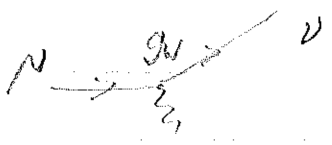
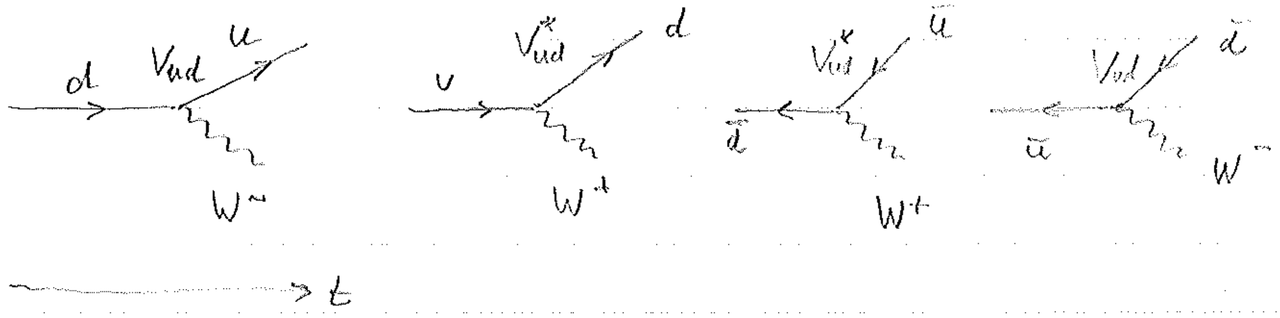


Pictorial explanation of CKM elements:



4) Properties of V_{CKM}

V_{CKM} is a complex and unitary matrix: $V_{CKM} V_{CKM}^\dagger = 1$

parameters of compl. 3×3 : 18 = 9 real + 9 phases
after unitarity condition : 9 = 3 real + 6 phases
(→ 9 parameters removed)
5 phases are absolute quark phases and are not observable : 4 = 3 real + 1 phase

Unobservable Quark phases:

Rephasing of quark fields possible: $q_L = e^{i\phi_q} \cdot q_L$
for $q = u, d, c, s, t, b$

Under this phase transformation:

$$V_{CKM} \mapsto \begin{pmatrix} e^{-i\phi_u} & 0 & 0 \\ 0 & e^{-i\phi_c} & 0 \\ 0 & 0 & e^{-i\phi_t} \end{pmatrix} V_{CKM} \begin{pmatrix} e^{i\phi_d} & 0 & 0 \\ 0 & e^{i\phi_s} & 0 \\ 0 & 0 & e^{i\phi_b} \end{pmatrix}$$

or

$$V_{\alpha\beta} \to \exp\left(i[\phi_\beta - \phi_\alpha]\right) V_{\alpha\beta}$$

this allows to rotate away 5 phases related to phase diff.

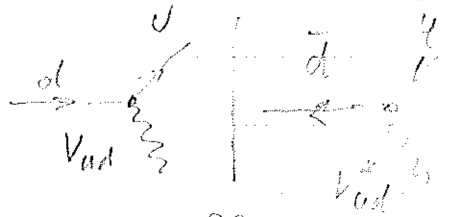
In general for N quark generations: $N \times N$ mixing matrix

$\frac{1}{2} N(N-1)$ real parameters

$\frac{1}{2}(N-1)(N-2)$ phases

N	# real param	# phases
2	1	0
3	3	1
4	6	3

- Phase in CKM matrix is the source of CPV:
- True also for neutrinos!



CP

5) Summary of Flavor sector in SM

Quark sector

6 masses

4 CKM parameter

Lepton sector

3 charged Lepton masses

+ extension: massive neutrinos

+ 3 neutrino masses

+ 4 mixing parameter

= PMNS matrix

= Pontecorvo - Maki - Nakagawa - Sakata

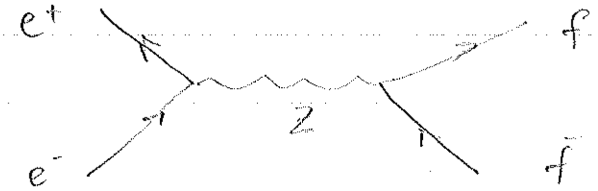
So far masses and mixing parameters are exp. inputs to theory. There is no deeper understanding of the mass hierarchy and the mixing parameters $\longrightarrow$ Is there a relation between Masses & mixing and leptons & quarks? (Fig)

6) Limits on possible „sequential" fourth generation

6.1 LEP bounds : additional heavy neutrinos

LEP studied the properties of the Z-Boson via resonant Z Boson production in e^+e^- annihilation:

