

4 B - Mesons

	$\tau = 1/\Gamma$	ΔM	$x = \frac{\Delta M}{\Gamma}$	$y = \frac{\Delta\Gamma}{2\Gamma}$
K^0 -System	$0.26 \cdot 10^{-9} \text{ s}$	5.29 ns^{-1}	0.477	-1
D^0 -System	$0.4 \cdot 10^{-12} \text{ s}$	0.0024 ps^{-1}	0.0097	0.0078
B^0 -System	$1.5 \cdot 10^{-12} \text{ s}$	0.507 ps^{-1}	0.78	0.0015
B_s System	$1.5 \cdot 10^{-12} \text{ s}$	17.7 ps^{-1}	26.8	0.07

Remember the Wolfenstein parametrization of V_{CKM} :

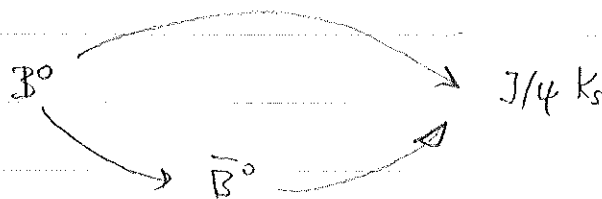
$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \cdot e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| \cdot e^{-i\beta} & -|V_{ts}| \cdot e^{i\beta_c} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

→ useful to evaluate the expected CP asym. in B-decays

It turns out that the phases β, β_s, γ as well as $V_{ub}, V_{cb}, V_{td}, V_{ts}$ can be measured in B-decays \Rightarrow Test of UT

4.1 Measurement of CKM phase β in $B^0 \rightarrow J/\psi K_S^0$ decays

The channel $B^0 \rightarrow J/\psi K_S$ allows an interference of the decay w/ and w/o mixing:



With the Master Eq. from Chap. 34E one obtains for small $\Delta\Gamma$ and for negligible (no) direct CPV (i.e. $|A_f| = |\bar{A}_f|$) no CPV in mixing:

$$A_{CP}(t) = \frac{\Gamma(B^0 \rightarrow f)(t) - \Gamma(\bar{B}^0 \rightarrow f)(t)}{\Gamma(B^0 \rightarrow f)(t) + \Gamma(\bar{B}^0 \rightarrow f)(t)} = -\text{Im}(\lambda_f) \sin(\Delta M t)$$

For the final-state $f = f_{CP}$ with $CP|f_{CP}\rangle = \eta_{CP}|f_{CP}\rangle$ and $\eta_{CP} = \pm 1$

one obtains for λ_{fcp} :

$$\lambda_{fcp} \equiv \frac{q}{p} \cdot \frac{\bar{A}_{fcp}}{A_{fcp}} = \frac{q}{p} \cdot \eta_{cp} \cdot \frac{\bar{A}_{fcp}}{A_{fcp}}$$

where we have rewritten λ_{fcp} as a function of \bar{A}_{fcp} .
 \bar{A}_{fcp} and A_{fcp} are related by CP conjugation and differ only in the signs of the weak phases $\rightarrow \bar{A}_{fcp}$ can be calculated from A_{fcp} by inverting the phases!

For the final state $J/\psi K_S$: $\eta_{cp} = -1$

Moreover for this final state one has to consider the following amplitudes:

$$A(B^0 \rightarrow J/\psi K_S) = A(B^0 \rightarrow J/\psi K^0) \times A(K^0 \leftrightarrow K_S)$$

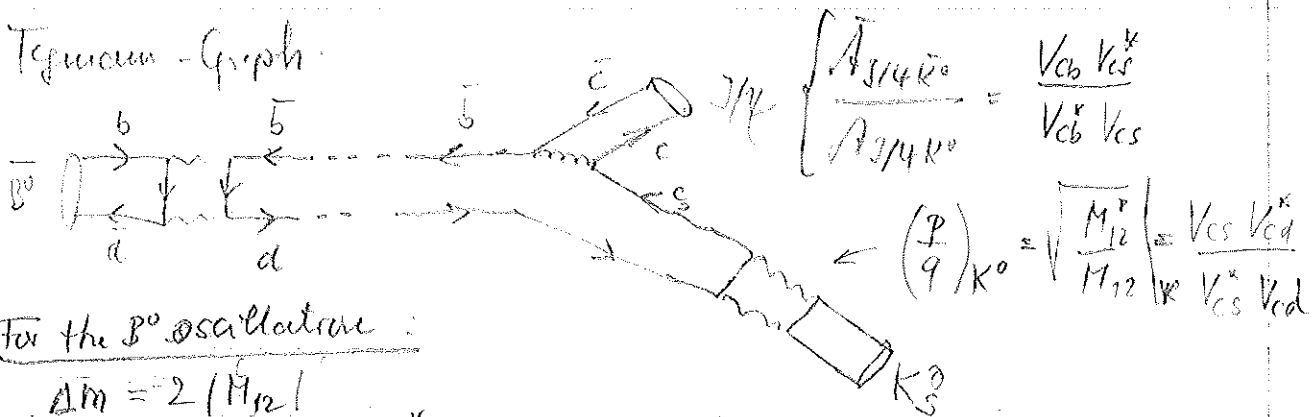
$$A(\bar{B}^0 \rightarrow J/\psi K_S) = A(\bar{B}^0 \rightarrow J/\psi \bar{K}^0) \times A(\bar{K}^0 \leftrightarrow K_S)$$

