

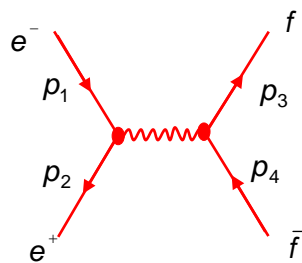
4. Fermion pair production in e⁺e⁻ annihilation

1. QED prediction
2. Experimental methods
3. $e^+e^- \rightarrow \mu^+\mu^-$
4. Bhabha scattering
5. Discovery of the Tau-Lepton
6. $e^+e^- \rightarrow$ hadrons
7. Hadronic resonances

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4. Fermion pair production in e⁺e⁻ annihilation

$$e^+e^- \rightarrow f\bar{f}$$

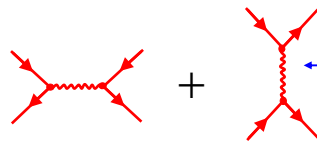


$$f\bar{f} = e^+e^-, \mu^+\mu^-, \tau^+\tau^-$$

$$= \underbrace{u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}}_{\text{Observed as hadrons}}$$

Observed as hadrons

In case of electron pair production there are two interfering amplitudes:



Nothing really new:
1/q⁴ term (Rutherford+spin)

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4.1 QED prediction

$e^+ e^- \rightarrow \mu^+ \mu^-$

flux
 $F = 2E_1 2E_2 \frac{|\vec{p}_1|}{E_1}$

$$d\sigma = \frac{|M_{fi}|^2}{F} \cdot (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \cdot \frac{d^3 p_3}{2E_3 (2\pi)^3} \cdot \frac{d^3 p_4}{2E_4 (2\pi)^3}$$

$$T_{fi} = \langle \mu^+(p_4) \mu^-(p_3) | \mathbf{S} | e^+(p_2) e^-(p_1) \rangle = -i \cdot (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) M_{fi}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot |M_{fi}|^2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \underbrace{\frac{|\vec{p}_3|}{|\vec{p}_1|}}_{\approx 1} \cdot |M_{fi}|^2$$

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Lorentz invariant transition amplitude M_{fi}

Reminder:
 $J^v = -ie \bar{\psi} \gamma^v \psi$

electron current
 $J_e^v = -ie \bar{v}_e \gamma^v u_e$

muon current
 $J_{muon}^v = -ie \bar{u}_\mu \gamma^v v_\mu$

$$M_{fi} = \underbrace{(ie)^2 \bar{u}_\mu(p_3) \gamma^v v_\mu(p_4)}_{J_{muon}^v} \frac{1}{q^2} \underbrace{\bar{v}_e(p_2) \gamma_v u_e(p_1)}_{J_e^v}$$

See next lecture by J.Pawlowski.

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Spin states

$$M_{fi} = (ie)^2 \bar{u}_\mu(p_3) \gamma^\nu v_\mu(p_4) \frac{1}{q^2} \bar{v}_e(p_3) \gamma_\nu u_e(p_4)$$

This result is true only for one specific spin configuration.

For non-polarized ingoing particles and for non-observation of final state spin one observes unpolarized cross sections \Rightarrow need to **average over possible initial spin states** and **sum over all final spin states**.

$$\overline{|M_{fi}|^2} = \frac{1}{(2s_e + 1)(2s'_e + 1)} \cdot \sum_{s_e, s'_e} \sum_{r_\mu, r'_\mu} |M_{fi}|^2$$

See next lecture by J.Pawlowski.

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See next lecture by J.Pawlowski.

Averaging and summing over spins of initial/final state:

$$\begin{aligned} \overline{|M_{fi}|^2} &= \frac{1}{4} \frac{e^4}{s^2} \sum_{s, s', r, r'} |\bar{u}_{\mu, s}(p_3) \gamma^\nu v_{\mu, s'}(p_4) \bar{v}_{e, r}(p_2) \gamma_\nu u_{e, r'}(p_1)|^2 \\ &\hspace{15em} \text{neglect masses} \\ &= 4 \frac{e^4}{s^2} \left[(p_1 p_4)(p_2 p_3) + (p_2 p_4)(p_1 p_3) \right] \hspace{2em} \text{Lorentz invariant!} \end{aligned}$$