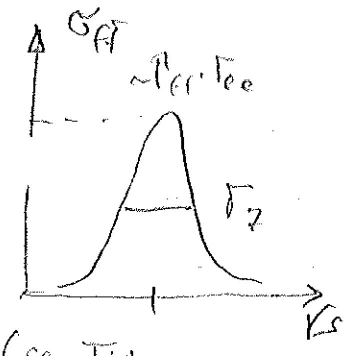
$$e^+e^- \to Z \to f\bar{f}$$



$f\bar{f} = q\bar{q} \to$ Hadrons
$= ee, \mu\mu, \tau\tau$: Leptons
$= \nu_x\bar{\nu}_x$, $x = e, \mu, \tau$: invisible

total Z decay width: $\Gamma_Z = \Gamma_{had} + 3\Gamma_{Lep} + \Gamma_{inv}$

↳ number of light neutrinos $\left(m_\nu < \frac{m_Z}{2}\right)$:



(see Fig

$$N_\nu = \frac{\Gamma_{inv}^{meas}}{\Gamma_\nu^{SM}} = \left(\frac{\Gamma_{inv}}{\Gamma_{lep}}\right)_{meas} \cdot \left(\frac{\Gamma_{lep}}{\Gamma_\nu}\right)_{SM}$$

$$N_\nu = 2.984 \pm 0.008.$$

i.e. perfect agreement with SM expectation 3
→ puts limit on $m_{\nu_4} > \frac{m_Z}{2}$!

Direct searches at LEP200 put constraints on heavy quarks & leptons:

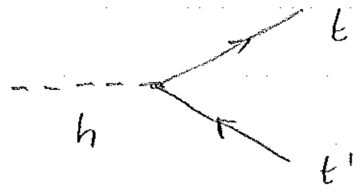
$$m_{b'},\ m_{t'},\ m_{\ell_4} > 100\ \text{GeV}$$

6.2 Direct searches at Tevatron and LHC

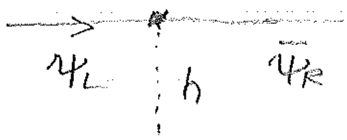
Tevatron & LHC: $m_{t'}, m_{b'} \gtrsim 400$ GeV

experimental signatures
$b'\bar{b}' \to tWtW$
↳ WbW ↳ WbW — leptons + jets
$t'\bar{t}' \to bWbW$ } b-jets + jets + lept

6.3 Constraints from Higgs production

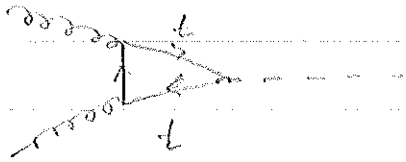


coupling to Higgs ~ $m_{t'}, m_{b'}, m_{\ell}, \ldots$



$\sim m_\psi \bar{\psi}_R \psi_L h$

Main Higgs production channel (at LHC) is Gluon-Fusion:

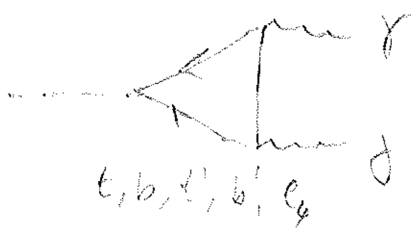


in SM main contribution from top

the presence of <u>additional heavy quarks</u> will increase the effective ggH coupling by roughly a factor 3 i.e. we would expect an enhanced Higgs production rate w.r. to the SM expectation and thus more detectable Higgs decays:

	$\sigma(gg\to H)\cdot BR(H\to \text{finals})$ / SM-expect.	
$gg\to H\to ZZ$	$\sim \times 5 \ldots 8$	in contradiction with observations: no enhancement in $H\to ZZ$ and $H\to\tau\tau$ seen!
$gg\to H\to f\bar{f}$	$\sim \times 5$	
$gg\to H\to \gamma\gamma$	~ 1 (*)	

*) Remark: $H\to\gamma\gamma$



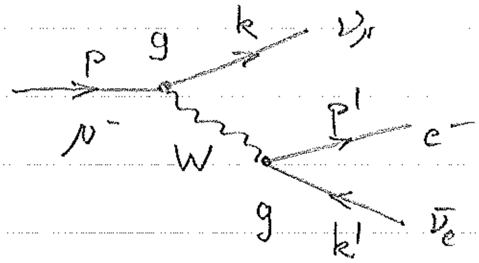
$\longrightarrow$ gives a reduction of $BR(H\to\gamma\gamma)$ for models with 4th gen. and therefore compensates enhanced cross section

$\Rightarrow$ $\boxed{\text{Observed Higgs} \to ZZ, \tau\tau \text{ Rates ruled out the existence of a } 4^{th} \text{ generation } (5.6\sigma)}$

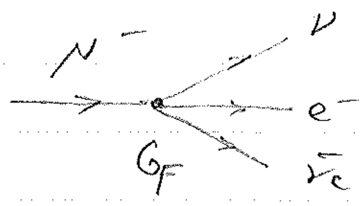
7. Weak decays:

