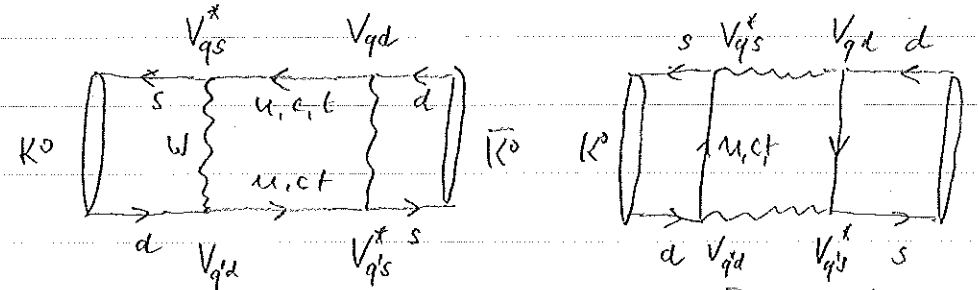
D. Standard model prediction for mixing

The short distance contribution to the $P^0 \leftrightarrow \bar{P}^0$ transition is described by M_{12} respectively Δm and can be calculated from the contributing box diagrams. For Kaons one obtains



Propagator q, q'

$$\mathcal{M} \sim \sum_{q,q'} V_{qs}^* V_{qd} \Pi_q \cdot V_{q'd} V_{q's} \Pi_{q'} \quad \text{with } qq' = \begin{matrix} uu & uc & ut \\ cu & cc & ct \\ tu & tc & tt \end{matrix}$$

$$\sim \int d^4k \, k_\mu k_\nu \left(\frac{V_{us}^* V_{ud}}{k^2 - m_u^2} + \frac{V_{cs}^* V_{cd}}{k^2 - m_c^2} + \frac{V_{ts}^* V_{td}}{k^2 - m_t^2} \right)^2$$

$$\sim \int d^4k \, k_\mu k_\nu \left(V_{cs}^* V_{cd} \left[\frac{1}{k^2 - m_c^2} - \frac{1}{k^2 - m_u^2} \right] + V_{ts}^* V_{td} \left[\frac{1}{k^2 - m_t^2} - \frac{1}{k^2 - m_u^2} \right] \right)^2$$

Unitarity relation: $V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0$

Neglecting in the further calculation the u-quark mass the loop integrals results to 3 terms described by the Inami-Lim-Fkt.

Internal quarks:

cc	$S_0(m_c^2/M_W^2)$	$\simeq 3.5 \cdot 10^{-4}$
tt	$S_0(m_t^2/M_W^2)$	$\simeq 2.5$
ct	$S_0(m_c^2/M_W^2, m_t^2/M_W^2)$	$\approx 3 \cdot 10^{-3}$

For the CKM factors one obtains: $|V_{cd} V_{cs}^*|^2 \gg |V_{td} V_{ts}^*|^2$

$$\sim \lambda^2 \qquad \sim \lambda^{10}$$

$$\sim 2.7 \cdot 10^{-2} \qquad \sim 1.1 \cdot 10^{-7}$$

$$\Rightarrow \quad M_{K\bar{K}^0} \sim S_0(m_c^2/M_W^2) \, |V_{cd} V_{cs}^*|^2$$

Taking the hadronic part into account;

$$\langle K^0 | J_\mu^\dagger J^\mu | \bar{K}^0 \rangle = \sum_X \langle K^0 | J_\mu | X \rangle \langle X | J^\mu | \bar{K}^0 \rangle$$

$$= \underset{\substack{\uparrow \\ \text{bag factor which accounts} \\ \text{for the vacuum insertion.}}}{B_K} \langle K^0 | J_\mu | 0 \rangle \underbrace{\langle 0 | J^\mu | \bar{K}^0 \rangle}_{\text{decay constant}} = B_K f_K^2 p_\mu p^\mu$$

one finally obtains:

$$\Delta m_K = 2 |M_{12}| = \frac{G_F^2 \cdot M_W^2}{6\pi^2} \cdot \underbrace{\eta_{QCD}}_{\text{perturbative QCD corrections}} B_K \cdot f_K^2 m_K \left[S_0(m_c^2/M_W^2) |V_{cd} V_{cs}^*|^2 \right]$$

For the B system one has $|V_{td} V_{tb}^*|^2 \sim |V_{cd} V_{cb}^*|^2$ (both $\sim A^2\lambda^6$) because of $m_t^2 \gg m_c^2$ it is now the top-loop which contributes:

$$\Delta m_{B_d} = \frac{G_F^2 m_W^2}{6\pi^2} \cdot \eta_{QCD} \underbrace{B_B f_B^2 m_B}_{\text{changes for } B_s} \left[S_0(m_t^2/M_W^2) \underbrace{|V_{td} V_{tb}^*|^2}_{B_s:\ V_{ts} V_{ts}} \right]^2$$

For D-mesons the mass of the heaviest internal quark (d-type: b-quark m_b) is not large enough to compensate the large CKM suppression $\sim |V_{ub} V_{cb}^*|^2$. As a result the light s-quark dominates the short range mixing:

$$\Delta m_D \sim |V_{us} V_{cs}^*|^2 \cdot \underbrace{S_0(m_s^2/M_W^2)}_{\sim m_s^2/M_W^2} \sim \lambda^2 \underbrace{S_0(m_s^2/M_W^2)}_{\sim m_s^2/M_W^2}$$

→ Mixing parameters are small (very slow mixing): most of the D's decay before they mix.