By using the Mandelstam variables in the relativistic limit

$$\left. \begin{aligned} s &= (p_1 + p_2)^2 = m^2 + m^2 + 2p_1 p_2 \approx 2p_1 p_2 \approx 2p_3 p_4 \\ t &= (p_1 - p_3)^2 = m^2 + M^2 - 2p_1 p_3 \approx -2p_1 p_3 \approx -2p_2 p_4 \\ u &= (p_1 - p_4)^2 = m^2 + M^2 - 2p_1 p_4 \approx -2p_1 p_4 \approx -2p_2 p_3 \end{aligned} \right\} \text{if masses neglected}$$

$$\overline{|M_{fi}|^2} = 2 e^4 \frac{t^2 + u^2}{s^2}$$

Remember: matrix element squared can be expressed in s, u, t!

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III. Introduction to QED

$$\overline{|M|^2}_{e^+e^- \rightarrow \mu^+\mu^-}(s, t, u) = 2e^4 \frac{t^2 + u^2}{s^2}$$

$$\downarrow \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot |M_{fi}|^2$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2} \cdot \frac{1}{s} \cdot \frac{t^2 + u^2}{s^2}$$

$$= \frac{e^4}{64\pi^2} \cdot \frac{1}{s} \cdot (1 + \cos^2 \theta)$$

$$\downarrow e^2 = 4\pi\alpha$$

$$\left. \frac{d\sigma}{d\Omega} \right|_{CMS} = \frac{\alpha^2}{4s} \cdot (1 + \cos^2 \theta)$$

Kinematics for high-relativistic particles

CMS

$s = (k + k')^2 \approx 4E_i^2$
 $t = (k - p)^2 \approx -2kp \approx -2E_i^2(1 - \cos \theta^*)$
 $\approx -\frac{s}{2}(1 + \cos \theta)$
 $u = (k - p')^2 \approx -2kp' \approx -2E_i^2(1 - \cos \theta)$
 $\approx -\frac{s}{2}(1 - \cos \theta)$

← 1/s dependence from flux factor

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$$\left. \frac{d\sigma}{d\Omega} \right|_{CMS} = \frac{\alpha^2}{4s} \cdot (1 + \cos^2 \theta)$$

$$\sigma_{tot} = \frac{4\pi\alpha^2}{3s} = \frac{86.86 \text{ nb GeV}^2}{s}$$

Fig. 6.6 The total cross section for $e^-e^+ \rightarrow \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

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4.2 Experimental methods

e⁺e⁻ accelerators (personal selection)

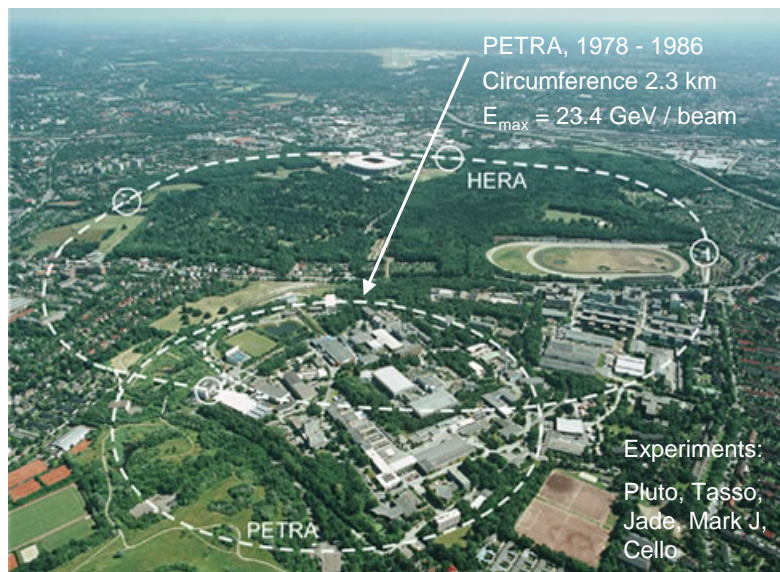
Accelerator	Lab	\sqrt{s}	$L_{\text{int}} / \text{Exper.}$	
SPEAR	SLAC	2 - 8 GeV		
PEP	SLAC	→29 GeV	220 - 300 pb ⁻¹	
PETRA	DESY	12 - 47 GeV	~20 pb ⁻¹	today
TRISTAN	KEK	50 - 60 GeV	~20 pb ⁻¹	
LEP	CERN	90 GeV	~200 pb ⁻¹	Z physics

