3.5 CP Violation in the neutral Kaon system

How can we explain the observed CPV in $K_L (CP=-1) \to 2\pi\ (CP=+1)$?

3.5.1 CP Violation in mixing

The key to the theoretical explaination is the observation of the time-dependent interference term in $K^0\ (\overline{K^0}) \to \pi^+\pi^-$ decays.*)

$$\Gamma(K^0 \to \pi^+\pi^-)(t) - \Gamma(\overline{K^0} \to \pi^+\pi^-)(t) = 2\,|\eta_{+-}|\, e^{-(\Gamma_S+\Gamma_L)\frac{t}{2}} \cos(\Delta m t - \phi_{+-})$$

$$\text{where } \eta_{+-} = |\eta_{+-}|\, e^{i\phi_{+-}} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$$

The oscillating interference term is CP violating and describes the different probabilities that a $K^0 \to \overline{K^0}$ and $\overline{K^0} \to K^0$

$$P(K^0 \to \overline{K^0}) \neq P(\overline{K^0} \to K^0) \Leftrightarrow \left|\frac{q}{p}\right| \neq 1 \quad \text{(see above)}$$

This experimental result is also confirmed by semi leptonic $K \to \pi l \nu$ decays. A detailed analysis reveals that only CPV in mixing is a possible explanation of the effect.

<u>Remember</u>:

$$K_1 = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K^0}\rangle\right) = \frac{1}{\sqrt{2}}\left(p|K^0\rangle + q|\overline{K^0}\rangle\right)$$
$$K_2 \quad \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K^0}\rangle\right) = \frac{1}{\sqrt{2}}\left(p|K^0\rangle - q|\overline{K^0}\rangle\right)$$

---

*) see the discussion in Chap 3.2 and in the exercises!

It is usual to rewrite $q/p = \frac{1-\epsilon}{1+\epsilon}$, ϵ being a complex parameter : $\epsilon = \frac{p-q}{p+q}$ $\Rightarrow$

$$K_{S,L} = \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \left[(1+\epsilon)|K^0\rangle \pm (1-\epsilon)|\bar{K}^0\rangle \right]$$

or equivalently:

$$K_{S,} = \frac{1}{\sqrt{1+\epsilon^2}} \left[|K_1\rangle - \epsilon|K_2\rangle \right]$$

$$K_L = \frac{1}{\sqrt{}} \left[|K_2\rangle + \epsilon|K_1\rangle \right]$$

Which means that K_S, K_L are not equivalent to the CP eigenstates anymore but have very small (ϵ) wrong admixture.

Using the λ Parameter as introduced in the last section :

$$\lambda_{\pi\pi} = \left(\frac{q}{p}\right)_{K^0} \frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}}$$

One can rewrite the CPV ratio of the amplitudes as :

$$\eta_{+-} = \frac{A(K_L \to \pi\pi)}{A(K_S \to \pi\pi)} = \frac{p\,A_{\pi\pi} - q\,\bar{A}_{\pi\pi}}{p\,A_{\pi\pi} + q\,\bar{A}_{\pi\pi}}$$

$$= \frac{1-\lambda_{\pi\pi}}{1+\lambda_{\pi\pi}} = \frac{1-q/p}{1+q/p} = \frac{p-q}{p+q} = \epsilon$$

where the last equality assumes that there is no direct CPV in the decay process itself, i.e. $A_{\pi\pi} = \bar{A}_{\pi\pi}$

From the time-dependent study of $K^0 \to \pi^+\pi^-$ decays on finds

$$|\eta_{+-}| = (2.236 \pm 0.018)\, 10^{-3}$$
$$\phi_{+-} = (43.4 \pm 1.2)^\circ$$

aus Sozzi S. 349
Ref.: Yao et al. 2006

$\rightarrow$

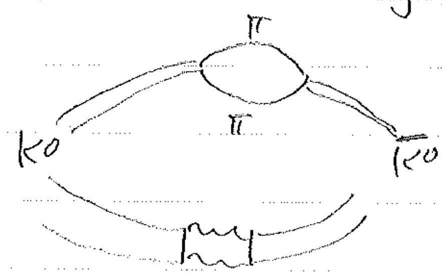
and for the ratio $|q/p| = \left|\frac{1-\varepsilon}{1+\epsilon}\right| = 0.995552 \pm 0.000029$ *)

