to 0$, the π^- is at rest and thus has no angular momentum. Thus $l = 0$, $J = L$ and $P = (-1)^{J+1}$. Hence if the tau is *not* in the sequence $0^-, 2^-, 4^-, \ldots$ there should be a depletion of events near $E_3 = 0$. As data accumulated in 1953 and 1954, it became apparent that there was no such depletion and thus it was established that τ^+ had J^P in the series $0^-, 2^-, \ldots$

The decay distribution for a two-body decay is given by Fermi's Golden Rule (which is actually due to Dirac) in relativistic form:

$$d\Gamma = \frac{1}{32\pi^2}|\mathcal{M}|^2 \frac{p_{cm}\,d\Omega}{M^2}. \tag{3.1}$$

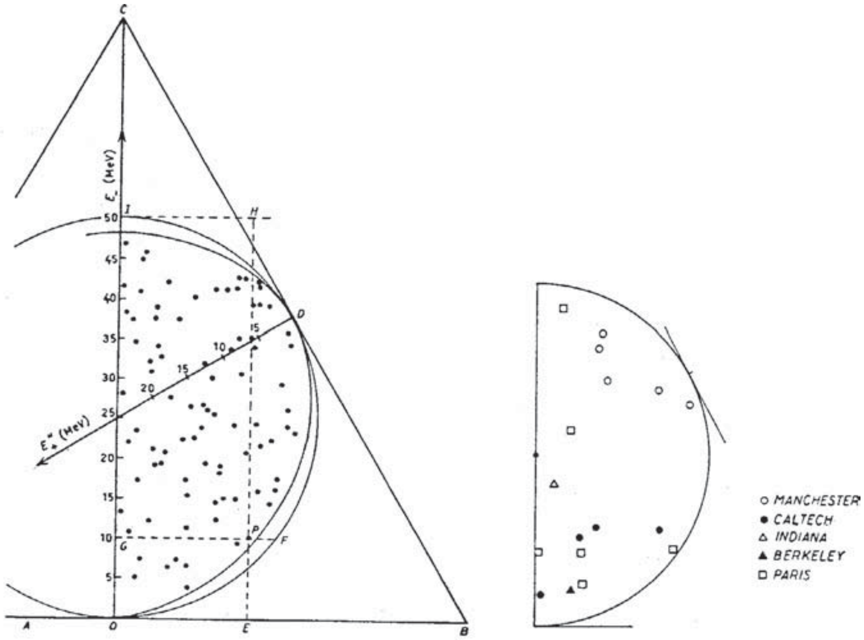


Figure 3.3. Dalitz plots showing worldwide compilations of tau meson decays ($\tau^+ \to \pi^+\pi^+\pi^-$) as reported by E. Amaldi at the Pisa Conference in June 1955 [*Nuovo Cimento Sup.* IV, 206 (1956)]. On the left, data taken in emulsions. On the right, data from cloud chambers. There is no noticeable depletion of events near $E_3 = 0$, *i.e.* near the bottom center of the plot. Parity conservation would thus require the tau to have $J^P = 0^-, 2^-$.

Here $d\Gamma$ is the decay rate, p_{cm} is the center-of-mass momentum of either final state particle, M is the mass of the decaying particle and $d\Omega$ is the solid angle element into which one final state particle passes. $\mathcal{M}$ is the Lorentz-invariant amplitude for the process. The amplitude $\mathcal{M}$ will involve the momenta of the various particles and factors to represent the spins of the particles.

For three-body decays there are more final state variables. If the particles are spinless or if polarization is ignored, however, there are only two variables necessary to specify the final state. They may be chosen to be the energies of the final state particles. The Golden Rule then takes the form

$$d\Gamma = \frac{1}{64\pi^3 M}|\mathcal{M}|^2 dE_1 dE_2. \tag{3.2}$$

Thus if $\mathcal{M}$ is constant, $d\Gamma \sim dE_1 dE_2$ and the events fall evenly on the Dalitz plot.

