1.1 Hadronic quantum numbers

To describe hadronic states 3 types of quantum number (QN) are used:

(i) exact QN:

There are only 2 exact QN: electr. charge Q and spin J *)
Mixing between two hadronic states is only possible if they exhibit the same exact quantum numbers.

*) Remark: Since baryons always have half-integer spin ($2J+1$ = even) and mesons have integer spin ($2J+1$ = odd) the baryon number is implicitly conserved if the spin is conserved.

(ii) QN exact only under QCD but not under weak interaction

Parity P, charge conjugation C and flavor number

a) P and C: In the PDG mesons are denoted by their spin J parity and C-conjugation: J^{PC} ← superscript: exact only in QCD

Examples: 1) $J^{PC}(\pi^0) = 0^{-+}$

negative internal parity?

→ π^0 is lowest $q\bar{q}$ state: $l=0$ ⤳ $\uparrow\downarrow$ to get $J=0$

This leads to a minus sign when P is applied:

$\uparrow\downarrow \longrightarrow \downarrow\uparrow$ (wave function picks up „−" sign)

2) $J^P(\pi^+) = 0^-$ — no C-value!

π^+ is not an eigenstate under C: $\pi^+ \xrightarrow{C} \pi^-$

For the π^+ (and also for other charged mesons) the PDG lists an additional QN: $I^G = 1^-$, where I is the isospin ($I=1$ $I_3=+1$) and the G parity is defined as:

G-parity: $G = C \cdot \underbrace{e^{i\pi I_2}}$

rotation in iso-spin space around the y-axis: $\pi^+ (I_3 = +1) \rightarrow \pi^- (I_3 = -1)$

The rotation in isospin space gives for the π^+ an additional phase π (i.e. a „–" sign): $G(\pi^+) = -\pi^+$

Gruppierung $G(\pi^+) = -1$

b) Flavor numbers

- for the light (historically called „flavorless") mesons composed of u and d quarks the isospin is used. (Isospin is in principle even in QCD only approximate, since it neglects the mass diff. between u and d quarks $\Delta m_{ud} \ll \Lambda_{QCD}$ ⇝ good approximation)

- Strangeness, charm, beauty / bottomness:

1.2 Masses and mixing :

Beside the problem that quarks are bound in strongly-coupled bound-states, there is a second problem associated with QCD: Once a quark of a given flavor is bound in a physical-state meson, the meson does not necessarily preserve that quark's flavor:
[Idea of mixing for systems which live in diff. basis can be seen already for light meson states.]

Example : u-quark which hadronize into a neutral π.

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

This state is 50% $u\bar{u}$ and 50% $d\bar{d}$!

Mixing : [A given quark pair describes a meson state with a definite flavor in the sense of $q\bar{q}$. As seen for the π^0 these mesons (flavor states) can mix and the physical mesons are linear combinations of diff. flavor states.

Question: Why exhibits the $|\pi^0\rangle = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$ maximum mixing while the ψ is a definite flavor state ?
(no contributions from light $u\bar{u}$, $d\bar{d}$ or heavy quarks)

The answer is related to the origin of the meson masses:
We saw for the ρ and B that masses are either predominately the effect of QCD binding ($\mathcal{O}(\Lambda_{QCD})$) or the effect of valence quarks masses.

For the light mesons $q\bar{q}$, $q = u, d, s$ isospin is a good symmetry and the masses come from the QCD binding!
(+ additional internal degrees of freedom, like spin)

<u>u, d states:</u> Each quark has isospin $\frac{1}{2}$, $q\bar{q}$ bound state has $I = 0$ or $I = 1$:

$$q\bar{q} : 2 \otimes \bar{2} = \underset{\text{isospin singlet}}{1} \oplus \underset{\text{triplet}}{3}$$

$$\pi : \begin{cases} u\bar{d} \\ \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \\ \bar{u}d \end{cases}$$

$$\eta : \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

- Within the triplet masses are nearly degenerate: m_3
- Singlet state has a different mass: m_1

→ QCD potential gives mass according to the isospin argument

$$\mathcal{L}_{mass,iso} \sim \underbrace{(\langle 1|, \langle 3|)}_{\text{isospin basis}} \begin{pmatrix} m_1 & 0 \\ 0 & m_3 \end{pmatrix} \begin{pmatrix} \langle 1| \\ \langle 3| \end{pmatrix}$$

<u>heavy states</u> e.g. charmonia $c\bar{c}$, bottomonia $b\bar{b}$

The masses are defined through the flavor content.

$$\mathcal{L}_{mass,flavor} \sim \underbrace{(\langle q| \; \langle q'|)}_{\text{flavor basis}} \begin{pmatrix} m_q & 0 \\ 0 & m_{q'} \end{pmatrix} \begin{pmatrix} \\ \end{pmatrix}$$

(mixing: $\frac{1}{\sqrt{2}}(c\bar{c} + b\bar{b})$)

For real mesons we need to take both effects into account.

The real mass matrix lies in general between the two bases:

Two-state system with oscillations.