7.1 Muon decay

effektive local IA



$m_\mu \ll m_W$ $\Longrightarrow$



$$\mathcal{M} \sim \; g\, J^{\text{Muon},\alpha} \left(\frac{g_{\alpha\beta}}{M_W^2 - q^2} \right) \cdot g \cdot J^{\text{electr},\beta}_{\ldots} \rightarrow \mathcal{M} \sim \frac{G_F}{\sqrt{2}} J^{\text{Muon}\,\alpha} J^{\text{Elect}}_{\alpha}$$

with $\boxed{\dfrac{G}{\sqrt{2}} = \dfrac{g^2}{8M_W^2}}$

If one writes out the fermion currents one obtains:

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} \left(\bar{\nu}_{\mu,L}\, \gamma^\alpha\, \mu_L \right) \left(\bar{e}_L\, \gamma_\alpha\, \nu_{e,L} \right)$$

After summing over all spin configs in the final state and averaging over the spins of the initial state one obtains

$$\overline{|\mathcal{M}|^2} = 64\, G_F^2 \cdot (k \cdot p')(k' \cdot p)$$

Taking phase space into account one finally obtains for the decay width Γ ($1/\tau$) of the muon:

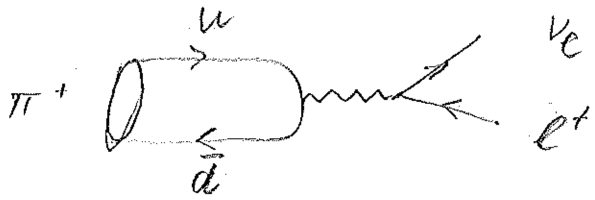
$$\Gamma = \frac{1}{\tau} = \frac{G_F^2 \cdot m_\mu^5}{192\,\pi^3}$$

From the muon lifetime and it's mass one obtains $G_F = 1{,}16 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$
(2,2 μs)

7.2 Pion decay

$\pi^+ \to \mu^+ \nu_\mu$

$\pi^+ \to e^+ \nu_e$



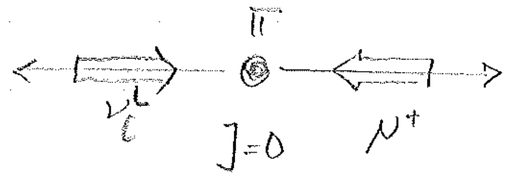
Text-book example of helicity suppression:

From phase space one would expect $BR(\pi^+ \to e^+\nu) \gg BR(\pi^+ \to \mu^+\nu)$
however one measures:
$$\frac{BR(\pi^+ \to e^+\nu)}{BR(\pi^+ \to \mu^+\nu)} \approx (1.230 \pm 0.004) \cdot 10^{-4}$$

(The Pion decays essentially only into μ, e decay suppressed!!)

Can be understood using angular momentum conservation

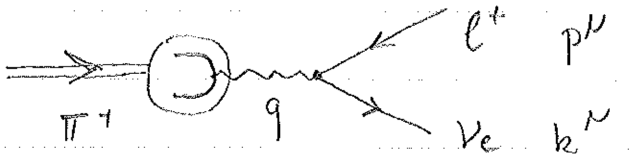
Spin $J(\pi) = 0$:



Due to angular momentum conservation and the LH ν
the μ^+ must have negative helicity (observable).
In the massless limit this is not possible.
However for massive leptons chirality flip is possible
and inserting a mass term would allow coupling to wrong chirality state

Reminder: [Helicity $H = \frac{\vec{S} \cdot \vec{p}}{|\vec{p}|} \rightsquigarrow H = \pm\frac{1}{2}$
Helicity of particle produced in weak IA: $-\frac{1}{2}\frac{v}{c}$]

<u>Problem:</u> Although quarks are the fundamental particles participating in the weak interaction it is the pion which is the asymptotic object.



Matrix element $\mathcal{M} \sim \frac{G_F}{\sqrt{2}} \cdot (\pi)_\mu \; (\text{Lepton})^\mu$

<u>Pion current:</u> With $J(\pi)=0$ the only relevant 4-Vector is the momentum transfer $q^\mu = p^\mu + k^\mu$ to the lepton system

$$(\pi)_\mu = q_\mu \cdot f_\pi$$

Normalisation is called pion decay constant (see below).

Matrix element considering the chirality flip:

$$\mathcal{M}(\pi \to \mu\nu) = -\sqrt{2}\, G_F \cdot f_\pi \cdot V_{ud}\; m_\mu \cdot (\bar{\mu}_R \nu_L)$$

Including the phase space one obtains for $\Gamma = \frac{1}{\tau}$:

$$\Gamma(\pi^+ \to \ell^+\nu_e) = \frac{G_F^2}{8\pi} \cdot |V_{ud}|^2 f_\pi^2 \cdot m_\ell^2 \cdot m_\pi \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2$$

$$\rightarrow \quad \frac{\Gamma(\pi^+ \to e^+\nu)}{\Gamma(\pi^+ \to \mu^+\nu)} = \left(\frac{m_e^2}{m_\mu^2}\right) \cdot \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.275 \cdot 10^{-4}$$

A theoretical calculation including radiative corrections leads to $(1.2354 \pm 0.0002)\,10^{-4}$ in perfect argument with measurement.