By examining the Dalitz plot, inferences can be drawn about spin and parity. Consider the $\tau \to 3\pi$. If the tau is spinless and the values of L and l are zero, $\mathcal{M}$ should be nearly constant. (Actually, it need not be absolutely constant. It may still depend on the Lorentz-invariant products of the momenta in the problem.) Suppose, on the contrary, tau has spin 1. Then it will be represented by a polarization vector, ϵ. The amplitude must be linear in ϵ. If we treat the pions as nonrelativistic, it suffices to consider just three-momenta rather than four-momenta. The amplitude, in order to be rotationally invariant, must be the dot product of ϵ with a vector made from the various pion momenta. In addition, because of Bose statistics, the amplitude must be invariant under interchange of the two

π^+'s, particles 1 and 2. Two examples are

$$\boldsymbol{\epsilon} \cdot \mathbf{p}_3$$
$$\boldsymbol{\epsilon} \cdot (\mathbf{p}_1 - \mathbf{p}_2) \times \mathbf{p}_3 \; (\mathbf{p}_1 - \mathbf{p}_2) \cdot \mathbf{p}_3$$

Both represent spin-1 decays. The parity of the decaying object, assuming parity is conserved in the decay, is determined by examining the behavior of the quantity dotted into $\boldsymbol{\epsilon}$. In the first case, the single momentum contributes (-1) to the parity since the momenta are reversed by the operation. In addition, the intrinsic parities of the three pions contribute $(-1)^3$. Altogether, the parity is even, so the state is $J^P = 1^+$. In the second instance, there are four factors of momentum and the parity is finally odd. In both cases, the amplitude vanishes as p_3 goes to zero in accordance with the earlier argument.

Dalitz's analysis led ultimately to the τ–θ puzzle: were the θ^+ (which decayed into $\pi^+\pi^0$) and τ^+, whose masses and lifetimes were known to be similar, the same particle? Of course this would require them to have the same spin and parity. But the parity of the $\theta^+ \rightarrow \pi^+\pi^0$ was necessarily $(-1)^J$ if its spin was J. These values were incompatible with the results for the tau showing that it had J^P in the sequence $0^-, 2^-, \ldots$. How this contradiction was resolved will be seen in Chapter 6.

Cosmic-ray studies had found evidence for hyperons besides the $\Lambda = V_1^0$. Positive particles of a similar mass were observed and initially termed V_1^+ or Λ^+. Evidence for this particle, now called the Σ^+, was observed by Bonetti *et al.* (Ref. 3.8) in photographic emulsion, and by York *et al.* (Ref. 3.9) in a cloud chamber. See Figure 3.4. Furthermore, a hyperfragment, which is a Λ or Σ^+ bound in a nucleus, was observed by Danysz and Pniewski in photographic emulsion (Ref. 3.10). See Figure 3.5. Working at Caltech, E. W. Cowan confirmed the existence of a negative hyperon (now called the Ξ^-) that itself decayed into $\Lambda^0\pi^-$ (Ref. 3.11).

By the end of the year 1953, the Cosmotron at Brookhaven National Laboratory was providing pion beams that quickly confirmed the cosmic-ray results and extended them. The existence of the V_1^+ (Σ^+) was verified and the V_1^- (Σ^-) was discovered. An especially important result was the observation of four events in which a pair of unstable particles was observed (**Ref. 3.12**). Such events were expected on the basis of theories that Abraham Pais and Murray Gell-Mann developed to explain a fundamental problem posed by the unstable particles. These unstable particles were clearly produced with a large cross section, some percent of the cross section for producing ordinary particles, pions and nucleons. The puzzle was this: The new particles were produced in strong interactions and decayed into strongly interacting particles, but if the decays involved strong interactions, the particle lifetimes should have been ten orders of magnitude less than those observed.

The first step in the resolution was made by Pais, who suggested that the new particles could only be made in pairs. One could assign a multiplicative quantum number, a sort of parity, to each particle, with the pion and nucleon carrying a value +1 and the new particles, K, Λ, etc. carrying -1. The product of these numbers was required to be the same in the initial and final state. Thus $\pi^- p \rightarrow K^0\Lambda$ would be allowed, but $\pi^- p \rightarrow K^0 n$ would be forbidden. The Cosmotron result on the production of pairs of unstable particles was