*) includes also the value for
$|\eta_{00}| = (2.232 \pm 0.025) \cdot 10^{-3}$
from $K^0 \to \pi^0\pi^0$
$\rightarrow |\epsilon| = (2.232 \pm 0.007) \cdot 10^{-3}$

Sozzi: 349
Yao et al. 2006

Explaination within the Standard Model

CP violating effects require the interference of amplitudes with different weak and strong phases:



$\leftarrow$ no weak phases becaus π consist only out of u, d

$\leftarrow$ interfering diagrams with diff. weak phase: internal heavy quarks

different strong phases

CP violating weak phases from internal c and t quarks.
$\rightarrow$ Theoretical precision is limited by the knowledge of the hadronic uncertainties.

(See CKM-Fitter)

$$|\epsilon| = \frac{G_F^2 m_W^2 m_K f_K^2}{12\sqrt{2}\pi^2 \Delta m_K} \cdot B_K \cdot \Big(\eta_{cc}\, S_0(x_c, x_c)\, \mathrm{Im}\big[(V_{cs}V_{cd}^*)^2\big] + \eta_{tt}\, S_0(x_t, x_t)\, \mathrm{Im}\big[(V_{ts}V_{td}^*)^2\big] + 2\eta_{ct}\, S_0(x_c, x_t)\, \mathrm{Im}\big[(V_{cs}V_{cd}^* V_{ts} V_{td}^*)\big]\Big)$$

$$x_i = \left(\frac{m_i}{m_W}\right)^2$$

η_{ij} NLO QCD corrections $\rightarrow$

S_0 = Inami-Lim Fkt.

3.5.2 Direct CP violation

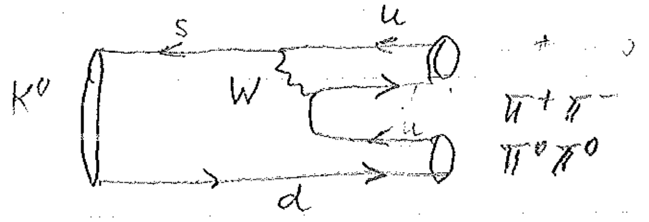
A long standing question of kaon physics was whether the CP asymmetry is also violated in the decay process itself, i.e. whether

e.g. $$\frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}} = \frac{A(\bar{K}^0 \to \pi\pi)}{A(K^0 \to \pi\pi)} \neq 1$$

If this is the case the amount of CP violation will depend on the specific decay process (final state) and might be different for $\overset{(-)}{K^0} \to \pi^+\pi^-$ and $\overset{(-)}{K^0} \to \pi^0\pi^0$ decays:

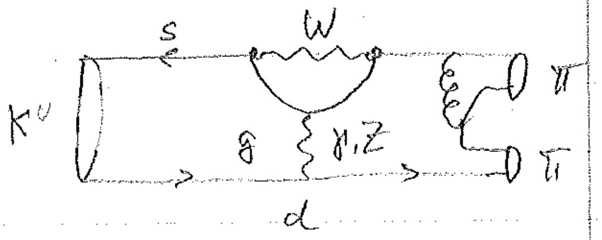
$$\eta_{+-} = \frac{A(K_L^0 \to \pi^+\pi^-)}{A(K_S^0 \to \pi^+\pi^-)} \neq \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}$$
$$\left(\approx \epsilon + \epsilon' \qquad\qquad \approx \epsilon - 2\epsilon' \right)$$