As mentioned earlier the mixing consists of 2 components M_{12} and $\frac{1}{2}\Gamma_{12}$. The calculation of Γ_{12} (real intermediate states) is difficult and is important for kaons and D mesons. ~~Within the SM~~ Γ_{12} can be approximated by the „on shell" (absorptive) part of the box diagrams
→ quark representation of the final state (very poor approx. for kaons)

To describe the mixing often, in addition to Δm, dimensionless parameters x and y are introduced:

$$x = \frac{\Delta m}{\Gamma} \qquad y = \frac{\Delta\Gamma}{2\Gamma}$$

Summary of mixing parameters for neutral mesons

System	τ	Δm	x	y
K^0	$0{,}26 \cdot 10^{-9}\,s$	$5.29\,ns^{-1}$	$0{,}477$	$+1$
D^0	$0.41 \cdot 10^{-12}\,s$	$0{,}0024\,ps^{-1}$	$0{,}0097$	$0{,}0078$
B_d^0	$1{,}53 \cdot 10^{-12}\,s$	$0{,}507\,ps^{-1}$	$0{,}78$	$0{,}0015$ *)
B_s	$1{,}47 \cdot 10^{-12}\,s$	$17.77\,ps^{-1}$	26.1	$0{,}06$ *)

*) theoretical values.

Figure with the mixing diagrams for the 4 neutral mesons,
→ Niels Tuning paper

E) Neutral meson decay: CPV through interference between decay & mixing

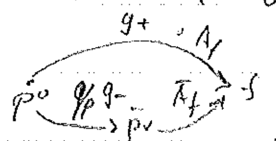
In addition to the oscillation we consider in the following also the subsequent decay of neutral mesons.
We consider 4 different decay amplitudes:

$$\mathcal{A}_f = \mathcal{A}(P^0 \to f) \qquad \bar{\mathcal{A}}_f = \mathcal{A}(\bar{P}^0 \to f)$$
$$\mathcal{A}_{\bar{f}} = \mathcal{A}(P^0 \to \bar{f}) \qquad \bar{\mathcal{A}}_{\bar{f}} = \mathcal{A}(\bar{P}^0 \to \bar{f})$$

and we define the complex parameter λ (observable): Modulus & phase of λ.

$$\lambda_f = \frac{q}{p}\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} \qquad \bar{\lambda}_f = \frac{1}{\lambda_f} \qquad \lambda_{\bar{f}} = \frac{q}{p}\frac{\bar{\mathcal{A}}_{\bar{f}}}{\mathcal{A}_{\bar{f}}} \qquad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}}$$

The time-dependent decay rate $\Gamma(P^0 \to f)(t) = |\mathcal{A}(P^0 \to f)|^2$ gives the probability that an initial P^0 decay at time t into the final state f:



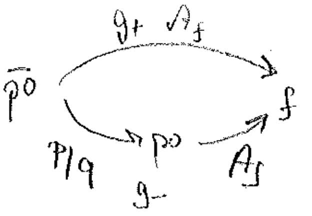
$$|\mathcal{A}(P^0 \to f)(t)|^2 = |\mathcal{A}(P^0 \to f)(t) + \mathcal{A}(P^0 \to \bar{P}^0 \to f)|^2$$

$$\Gamma(P^0 \to f)(t) = |\mathcal{A}_f|^2 \left[|g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2\,\mathrm{Re}\{\lambda_f\, g_+^*(t)\, g_-(t)\} \right]$$

analog ⊛
$$\Gamma(\bar{P}^0 \to f)(t) = |\mathcal{A}_f|^2 \left|\frac{p}{q}\right|^2 \left[|g_-(t)|^2 + |\lambda_f|^2 |g_+(t)|^2 + 2\,\mathrm{Re}\{\lambda_f\, g_+(t)\, g_-^*(t)\} \right]$$

with
$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left(\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) \pm \cos(\Delta m t)\right)$$

$$g_\pm^*(t)\, g_\mp(t) = \frac{e^{-\Gamma t}}{2}\left(\sinh\left(\tfrac{1}{2}\Delta\Gamma t\right) \pm i \sin(\Delta m t)\right)$$