$$\text{Remember: } |K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$$

Considering the K^0 mixing one obtains:

$$\lambda_{J/\psi K_S} = \left(\frac{q}{p}\right)_{B^0} \cdot \eta_{cp} \frac{\bar{A}_{J/\psi K_S}}{A_{J/\psi K_S}} = \left(\frac{q}{p}\right)_{B^0} \eta_{cp} \frac{\bar{A}_{J/\psi \bar{K}^0}}{A_{J/\psi K^0}} \cdot \left(\frac{p}{q}\right)_{K^0}$$

Triangle Graph:



For the B^0 oscillation:

$$\Delta m = 2 |M_{12}|$$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^*}{M_{12}}} = \frac{V_{cb}^* V_{td}}{V_{cb} V_{td}^*} = e^{-i2\beta}$$

^a pure phase,
 $|(q/p)| \approx 1$

Putting everything together one obtains for $\lambda_{J/\psi K_S}$:

$$\begin{aligned}\lambda_{J/\psi K_S} &= - \left(\frac{V_{cb}^* V_{cd}}{V_{cb} V_{cd}^*} \right) \cdot \left(\frac{V_{cs} V_{cs}^*}{V_{cb}^* V_{cs}} \right) \left(\frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \right) \\ &= - \left(\frac{V_{cb}^* V_{cd} V_{cb} V_{cd}^*}{V_{cb} V_{cd}^* V_{cb}^* V_{cd}} \right)\end{aligned}$$

$$\begin{aligned}\text{Im}(\lambda_{J/\psi K_S}) &= -\sin\left(\arg\left(\frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}}\right)\right) = -\sin\left(2 \arg\left(\frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}}\right)\right) \\ &= -\sin 2\beta\end{aligned}$$

$$\rightarrow A_{CP}(t) = \underbrace{-\sin(2\beta)}_{2 \times \text{phase of } V_{td}} \sin(\Delta m t)$$

One often shortcuts the argumentation by saying that the $B^0 \rightarrow \bar{B}^0$ mixing leads to the factor $(q/p)_{B^0} = e^{-2i\beta}$ and all other amplitudes are essentially real!

For different final states e.g. $B^0 \rightarrow J/\psi K_L$ one has to consider the different CP eigenvalues. In general one therefore often writes:

$$A_{CP}(t) = \eta_{CP} \cdot \sin(2\beta) \sin(\Delta m t)$$

The measurement of 2β was performed by BABAR and Belle, both experiments at a high-luminosity e^+e^- B-factory:

e^+e^- B-factories operate at the center-of-mass energy of $(13.0, 5.6)$ GeV the $\Upsilon(4S)$ resonance which decays nearly entirely to $B^0\bar{B}^0$ or $B^+\bar{B}^-$: $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^0\bar{B}^0, B^+\bar{B}^-$

Measurement of $\sin(2\beta)$ was the first observation of CPV outside K^0 -system \rightarrow correctly predicted by SM.

Experimental problems of the measurement:

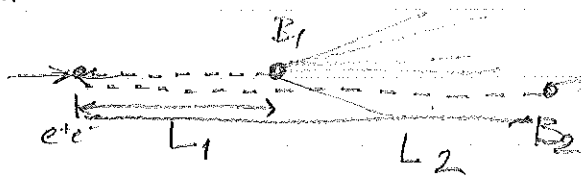
$$1) M_{\psi(4S)} = 10.56 \text{ GeV}/c^2 \approx 2 m_B$$

→ If the $\psi(4S)$ is produced at rest the 2 B 's have very little momentum. A lifetime measurement which is based on the measurement of the flight length is not possible (B 's don't fly!).