Direct CPV requires the interference of at least 2 amplitudes with diff. weak and strong phases. For the process $K^0 \to \pi\pi$ there exists beside the dominating Tree-amplitude also a penguin contribution:



no weak phase

$\pi\pi$ System always in Isospin $I=0$ State



internal quarks → weak phase

$\pi\pi$ system can be in $I=0, 2$ state

Gluon contribution → only $I=0$

γ, Z — " — → $I=0,2$

→ different strong phase

Since the pion has Isospin $I=1$ the $\pi\pi$-System can be in a $I=0$ and a $I=2$ state. A Clebsch-Gordon Spin Composition gives:

$$\left.\begin{aligned}|\pi^+\pi^-\rangle &= \sqrt{\tfrac{2}{3}}\,|\pi\pi; I=0\rangle + \sqrt{\tfrac{1}{3}}\cdot|\pi\pi; I=2\rangle \\ |\pi^0\pi^0\rangle &= -\tfrac{1}{\sqrt{3}}\,|\pi\pi; I=0\rangle + \sqrt{\tfrac{2}{3}}\,|\pi\pi, I=2\rangle\end{aligned}\right\} I_3=0$$

($\pi\pi$ obtained from K^0 with $\Delta I_3 = \frac{1}{2}$ rule)

For the decay amplitudes one obtains:

$$A(K^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{3}}\left(\sqrt{2}\,A_0 + A_2\right)$$

$$A(K^0 \to \pi^0\pi^0) = -\frac{1}{\sqrt{3}}\left(A_0 - \sqrt{2}\,A_2\right)$$

The two pions in the final state interact differently if they are in the $I=0$ or $I=2$ state. This introduces a strong phase difference:

$$A_I = a_I \cdot e^{i\delta_I} \quad \text{or} \quad \bar{A}_I = a_I^* \, e^{i\delta_I}$$

The above penguin diagrams with amplitudes which have different CKM phases and different strong phases can lead to direct CPV.

A careful evaluation of the $K_S, K_L \to \pi\pi$ decay amplitude ratio leads to:

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \epsilon + \frac{\epsilon'}{1+\Delta} \approx \epsilon + \epsilon'$$

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \epsilon - \frac{2\epsilon'}{1-2\Delta} \approx \epsilon - 2\epsilon'$$

ϵ' describes direct CPV

with $F = e^{i(\delta_2 - \delta_0)}$ $\quad \Delta = \frac{F}{\sqrt{2}} \cdot \frac{\mathrm{Re}\, A_2}{A_0}$ $\quad \epsilon' = i \frac{F}{\sqrt{2}} \frac{\mathrm{Im}\, A_2}{A_0}$

With very small direct CPV and $|\Delta| \ll 1$ and $|\epsilon'| \ll 1$ one finds for the double ratio:

$$\left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 = \frac{\Gamma(K_L \to \pi^0\pi^0)\ \Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^0\pi^0)\ \Gamma(K_L \to \pi^0\pi^0)}$$

$$= 1 - 6\,\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) \quad (*)$$

Two experiments KTeV (Fermilab) and $\left\{ \begin{array}{l} \text{NA31} \\ \text{NA48 (CERN)} \end{array} \right.$ have precisely measured the above double ratio. To limit the syst. uncertainties as much as possible the 4 decays were measured simultaneously using at the same time a K_L and a K_S beam, and measuring $\pi^0\pi^0$ as well as $\pi^+\pi^-$.

$$\mathrm{Re}(\epsilon'/\epsilon) = (1.67 \pm 0.23)\, 10^{-3}$$

i.e. direct CPV in Kaon decay is a very tiny (10^{-6}) effect
In B-mesons direct CP is of $\mathcal{O}(20\%)$!

$$* \quad \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 = \frac{|\epsilon - 2\epsilon'|^2}{|\epsilon + \epsilon'|^2} = \frac{|\epsilon|^2 - 4\,\mathrm{Re}(\epsilon^*\epsilon') + |\epsilon'|^2}{|\epsilon|^2 + 2\,\mathrm{Re}(\epsilon^*\epsilon') + |\epsilon'|^2}$$

r. a. Sozzi
S. 351

$$= 1 - \frac{6\,\mathrm{Re}(\epsilon^*\epsilon')}{|\epsilon|^2 + 2\,\mathrm{Re}(\epsilon^*\epsilon') + |\epsilon'|^2} \approx 1 - \frac{6\,\mathrm{Re}(\epsilon^*\epsilon')}{|\epsilon|^2}$$

