I. Flavor in the Standard Model

1. Fundamental particles

3 Generations of fundamental particles

	I	II	III	Q [e]
Quarks	u	c	t	+2/3
	d	s	b	-1/3
Leptons	ν_e	ν_μ	ν_τ	0
	e	μ	τ	-1

The 3 generations carry the same charges under the SM gauge group $SU(3)_c \times SU(2)_L \times U_Y(1)$.

Flavor physics: Difference between the 3 Generations and the phenomenology related to it. } e.g. Fermion masses

Remark: Chirality

Exp. observation: Only left-handed fermion components ψ_L participate in the weak charged current interaction.

→ W bosons only couple to LH fermion components (RH antifermions)

Parity is violated, maximally!

Fermion components:
$$\psi_L = P_L \psi = \frac{1}{2}(1-\gamma^5)\psi$$
$$\psi_R = P_R \psi = \frac{1}{2}(1+\gamma^5)\psi$$

with projectors P_L and P_R:
$$P_L^2 = P_L \qquad P_L P_R = P_R P_L = 0$$
$$P_R^2 = P_R \qquad P_L + P_R = 1$$

One finds the following decomposition:

(*) $\bar{\psi}\gamma^\mu\psi = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R$ fermion currents

$\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$ mass terms

with $\bar{\psi} = \psi^\dagger\gamma^0$

Exercise: Show (*)

[For QCD with n massless quarks:
Global chiral symmetry: $U(n)_L \times U(n)_R$ } broken by quark masses

In order to accomodate observed parity violation, left and right-handed quarks are assigned to different representations of $SU(2)_L$: LH fermions are doublets with weak isospin $\frac{1}{2} = T$
RH fermions are singlets; no weak charge

Exercise - Solution

1) $\bar{\psi}_L = \psi_L^\dagger\gamma^0 = \left(\psi^\dagger\frac{1}{2}(1-\gamma^5)\right)\gamma^0 = \psi^\dagger\gamma^0\frac{1}{2}(1+\gamma^5) = \bar{\psi}\,\frac{1}{2}(1+\gamma^5)$

$\gamma^5 = \gamma^{5\dagger}$; $\gamma^5\gamma^0 = -\gamma^0\gamma^5$

damit hat man: $\bar{\psi}_L\gamma^\mu\psi_R = \frac{1}{4}\bar{\psi}(1+\gamma^5)\gamma^\mu(1+\gamma^5)\psi$

$\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5 \rightarrow = \frac{1}{4}\bar{\psi}\gamma^\mu\underbrace{(1-\gamma^5)(1+\gamma^5)}_{=0}\psi = 0$

damit wird $\bar{\psi}\gamma^\mu\psi = (\bar{\psi}_L + \bar{\psi}_R)\gamma^\mu(\psi_L+\psi_R) = \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R$

2. Particle representations and charges under $SU(3)_c \times SU(2)_L \times U_Y(1)$

$Y = Q - T_3$

Each generation consists of the following five fermion multiplets

$Q_{Li}\ (3, 2, +1/6) = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$ LH Quark multiplett
↑ generation index

Triplet under $SU(3)_c$
Doublett under $SU(2)_L$ $T = \frac{1}{2}$
$U(1)$ Hypercharge $Y = +\frac{1}{6}$

$U_{Ri}\ (3, 1, +\frac{2}{3})$ — $SU(2)_L$ singlet $T = 0$

$d_{Ri}\ (3, 1, -\frac{1}{3})$

$L_{Li}\ (1, 2, -\frac{1}{2}) = \begin{pmatrix} \nu_{Li} \\ \ell_{Li} \end{pmatrix}$ Leptons are $SU(3)_c$ singlets
Isospin doublet and Hypercharge $-\frac{1}{2}$

$\ell_{Ri}\ (1, 1, -1)$

3) Yukawa coupling to Higgs and origin of mass

SM Lagrangian: $\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$

a) Kinetic term describes dynamics of fermions

$$\mathcal{L}_{Kinetic} = i \bar{\Psi} D^{\mu} \gamma_{\mu} \Psi$$

with the kovariant derivative

$$D^{\mu} = \partial^{\mu} + i g_s G_a^{\mu} \lambda_a + i g W_b^{\mu} \sigma_b + i g' B^{\mu} Y$$

G_a^{μ}, W_b^{μ} and B^{μ} are the $8 + 3 + 1$ gauge fields

- Kinetic term for LH quarks and leptons

$$\mathcal{L}^{quark}_{kinetic} = i\,\overline{Q}_{Li}\,\gamma_\mu\Big(\partial^\mu + i g_s G^\mu_a \lambda_a + \underset{T}{\tfrac{i}{2}} g W^\mu_b \tau_b + \underset{Y}{\tfrac{i}{6}} g' B^\mu\Big) Q_{Li}$$

$$\mathcal{L}^{Lepton}_{kinetic} = i\,\overline{L}_{Li}\,\gamma_\mu\Big(\partial^\mu + \tfrac{i}{2} g W^\mu_b \tau_b + \tfrac{i}{2} g' B^\mu\Big) L_{Li}$$

(expression for RH Quarks + Leptons less interesting: only NC!)

b) Higgs-Potential describing the scalar self-interactions:

$$\mathcal{L}^{Self}_{Higgs} = \underbrace{\mu^2 \phi^\dagger\phi - \frac{\lambda}{4}\left(\phi^\dagger\phi\right)^2}_{-V(\phi)}$$

