1.2 Neutrino masses in the SM and Majorana mass terms

The observation of flavor oscillation of neutrinos:

$$\nu_e \leftrightarrow \nu_\mu$$
$$\searrow \; \nu_\tau \; \nearrow$$

implies the existence of mass states which are different from flavor states and thus massive neutrinos.

In the SM neutrino masses are set to zero because of the missing RH ν-singlet. The observation of non-vanishing neutrino masses thus indicates physics beyond the SM.

Remark: Nowadays massive neutrinos are often treated as „part of the SM" assuming the existence of RH ν. In a certain sense the additional new particle is not modifying the gauge structure of the theory which is what often is called SM

a) Dirac mass terms

Neutrino masses can be created in the SM by extending the particle content and by adding ν_R, i.e. RH ν-singlets.

$$-\mathcal{L}^{\text{Neutrino}}_{\text{Yukawa}} = Y^{\nu}_{ij}\, \underbrace{\bar{L}_{Li}}_{\begin{pmatrix}\nu_L \\ l_L\end{pmatrix}_i}\, \tilde{\Phi}\, \nu_{Rj} + h.c.$$

resulting in a Dirac mass term $m\bar{\nu}\nu$:

$$\mathcal{L}_{mass} = m_\nu\,(\bar{\nu}\nu) = m_\nu\,(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L)$$

The mass term mixes LH $\leftrightarrow$ RH neutrino states

$$\rightarrow \frac{v}{\sqrt{2}} g_\nu \bar{\nu}_L \nu_R \equiv M_\nu \bar{\nu}_L \nu_R$$
$w/\ g_\nu = 10^{-12}$. why so tiny

The smallness of the neutrino masses are explained by very small Yukawa-coupling. However it is not clear why compared to the quark sector the difference between ν masses and charged lepton masses are so large. The RH neutrinos have Hypercharge $Y=0$ and $I, I_3=0$ $\rightarrow$ they would not interact with anything $\Rightarrow$ Sterile neutrinos.

5) Majorana masses

We need charge-conjugation C [in fact we need particle – antiparticle transformation]:

Dirac Eq for e^- $\quad [\gamma_\mu(i\partial_\mu + eA_\mu) - m]\psi = 0$

invented for e^+ $\quad [\gamma_\mu(i\partial_\mu - eA_\mu) - m]\psi^c = 0$

One finds (see exercise) for ψ^c and the C-operator:

$$\boxed{\psi^c = i\gamma_2\psi^* = i\gamma_2\gamma_0\bar{\psi}^T \equiv C\bar{\psi}^T}$$

The C-operator flips all charge-like quantum numbers.

$$\left.\begin{aligned} C^+ = C^T = C^{-1} &= -C \\ C\gamma_\mu C^{-1} &= -\gamma_\mu^T \\ C\gamma_5 C^{-1} &= \gamma_5^T \\ C\gamma_\mu\gamma_5 C^{-1} &= (\gamma_\mu\gamma_5)^T \end{aligned}\right\} \Rightarrow \begin{aligned} (\psi^c)^c &= \psi \\ \overline{\psi^c} &= \psi^T C \\ \bar{\psi}_1\psi_2^c &= \bar{\psi}_2^c\psi_1 \end{aligned}$$

One finds:

$\bar{\psi} = \psi^\dagger\gamma^0$

and further: $$(\psi_{L,R})^c = \left(\frac{1\pm\gamma^5}{2}\psi\right)^c = \frac{1\mp\gamma^5}{2}\psi^c = (\psi^c)_{R,L}$$

$\Rightarrow$ C operator flips chirality!

also $(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$

Mass terms require LH and RH terms: $m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L$

2 principle possibilities:

(i) ψ_R and ψ_L are independent: Dirac particle

(ii) $\psi_R = (\psi_L)^c$: Majorana particle

$\hookrightarrow \psi^c = (\psi_L + \psi_R)^c = (\psi_L)^c + (\psi_R)^c = \psi_R + \psi_L = \psi$

$\boxed{\psi^c = \psi}$ particle and antiparticle are identical

(only works for neutral fermions)