$\epsilon \gg \epsilon'$

$$= 1 - 6\,\mathrm{Re}\left(\frac{\epsilon^*\epsilon'}{\epsilon^*\epsilon}\right) = 1 - 6\,\mathrm{Re}(\epsilon'/\epsilon)$$

3.6 Rare Kaon Decays

FCNC decays such as $K_L \to \mu^+\mu^-$ or $K \to \pi \nu\bar{\nu}$ are in the SM forbidden at tree-level and can only happen at loop level. Due to the unitarity of the CKM matrix the decays are strongly suppressed and have typical branching ratios in the order of $10^{-9} \ldots 10^{-11}$, thus they are extremly rare!

3.6.1 $K_L \to \mu\mu$ decay

Semi-leptonic kaon decays ($\Delta S = 1$) have BR of $O(1)$:

$$BR(K^+ \to \mu^+\nu) = 0.64$$

The similar looking „neutral current" decay $K_L \to \mu^+\mu^-$ is however heavily suppressed:

$$BR(K_L \to \mu^+\mu^-) = 7 \cdot 10^{-9}$$

At the end of the 1960's (beginning of the 1970s) in the times of the „3 Quark model" this tiny BR was not understandable:

- In the 3-quark model one expects a tree-level $\bar{s}dZ$ coupling with strength $G_F \to BR(K_L \to \mu\mu) \sim BR(K^+ \to \mu\nu)$
- even if the flavor violating Z coupling were forbidden the problem would appear at 1-loop level

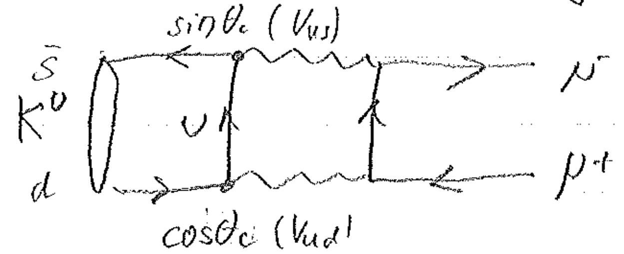
3-Quark Model

$$\begin{pmatrix} |u\rangle \\ |d'\rangle \end{pmatrix} = \begin{pmatrix} |u\rangle \\ \cos\theta_c |d\rangle + \sin\theta_c |s\rangle \end{pmatrix}$$

with $\theta_c \approx 13^0$ = Cabibbo angle

suppression of $s \to u$ decays by $\sin^2\theta_c \approx 0.05$

within this model the decay $K_L \to \mu\mu$ could proceed via loops:



$$A_u \sim \frac{m_u^2}{M_W^2} \ln\left(\frac{M_W}{m_u}\right) \sin\theta_c \cos\theta_c$$

$\to$ large BR expected

(Note: the 3-quark theory is not renormizable)

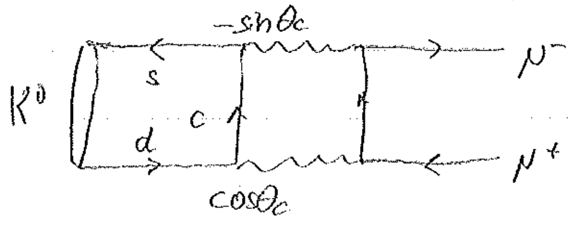
Glashow, Iliopoulos and Maiani (GIM) proposed in 1970 the introduction of a new u-type Quark (c-quark) to solve the problem of large expected BR.