<u>if iso-spin splitting is more important:</u>

iso-spin basis is the natural basis and flavor effect acts as perturbation
→ flavor effect leads to off-diagonal element and thus to mixing between the iso spin states :

<u>if flavor-splitting is dominant:</u>

flavor basis is the natural basis and iso-spin splitting acts as perturbation
→ leads to mixing

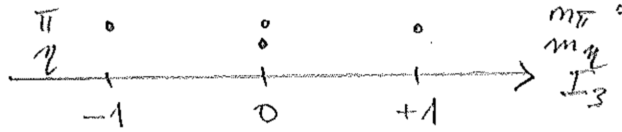
e.g.: Hypothetical $\cos\theta\,|b\bar{b}\rangle + \sin\theta\,|c\bar{c}\rangle$ state

$$\sin\theta \sim \frac{\Lambda_{QCD}}{m_b^2 - m_c^2} \ll 1$$

→ reason why this kind of mixing does not appear.

a) <u>Pseudo-scalar meson(s) multiplett:</u>

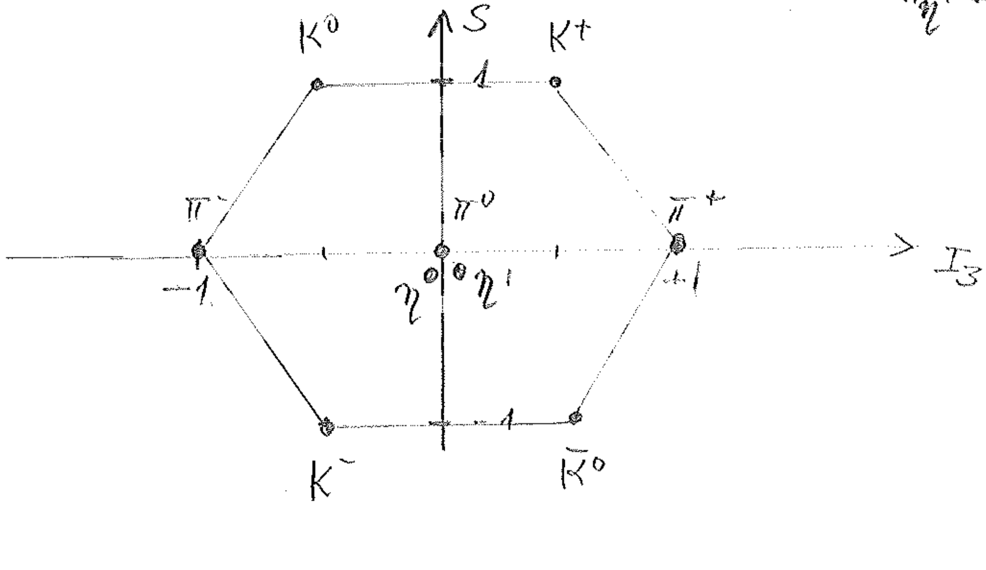
<u>u, d quarks:</u>



$m_\pi \approx 135$ MeV
$m_\eta \approx 550$ MeV

<u>+ s quark</u> : $SU(3)_{\text{iso + strangeness}}$ (often „flavor SU(3)")

$3 \otimes \bar{3} = 1 \oplus 8$ <u>Singlet</u>: η'
$m_{\eta'} \approx 960$ MeV



SU(3) group Theory:

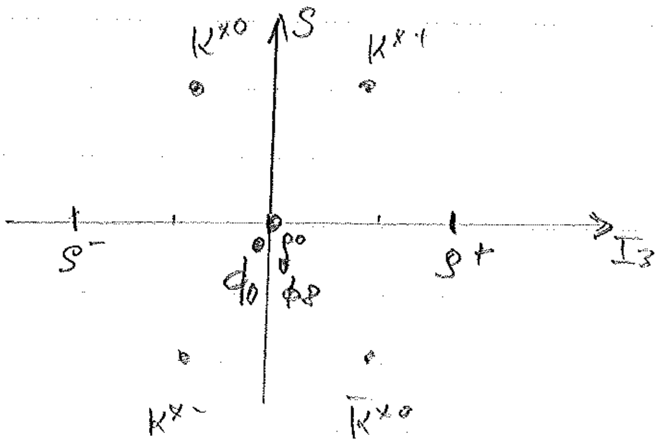
Octet
$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \qquad I=1\ I_3=0$$
$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \qquad I=0\ I_3=0$$

Singlet
$$\eta_0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \qquad I=0\ I_3 0$$

→ Isospin basis, but the mass diff $|m_s - m_{u,d}|$ leads to mixing between the two state → 10% correction
(physical states are mixture of the isospin states)
$\eta_8, \eta_0 \rightarrow$ physical states η, η'

b) Vector mesons $J^P = 1^-$ (spin aligned different w/r to pseudo-scalars)

SU(3) group theory:
$1 \oplus 8$



In principle the same as for pseudo-scalar mesons but the mixing is large:
Physical states ϕ, ω differ significantly from isospin state $\phi_0\ \phi_8$:

$$\phi = \cos\theta\ \phi_8 - \sin\theta\ \phi_0 \qquad \text{with } \theta = 37^0$$
$$\omega = \sin\theta\ \phi_8 + \cos\theta\ \phi_0$$

Ideal mixing $\tan\theta = 2\sqrt{2} \approx 35.5^0$;
$$\phi' = s\bar{s}$$
$$\omega' = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$
Real ϕ is nearly a pure $s\bar{s}$ state!