Contrary to a Dirac spinor (4 degrees of freedom $\begin{pmatrix}\phi\\ \xi\end{pmatrix}$ with ϕ, ξ = Weyl spinor) the Majorana Spinor has only 2 degrees of freedom:

$$\psi = \begin{pmatrix}\phi \\ -i\sigma_2\phi^*\end{pmatrix}$$

One can now write down a Mass term for Majorana neutrinos:

$$\mathcal{L}_M = \frac{1}{2}m_L\bar{\nu}\nu = \frac{1}{2}m_L(\bar{\nu}_L + \bar{\nu}_L^c)(\nu_L + \nu_L^c)$$
$$= \frac{m_L}{2}\left(\bar{\nu}_L\nu_L^c + \bar{\nu}_L^c\nu_L\right) = \frac{m_L}{2}\bar{\nu}_L\nu_L^c + h.c.$$

However the mass terms couple particle + anti-particle $\Rightarrow$
Lepton-Flavor-Violation: $\Delta L = 2$

Additional problem: With $\nu_L = \begin{cases} I_3 = \frac{1}{2} \\ Y = -1\end{cases}$ $\bar{\nu}_L^c\nu_L = \begin{cases} I_3 = 1 \\ Y = -2\end{cases}$

To generate such a mass term via a Higgs-coupling the Standard Higgs is not sufficient. We need a Higgs Triplett with $Y=2$, $I_3=1$ $\Rightarrow$ not possible in SM

$\Rightarrow$ We are forced to consider existance of an additional RH neutrino field even in case of the Majorana mass terms: ν_R-singlett $\begin{cases} I_3 = 0 \\ Y = 0\end{cases}$

Neutrino Masses (Dirac or Majorana) requires BSM Physics $\rightarrow$

BSM physics: ν_R or Higgs-Triplett or new mass generation.

c) Most general mass terms and seesaw model

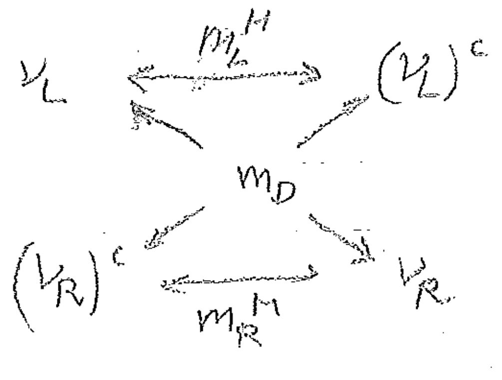
It is instructive to consider the simplest case of one flavor and two neutrino fields ν_L and $(\nu_R)^c$.
Most general mass term considers Dirac and Majorana terms:

$$\mathcal{L}^{D+M} = -\frac{1}{2} m_L \bar{\nu}_L (\nu_L)^c - \frac{1}{2} m_D \left(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L \right) - \frac{1}{2} m_R \overline{(\nu_R)^c} \nu_R + h.c.$$

which can be rewritten if one uses:

$$n_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} = \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} \quad \text{with } (n_L)^c = \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix} = \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix} = n_R^c$$

$$\Rightarrow \quad \mathcal{L}^{D+M} = -\frac{1}{2} \bar{n}_L \cdot \underbrace{\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}}_{M^{D+M}} (n_L)^c + h.c.$$



Coupling scheme for neutrinos via masses

The helicity fields ν_L and $(\nu_R)^c = \nu_L^c$ are not the mass eigenstates – these are found by diagonalizing the mass matrix M^{D+M} $\Rightarrow$ can be easily diagonalized using the orthogonal matrix $\mathcal{O}$

$$\mathcal{O} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \qquad M^{D+M} = \mathcal{O} M' \mathcal{O}^T$$

$$M' = \mathrm{diag}(m_1', m_2')$$

with $\tan 2\theta = \dfrac{2 m_D}{m_R - m_L}$

and $m'_{1,2} = \frac{1}{2}(m_R + m_L) \mp \frac{1}{2}\sqrt{(m_R - m_L)^2 + 4 m_D^2}$

where $m'_{1,2}$ can be positive and negative
$\rightarrow$ rewrite $m'_i = |m'_i| \cdot \eta_i = m_i \cdot \eta_i$ w/ $\eta_i = \pm 1$

Taking this into account one can express the diagonalization of M^{D+M} as:

$$(*) \quad M^{D+M} = \mathcal{O}\, M \cdot \eta\, \mathcal{O}^T = U\, M \cdot U^T$$

with $M = \mathrm{diag}(m_1, m_2)$
and $U = \mathcal{O} \sqrt{\eta}$ = unitary matrix.