They proposed 2 complete generations of quarks with a 2×2 matrix describing the quark mixing:

$$\begin{pmatrix} u \\ d' \end{pmatrix} \begin{pmatrix} c \\ s' \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} |d'\rangle \\ |s'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_c |d\rangle + \sin\theta_c |s\rangle \\ -\sin\theta_c |s\rangle + \cos\theta_c |d\rangle \end{pmatrix}$$

The 2×2 unitary matrix is called Cabibbo Matrix

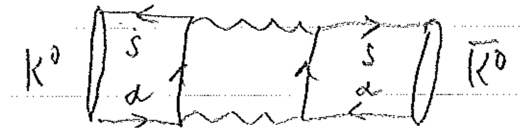
In addition to the amplitude A_u there is now also A_c



$$A_c \sim -\frac{m_c^2}{M_W^2} \cdot \ln\left(\frac{M_W}{m_c}\right) \cdot \sin\theta_c \cos\theta_c$$

For $m_c = m_u$ this additional amplitude leads (in the 2 Generation model) to a perfect cancellation. For $m_c \neq m_u$ the total amplitude is strongly suppressed: GIM suppression.

It is very interesting to note that already GIM in their original paper stressed the connection to the $K^0\bar{K}^0$ mixing:



where the additional c-quark also strongly influences the observable mixing effect. From the potential influence they concluded on a mass term violating the quark flavor symmetry: $\Delta < 3 \ldots 4$ GeV

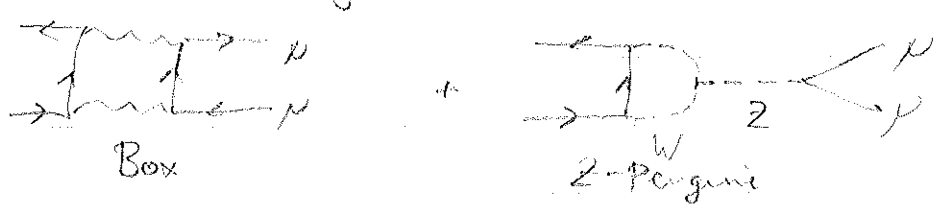
A careful analysis of the $\bar{K}^0$-mixing in the light of a new up-type quark was performed by Gaillard & Lee (1974):

$$\Delta m_K \sim |V_{cs}^* V_{cd}|^2 \cdot \underbrace{S_0\left(\left(\frac{m_c}{M_W}\right)^2\right)}_{\sim m_c^2/M_W^2}$$

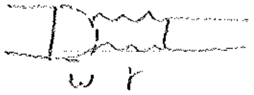
They obtained for m_c: $m_c \sim 1.5$ GeV which (accidently) fits very well today's c-quark mass! (*)

The prediction of the 4th quark together with its mass is certainly one of the triumphs of the SM.

(*) Today we know thus the theoretical calculation of the BR($K_L \to \mu^+\mu^-$) is difficult. In addition to the short range contributions



there are contributions from a long-distance amplitude with a 2-γ intermediate state



⇒ Makes the exact theoretical calculation difficult!

3.6.2 $K \to \pi\nu\nu$

(Rare decay which is theoretical much better under control)

The decay $K \to \pi\nu\nu$ is a genuine loop-process in SM;

Beside the loop- and the helicity suppression there exists a strong GIM suppression $\Rightarrow$ BR $\sim \mathcal{O}(10^{-10})$

Because of the GIM mechanism a very large contribution comes from the top $\rightarrow$ m_t, V_{ts}, V_{td}.
In addition this decay is very sensitive to new physics effects.

Since $K \to \pi\nu\nu$ is reliably calculable the precise measurement of BR$(K \to \pi\nu\nu)$ provides an excellent test of the flavor sector.
For the amplitude one finds:

$$A \sim \sum_{i=u,c,t} \lambda_i \cdot F(x_i) \underset{\text{unitarity}}{=} \lambda_c \left(F(x_c) - F(x_u) \right) + \lambda_t \left(F(x_t) - F(x_u) \right)$$

$$\text{with } \lambda_i = V_{is}^* V_{id} \quad x_i = m_i^2 / M_W^2$$

One finds:

$$F(x_u) \sim 10^{-5} \ll \frac{m_c^2}{M_W^2} \cdot \ln\left(\frac{M_W}{m_c}\right) \sim 10^{-3} \ll F(x_t) \sim \mathcal{O}(1)$$