(for $\mu^2 > 0$ and $\lambda > 0$ → Mexican hat potential)
$V(\phi) = -\mu^2 \phi^\dagger\phi + \frac{\lambda}{4}(\phi^\dagger\phi)^2$ ↳ Symmetric Breaking

c) Yukawa interaction with Higgs-Field:

→ Split into leptonic and baryonic (quark) part:

$$-\mathcal{L}^{lept}_{Yukawa} = Y^{\ell}_{ij} \cdot \overline{L}_{Li}\, \phi \cdot \ell_{Rj} + h.c.$$

After symmetry breaking Higgs acquires VEV $\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v + h(x)\end{pmatrix}$
→ lead to masses for the charged leptons

Lepton Yukawa matrix diagonal (no mixing) and involve 3 parameters: charged lepton masses. $m_{\ell i} = Y^{\ell}_{ii} \cdot \frac{v}{\sqrt{2}}$
Note that neutrinos stay massless in SM.

→ For the quarks one obtains:

$$-\mathcal{L}^{quarks}_{Yukawa} = Y^d_{ij}\, \bar{Q}_{Li}\, \phi\, d_{Rj} + Y^u_{ij}\, \bar{Q}_{Li}\, \tilde{\phi}\, u_{Rj} + h.c.$$

This is where the quark masses and the flavor mixing arises: The Yukawa matrices describing the Yukawa interaction are in general <u>complex</u> and <u>non-diagonal</u> (→ flavor structure)

After symmetry breaking and with the doublet $Q_{Li} = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$ one can rewrite the Yukawa interaction as:

$$-\mathcal{L}^{quarks}_{Yukawa} = -\frac{v}{\sqrt{2}} \left(\bar{d}_{Li}\, Y^{d}_{ij}{}'\, d_{Rj} + \bar{u}_{Li}\, Y^u_{ij}\, u_{Rj} \right) + h.c.$$

Mass eigenstates $\tilde{u}$ and $\tilde{d}$ of quarks can be obtained by a unitary transformations:

$$\tilde{u}_L = V_{L,u}\, u_L \qquad \tilde{d}_L = V_{L,d}\, d_L$$

$$\tilde{u}_R = V_{R,u}\, u_R \qquad \tilde{d}_R = V_{R,d}\, d_R$$

with all matrices: $V_{A,q} \cdot V^{+}_{Aq} = 1$

The matrices V_{Aq} are chosen such that the matrices

$$M_d = \frac{v}{\sqrt{2}} \cdot V_{Ld} \cdot Y^d \cdot V^{+}_{Rd} = \mathrm{diag}(m_d, m_s, m_b)$$

$$M_u = \frac{v}{\sqrt{2}}\, V_{Lu} \cdot Y^u \cdot V^{+}_{Ru} = \mathrm{diag}(m_u, m_c, m_t)$$

are diagonal.

After this transformation quark mass terms appear as usual Dirac terms:

$$\mathcal{L}^{quark}_{Yukawa} = \bar{\tilde{d}}_{Li} (M_d)_{ii} \tilde{d}_{Ri} + \bar{\tilde{u}}_{Li} (M_U)_{ii} \tilde{u}_{Ri}$$

→ leads to 6 quark masses: $m_u, \dots m_t$

However there is a non-vanishing effect on the charged currents:

→ u, d quarks appear together and are transformed by diff. $V_{L,q}$

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \left(\bar{u}_{Li} \gamma^\mu W^+_\mu d_{Li} + \bar{d}_{Li} \gamma^\mu W^-_\mu u_{Li} \right)$$

$u_L \rightsquigarrow \tilde{u}_{Li} = (V_{L,U})_{ij} u_{Lj}$ und $d_L \rightsquigarrow \tilde{d}_L = V_{L,d}\, d$

$u_{Li} = (V_{L,U})^+ \tilde{u}_{Li}$

↓

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \Big(\bar{\tilde{u}}_{Li} \gamma^\mu W^+_\mu \underbrace{(V_{L,U} \cdot V^+_{L,d})}_{V_{CKM}}{}_{ij} \tilde{d}_{Lj} + \dots$$

$$\bar{\tilde{d}}_{Li} \gamma^\mu W^-_\mu \underbrace{(V_{L,d} V^+_{L,U})}_{V^+_{CKM}}{}_{ij} \tilde{u}_{Lj}$$

$\bar{\tilde{u}} = \tilde{u}^+ \gamma^0$

There is no net effect on the neutral currents.

With $(u_1, u_2, u_3) = (u, c, t)$ und $(d_1, d_2, d_3) = (d, s, b)$

This choice: $d_i = (V_{CKM})_{ij} \tilde{d}_j$ → weak state are mixtures of mass; $u_i = \tilde{u}_i$

$$\underset{\text{interaction eigenstates}}{\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} \tilde{d}_L \\ \tilde{s}_L \\ \tilde{b}_L \end{pmatrix} \leftarrow \text{Mass eigenstates}$$

Since V_{CKM} is product of unitary matrices V_{CKM} is unitary: $V_{CKM} V^+_{CKM} = 1$

Bem.: For the rest of the lecture I will use d and u for the mass (flavor) eigenstates and d′ for the weak eigenstate.