To overcome this problem the $\psi(4S)$ is instead produced with a boost:

$$\xrightarrow{3.1 \text{ GeV}} e^+ + e^- \xleftarrow{9 \text{ GeV}} \Rightarrow \beta_{\psi} = 0.56$$

Decay kinematics:



$$\left. \begin{array}{l} \text{typical } \beta_{\psi} c \tau \approx 250 \mu\text{m} \\ w/ \tau \approx 1.5 \text{ ps} \end{array} \right\}$$

2) Flavor-tagging:

To measure $A_{CP}(t)$ one has to know whether a $B^0 \rightarrow J/\psi K_s$ was produced originally as B^0 or as \bar{B}^0 .

In general one uses the "2nd B " in the event which always contains two B 's w/ opposite flavor:

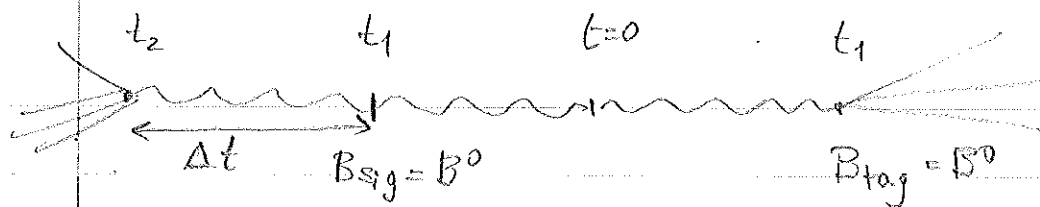
$$B_{\text{tag}} = B/b \text{ or } \bar{B}/\bar{b} \Rightarrow B_{\text{signal}} = B^0/b \text{ or } \bar{B}^0/\bar{b}$$

However for $B_{\text{tag}} = B^0$ one has the problem of oscillation, i.e. when decaying the B_{tag} might have changed its flavor!

→ wrong tag!!

At $\psi(4S)$ the problem can be solved:

Both B -mesons are produced coherently and are entangled: one does not know which one is B^0/\bar{B}^0 .



If one of the two decays in a given state it defines
at that time also the flavor of the other one!
(opposite)

↳ After the decay of the tagging B (e.g. $B_{tag} = B^0$) the
clock of the signal B ($B_{sig} = \bar{B}^0$ at t_2) starts to
tick ($\Delta t = t - t_2$).

Instead of measuring A_{CP} as function of the
absolute time t , it is measured as function of
 Δt (time after the tagging B has decayed!):

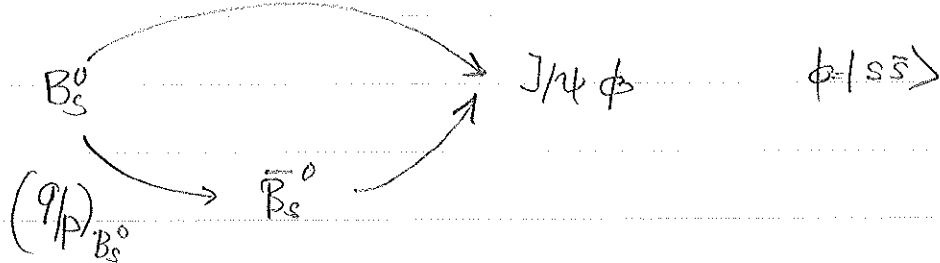
$$\begin{aligned} A_{CP}(\Delta t) &= \frac{\Gamma(B^0(\Delta t) \rightarrow f) - \Gamma(\bar{B}^0(\Delta t) \rightarrow f)}{\Gamma + \Gamma} \\ &= \frac{\Gamma(B_{tag} = \bar{B}^0, B \rightarrow f) - \Gamma(B_{tag} = B^0, B \rightarrow f)}{\Gamma + \Gamma}(\Delta t) \\ &= -\sin(2\beta) \sin(\Delta m t) \end{aligned}$$

Of course the signal B can also decay before the
tagging B . In this case $\Delta t < 0$.

BABAR plots for $\sin(2\beta)$ in $J/\psi K_S$ and $J/\psi K_L$!

2. Measurement of β_S in $B_S \rightarrow J/\psi \phi$

Changing spectator quark $d \rightarrow s$ one obtains the B_S counterparts:



Differences
w/r to $J/\psi K_S$

$$1) \quad \left(\frac{q}{p}\right)_{B_S^0} = \left(\frac{V_{ts} V_{tb}^*}{V_{ts}^* V_{tb}}\right) \approx e^{i2\beta_S}$$

$$\text{w/ } \beta_S = \arg\left(-\frac{V_{ts} V_{tb}^*}{V_{ts}^* V_{tb}}\right) \approx \arg(-V_{ts})$$

$$\rightarrow \text{Im}(\lambda_{J/\psi \phi}) = \sin(-2\beta_S)$$

$2\beta_S$ is very small
 $\sim \eta \lambda^2$

\rightarrow very small CPV

2) ϕ is not oscillating

$$3) \quad \Delta\Gamma \neq 0: \quad A_{CP}(t) = \frac{\Gamma(B_S \rightarrow J/\psi \phi)(t) - \Gamma(\bar{B}_S \rightarrow J/\psi \phi)(t)}{\Gamma(\dots) + \Gamma(\dots)}$$

$$= \frac{-\text{Im}(\lambda_{J/\psi \phi}) \sin(\Delta m_S t)}{\cosh\left(\frac{\Delta\Gamma t}{2}\right) + \text{Re}(\lambda_{J/\psi \phi}) \sinh\left(\frac{\Delta\Gamma t}{2}\right)}$$