Cross section (experimental definition)

$$\sigma(e^+e^- \rightarrow f\bar{f}) = \frac{N_{ff}(1-b)}{\varepsilon L_{\text{int}}}$$

- N_{ff} number of detected $e^+e^- \rightarrow ff$ events
- b background fraction
- ε acceptance / efficiency
- L_{int} integrated luminosity of collider

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III. Introduction to QED

Particle detectors

A detector cross-section, showing particle paths

- Beam Pipe (center)
- Tracking Chamber
- Magnet Coil
- E-M Calorimeter
- Hadron Calorimeter
- Magnetized Iron
- Muon Chambers

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MAGNETDETEKTOR JADE
MAGNET DETECTOR

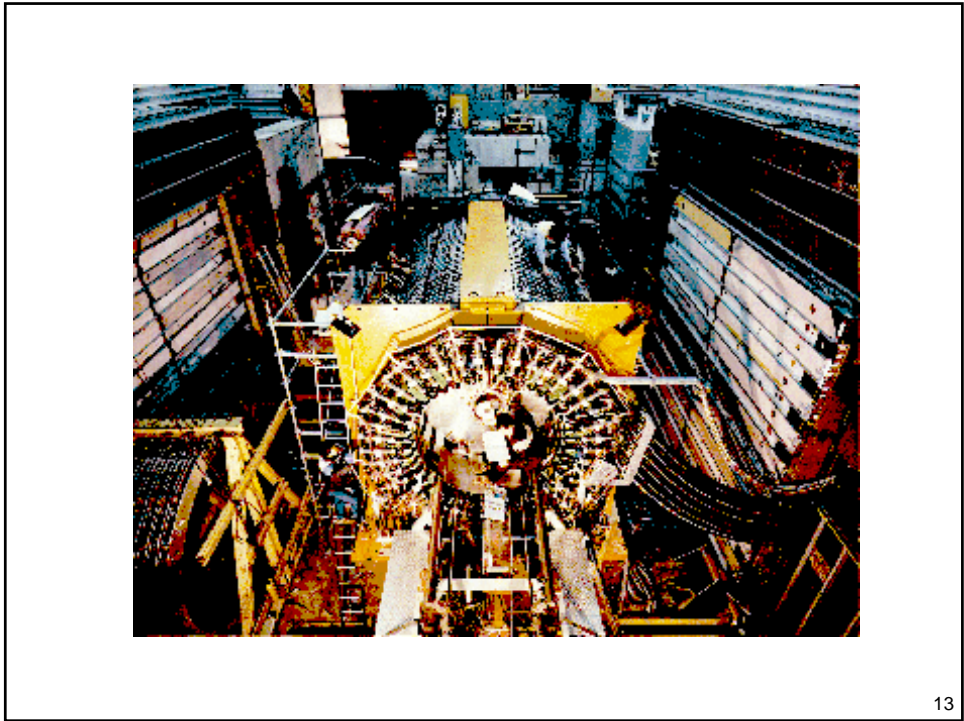
Japan – Deutschland – England

- 1 Strahlrohrzähler BEAM PIPE COUNTERS
- 2 Endseitige Bleiglaszähler END PLUG LEAD GLASS COUNTERS
- 3 Drucktank PRESSURE TANK
- 4 Myon-Kammern MUON CHAMBERS
- 5 Jet-Kammern JET CHAMBERS
- 6 Flugzeit-Zähler TIME OF FLIGHT COUNTERS
- 7 Spule COIL
- 8 Zentrale Bleiglaszähler CENTRAL LEAD GLASS COUNTERS
- 9 Magnetjoch MAGNET YOK
- 10 Myon-Filter MUON FILTERS
- 11 Beweglicher Endstopfen REMOVABLE END PLUG
- 12 Strahlrohr BEAM PIPE
- 13 Vorwärts-Detektor TAGGING COUNTER
- 14 Mini-Beta Quadrupol MINI BETA QUADRUPOLE
- 15 Fahrwerk MOVING DEVICES