⊛ 

In this way one obtains the „Master Equations" for time-dependent neutral meson decays:

$$\Gamma(P^0 \to f)(t) = |A_f|^2 \, \frac{e^{-\Gamma t}}{2} \cdot$$

$$\left((1+|\lambda_f|^2)\cosh\left(\tfrac{\Delta\Gamma}{2}t\right) + 2\,\mathrm{Re}\,\lambda_f \sinh\left(\tfrac{\Delta\Gamma}{2}t\right) + (1-|\lambda_f|^2)\cos(\Delta m t) - 2\,\mathrm{Im}\,\lambda_f \sin(\Delta m t) \right)$$

$$= |A_f|^2 \, (1+|\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left(\cosh\left(\tfrac{\Delta\Gamma}{2}t\right) + D_f \sinh\left(\tfrac{\Delta\Gamma}{2}t\right) + C_f \cos \Delta m t - S_f \cdot \sin \Delta m t \right)$$

with $D_f = \dfrac{2\,\mathrm{Re}\,\lambda_f}{1+|\lambda_f|^2}$ $\quad C_f = \dfrac{1-|\lambda_f|^2}{1+|\lambda_f|^2}$ $\quad S_f = \dfrac{2\,\mathrm{Im}\,\lambda_f}{1+|\lambda_f|^2}$

analog:

$$\Gamma(\bar{P}^0 \to f)(t) =$$

$$|A_f|^2 \cdot \left|\frac{p}{q}\right|^2 (1+|\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left(\cosh\left(\tfrac{\Delta\Gamma t}{2}\right) + D_f \sinh\left(\tfrac{\Delta\Gamma t}{2}\right) - C_f \cos \Delta m t + S_f \sin \Delta m t \right)$$

i.e. CP differences in the time-dependent decay rate (CPV):

$$A_{CP}(t) = \frac{\Gamma(P^0 \to f)(t) - \Gamma(\bar{P}^0 \to f)}{\Gamma + \Gamma}$$

$$= \frac{2 C_f \cos(\Delta m t) - 2 S_f \sin(\Delta m t)}{2\cosh\left(\frac{\Delta\Gamma t}{2}\right) + 2 D_f \sinh\left(\frac{\Delta\Gamma t}{2}\right)} \neq 0$$

if $\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f} \neq 1$

= CP violation in the interference between mixing and decay.

→ Master Eq for time dependent CP V in neutral meson decays!

(e.g. see BABAR physics book etc)

F) Classification of CP violation

Usually the observed CP violating effects in meson decays are classified in the following way:

(i) CPV in decay:

$$\Gamma(P^0 \to f) \neq \Gamma(\bar{P^0} \to \bar{f})$$

implies $\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| \neq 1$

e.g. $\Gamma(B^0 \to K^+\pi^-) \neq \Gamma(\bar{B^0} \to K^-\pi^+)$

In charged mesons where no mixing is possible this is the only type of CPV which can occure.

(ii) CPV in mixing

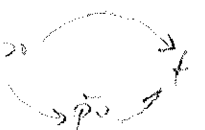
$$P(P^0 \to \bar{P^0}) \neq P(\bar{P^0} \to P^0)$$

this implies $\left|\frac{q}{p}\right| \neq 1$ (see mixing equations)

While for B mesons $\left|\frac{q}{p}\right| = 1 + \mathcal{O}(10^{-4} \cdot 10^{-5}) \simeq 1$

this is the dominating effect for kaons. $(\mathcal{O}(10^{-3}))$

(iii) CPV in interference between a decay w/ and w/o mixing:



→ time-dependent effect (see above)

no effect in time-integrated measurements

can only occure if $\mathrm{Im}(\lambda_f) = \mathrm{Im}\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0$

i.e. if either q/p or $\bar{A}_f/A_f$ has a non trivial phase.

An alternative classification distinguishes between direct and indirect CPV.

<u>Direct CPV</u> : $\mathcal{A}(P^0 \to f) \neq \mathcal{A}(\bar{P}^0 \to \bar{f})$

<u>Indirect CPV</u> : CPV that involves the mixing phenomenon in any way

<u>Final remarks</u>
CP violating effects all depend on J_{CP} and should be of the same order in the SM.
The observable asymmetries = ratios of CP violating to CP conserving quantities are <u>enhanced</u> for <u>suppressed</u> quantities: observable asymmetries larger in B decays than in kaons → B's have smaller CKM couplings and larger lifetimes (suppressed w/r to kaons)

To exhibit a CP violating phase the process must involve at least 4 CKM matrix elements (definition of J_{CP})

→ Below the charm threshold on-shell processes cannot violate CP as only V_{ud} and V_{us} are involved.

→ CPV in kaon sector only through virtual processes to which also heavier quarks can contribute: K^0-$\bar{K}^0$ mixing or penguin decays.

→ Above the charm-threshold CPV also in on-shell processes possible!