4) Additional problem:

$J/\psi \phi$ is not a pure CP eigenstate but a mixture of $\eta_{CP} = \pm 1$ states.

Reason: J/ψ and ϕ are both vector (Spin=1) particles

$$\begin{array}{ccc} B_S \rightarrow J/\psi + \phi & \text{implies relative angular} \\ \sqrt{0}^{PC} & \begin{array}{cc} + & + \\ - & - \end{array} & \text{momenta } (l=0, 1, 2 \text{ possible}) \end{array}$$

$$\Rightarrow \eta_{CP}(J/\psi \phi) = \eta_{CP}(J/\psi) \cdot \eta_{CP}(\phi) \begin{array}{cc} (-1)^l & \\ (+1) & (+1) \end{array} \begin{array}{cc} (-1)^l & \\ (-1)^l & \end{array}$$

$$l=1 \rightarrow \text{CP-odd}$$

$$l=0, 2 \rightarrow \text{CP-even}$$

If one does not separate the $\ell=0, 1, 2$ states from each other one might not observe any CPV even if there are large effects.

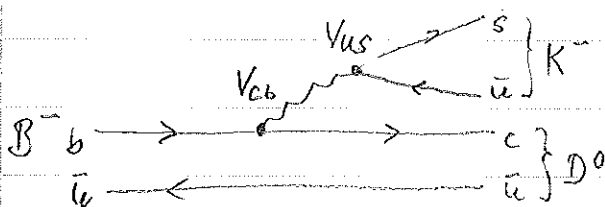
→ Solved by performing an angular analysis of the final states which allow to separate the CP = +1/-1 stbs

Measurement : LHCb 2013 $\phi_M = -2\beta_s = 0.01 \pm 0.07$
 SM $\phi_s^{SM} = -2\beta_s = -0.036 \pm 0.002$

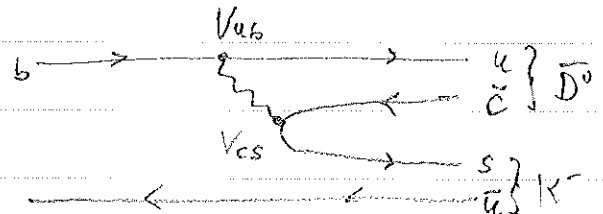
4.3 Measurement of the CKM phase γ using $B^- \rightarrow \bar{D}^0 K^-$

To measure γ one exploits "direct CPV", i.e. CPV in the decay. However to get the interference is a bit more tricky.

Cabibbo favored $B^- \rightarrow D^0 K^-$



Cabibbo suppressed $B^- \rightarrow \bar{D}^0 K^-$



Eriny: $V_{ub} = |V_{ub}| e^{-i\gamma}$

In general the 2 amplitudes lead to different final states and thus cannot interfere, however if D^0 and \bar{D}^0 decay to the same final state:

e.g. $D^0/\bar{D}^0 \rightarrow KK, \pi\pi$ (there are others)

the interference of the 2 amplitudes can result in CPV and allows to measure the weak phase $\gamma = -\arg(V_{ub})$

The extraction of the weak phase γ from the CPV in $B^- \rightarrow \bar{D}^0 K^-$ decays is rather complicated.

The most precise determination of γ is from LHCb.

$$\text{LHCb 2013: } \gamma = (67 \pm 12)^\circ$$

4.4 The unitarity triangle:

CKM phase β : $B^0 \rightarrow J/\psi K_s$

CKM phase γ : $B^0 \rightarrow D^0 K^0$

CKM phase β_s : $B_s^0 \rightarrow J/\psi \phi$

(CKM phase α : $B^0 \rightarrow \pi^+ \pi^-$: Measures $(\pi - \beta - \gamma = \alpha)$
not discussed)

The sides:

$V_{cb}, V_{ub} \rightarrow$ measured from singlept. BR

$b \rightarrow c \ell \nu$ $b \rightarrow u \ell \nu$

$V_{cs}/V_{cd} \rightarrow$ measured from oscillation.