Gesamtgewicht TOTAL WEIGHT: ~1200 t
 Magnetfeld MAGNETIC FIELD: 0.5 T

Beteiligte Institute PARTICIPANTS
 DESY, Hamburg, Heidelberg,
 Lancaster, Manchester,
 Rutherford Lab., Tokio

III. Introduction to QED



Experimental Signatures:

e^- \rightarrow f
 e^+ \rightarrow \bar{f}

$f \bar{f} = e^- e^+$

$\mu^- \mu^+$
 $\tau^- \tau^+$

$q \bar{q}$ mit
 $q = u, d, s, c, b, (t)$

Hadron jets

$\mu^- \mu^+$
 $q \bar{q}$

OPAL / LEP

$e^- e^+$
 $\mu^- \mu^+$
 $q \bar{q}$

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III. Introduction to QED

Determination of integrated luminosity

$$L_{\text{int}} = \int L_{ee}(t) dt$$

small angle Bhabha scattering
(low momentum transfer, QED works !!):

$e^+e^- \rightarrow e^+e^-(\gamma)$

$L_{\text{int}} = \frac{N_{ff}(1-b)}{\mathcal{E} L_{\text{int}}}$

Small angle Bhabha scattering is t channel dominated: theoretical cross section σ_{theo} well known.

$$L_{\text{int}} = \frac{N_{ee}}{\sigma_{\text{theo}} \mathcal{E}}$$

At LEP:
typ. errors < 0.5%

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4.3 $e^+e^- \rightarrow \mu^+\mu^-$

Good agreement with QED!
Quantitative limit for new physics ?

Effect of bremsstrahlung:

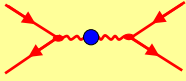
There will always be additional photons

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III. Introduction to QED

Possible deviation from QED:

- additional heavy photon



Modifies propagator

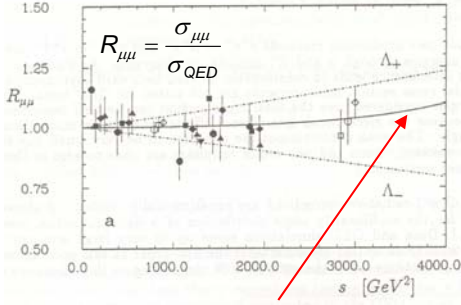
$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} - \frac{1}{q^2 - \Lambda^2} = \frac{1}{q^2} \left(1 - \frac{q^2}{q^2 - \Lambda^2}\right) \approx \frac{1}{q^2} \left(1 + \frac{q^2}{\Lambda^2}\right)$$

Λ corresponds to the mass of new photon

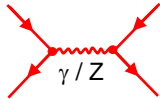
To also account for possible lower cross sections:

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} \left(1 \mp \frac{q^2}{q^2 - \Lambda_{\pm}^2}\right)$$

Meaning of Λ_{\pm} not clear



$R_{\mu\mu} = \frac{\sigma_{\mu\mu}}{\sigma_{QED}}$



$\frac{\sigma_{\mu\mu}^{\gamma+Z}}{\sigma_{QED}}$

Additional heavy photon:

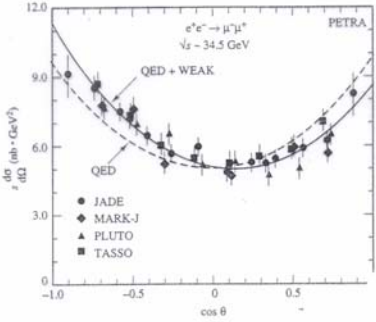
$$\sigma_{\mu\mu} = \frac{4\pi\alpha^2}{s} \left(1 \mp \frac{s}{s - \Lambda_{\pm}^2}\right)^2$$

$\rightarrow \Lambda_{\pm} > 200 \text{ GeV}$

Confirms "Coulomb law" down to 10^{-18} m

Effect of Z boson exchange

„heavy photon w/ different couplings“



$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \cdot (1 + \cos^2 \theta) \Big|_{QED}$

$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \cdot (1 + \cos^2 \theta + A \cos \theta) \Big|_{\gamma+Z}$

Clear deviation from QED:
 \Rightarrow Effect of electro-weak γ/Z interference

The effect of the "heavy" Z boson is already seen at low energies!