With the CKM factors λ_i included:

$$\lambda_c \cdot F(x_c) \sim 10^{-4} \qquad \lambda_t \cdot F(x_t) \sim 10^{-4}$$

(and u-quark contribution can be neglected)

A careful analysis gives (see Buchalla)

Channel	$K^+\to\pi^+\nu\bar\nu$	$K^0\to\pi^0\nu\bar\nu$
CP	conserving	CP violating
CKM	V_{td}	$\mathrm{Im}(V_{ts}^* V_{td}) \sim J_{CP} \sim \eta$
	c & t	only t
BR*)	$(0.85 \pm 0.07) \times 10^{-10}$	$(2.6 \pm 0.4)\, 10^{-11}$

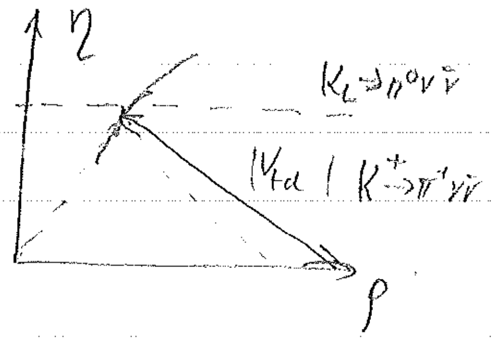
*) Talk by Cecchini + Ref. in this talk

The measurements of $BR(K^+\to\pi^+\nu\bar\nu)$ and $BR(K^0\to\pi^0\nu\bar\nu)$ allow an interesting phenomenological study.

→ the two BR define by their own the UT

$$BR(K_L\to\pi^0\nu\bar\nu) \to \mathrm{Im}(V_{ts}^* V_{td})$$

$$\left[\begin{array}{l} J_{CP} = \mathrm{Im}(V_{ts}^* V_{td} V_{us}^* V_{ud}) \\ \quad = \lambda(1-\lambda^2/2)\, \mathrm{Im}(V_{ts}^* V_{td}) \end{array}\right.$$



Experimental Status:

$K^+\to\pi^+\nu\bar\nu$ $\quad 1.73^{+1.15}_{-1.05}$ (AGS E787/E949, 2008) *)

$K_L^0\to\pi^0\nu\bar\nu$ $\quad < 2.6\cdot 10^{-8}$ @ 90% CL (KEK E391a, 2009)

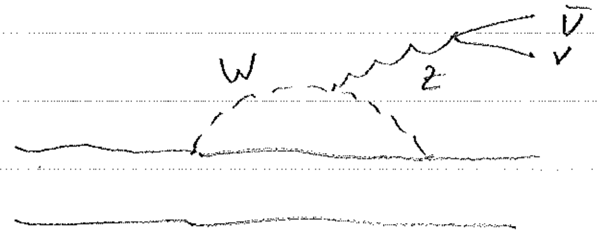
*) observed 7 events

Future Exp: NA62 → K^+
KOTO at JPARC → K^0
} decays in flight!

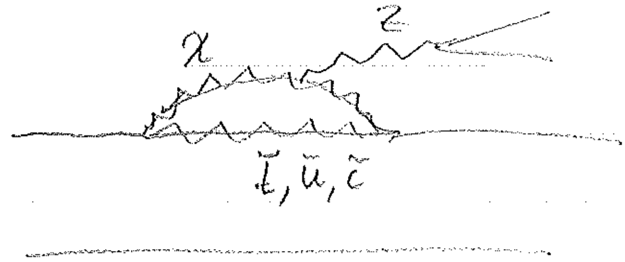
New Physics sensitivity: see Slide from Augusto.

The decay $K \to \pi \nu \bar{\nu}$ is an ideal place to look for new physics. New particles (e.g. SUSY) could significantly enhance / modify the BR:

SM



SUSY



NP can alter the helicity suppression / add additional amplitude.

~~$K \to \pi \nu \bar{\nu}$ decays~~