For the neutrino mass eigenstates one finds from $(*)$:

$$(**) \quad \nu^M = U^+ n_L + (U^+ n_L)^c = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\rightarrow \mathcal{L}^{D+M} = -\frac{1}{2}\, \bar{\nu}^M \cdot M \cdot \nu^M = -\frac{1}{2} \sum_{i=1,2} m_i \bar{\nu}_i \nu_i$$

Evidently $(\nu_i)^c = \nu_i \Rightarrow$ Mass eigenstates are Majorana ν.

from $(**)$ one obtains the following mixing eq.:

$$\nu_L = \cos\theta \sqrt{\eta_1} \cdot \nu_{1L} + \sin\theta \sqrt{\eta_2} \cdot \nu_{2L}$$
$$(\nu_R)^c = -\sin\theta \sqrt{\eta_1} \cdot \nu_{1L} + \cos\theta \sqrt{\eta_2} \cdot \nu_{2L}$$

The parameters η_i determine the CP parity of the Majorana ν_i

d) Seesaw (simplest case for 1 family)

The seesaw mechanism was proposed at the end of the 1970ies and is based on the Dirac and Majorana mass terms. It is a natural and viable mechanism to generate neutrino masses.

The three parameters m_L, m_D and m_R characterize left-handed Majorana, Dirac and RH-Majorana mass terms. The mass eigenstates characterized by m_1, m_2 are Majorana states (see above).

Assumptions:

1) no LH Majorana mass term $m_L = 0$
2) Dirac mass term generated by SM Higgs-coupling
   $\rightarrow$ m_D is of the order of a lepton or quark mass.
3) RH Majorana mass term $\neq 0$ breaks Lepton number conservation. We assume that this happens at a mass scale much larger than the e.w. scale.

$$\rightarrow \quad m_R \doteq M_R \gg m_D$$
$$\gg M_1, M_2$$

One obtains for the mass eigenvalues

$$m_1 \simeq \frac{m_D^2}{m_R} = \frac{m_D^2}{M_R} \ll m_D$$

$$m_2 \simeq M_R \gg m_D$$

Mixing angle $\theta \simeq \frac{m_D}{M_R} \ll 1$ and w/ $\eta_1 = -1$ $\eta_2 = +1$:

→ Mixing relations: $\nu_L \simeq i\nu_{1L} + \frac{m_D}{M_R}\cdot \nu_{2L}$

$$(\nu_R)^c = -i\frac{m_D}{M_R}\cdot \nu_{1L} + \nu_{2L}$$

Expressed as corresponding Majorana fields:

$\nu_1 \simeq \nu_L - (\nu_R)^c$: LH component w/ low mass → active

$\nu_2 \simeq (\nu_L)^c + \nu_R$ RH component w/ very high mass → sterile

Estimates of M_R:

$m_D \overset{?}{\simeq} m_t \simeq 170\,\text{GeV}$

$m_1 \simeq \sqrt{\Delta M^2}\big|_{\text{heaviest neutrino}} \simeq 5\cdot 10^{-2}\,\text{eV}$

$\Big\}\ \underline{M_R \simeq \frac{m_D^2}{m_1} \simeq 10^{15}\,\text{GeV}}$

If Seesaw is realized in nature:

- neutrinos are Majorana particles
- neutrinos masses are much smaller than lepton + quark masses
- heavy Majorana particles – seesaw partner – must exist

$\bar{\nu}_R$ =

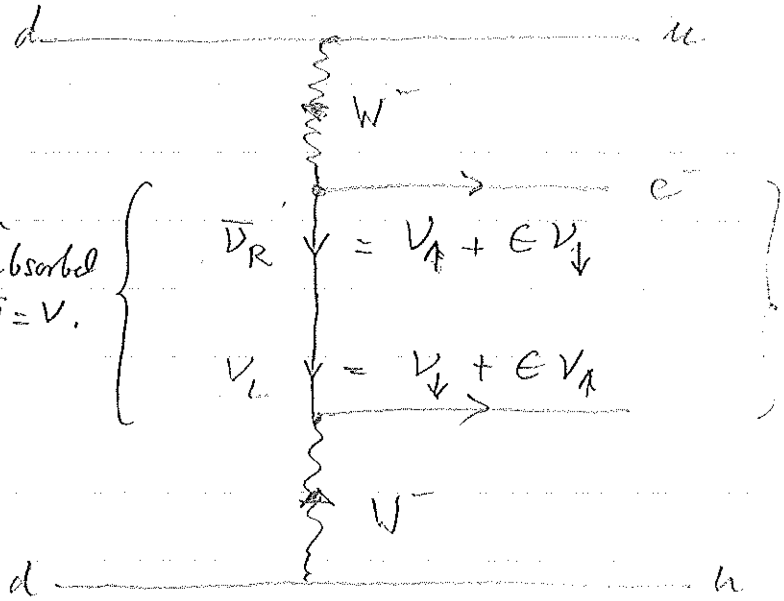
Remark / Question

If $\nu = \bar{\nu}$ why does the reaction $\bar{\nu} + n \to e^- + p$ not exist?
A: Only the RH component can interact → helicity suppressed!!

Experimental test of Majorana character ($\Delta L = 2$):

Neutrinoless double Beta decay : $(A, Z) \rightarrow (A, Z+2) + 2e^-$
e.g. $^{76}Ge \rightarrow {}^{76}Se + 2e^-$, $T_{1/2} \geq 2 \times 10^{25}$ yrs $(0\nu\beta\beta)$

can be absorbed if $\bar{\nu} = \nu$.



one needs to consider angular momentum conservation and thus the ν-helicity
$\nu_D = (\nu_\uparrow, \nu_\downarrow, \bar{\nu}_\uparrow, \bar{\nu}_\downarrow)$ 4 dof
$\nu_M = (\nu_\uparrow, \nu_\downarrow)$ 2 dof

For a $0\nu\beta$ - decay: (1) $\bar{\nu} = \nu$ i.e. neutrinos must be Majorana
(2) $\epsilon \neq 0$

What is ϵ? Remember: $u_\downarrow(p) = u_L(p)\,|p| + \frac{m}{2E} u_R(p)$

Helicity = Chirality for $m = 0$, i.e.
for vanishing neutrino masses $\nu_R = \nu_\uparrow$
$\rightarrow$ no $0\nu 2\beta$! $\nu_L = \nu_\downarrow$
ϵ - Parameter $\sim \mathcal{O}\left(\frac{m}{2E}\right)$ which means that probability to observe Majorana nature is always suppressed by $\left(\frac{m}{2E}\right)^2$.

If the flavor state ν_e is a mixture of the mass state ν_i described by U_{ei} : $\nu_e = U_{ei}\nu_i$ one obtains
$$\Gamma(0\nu 2\beta) \sim \left|\frac{\sum U_{ei}^2 m_i}{E^2}\right|^2 \text{ with } \sum U_{ei}^2 m_i = \langle m \rangle$$

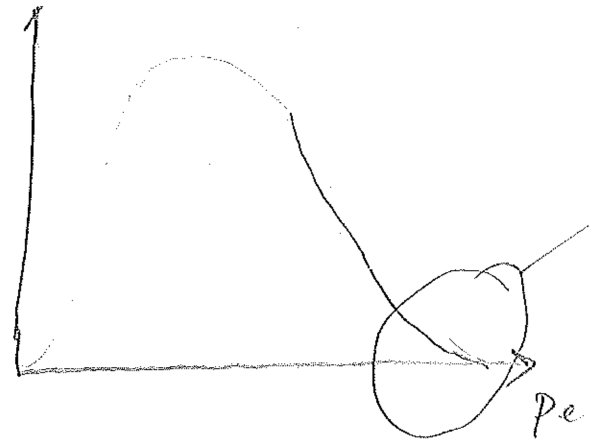
Experiments: (1) Heidelberg Moskau
(s. Slides) (2) Gerda

Neutrino Mass Measurement:

Measure e^--Spectrum in nuclear β-decays:

$n \to p + e^- + \bar{\nu}_e$

${}^3H \to {}^3He + e^- + \nu$



Study the endpoint (due to energy conservation endpoint depends on m_ν)

Experiments: - Mainz-Exp

- KATRIN