4. Fermion pair production in e+e- annhihilation

1. QED prediction
2. Experimental methods
3. $e^+e^- \to \mu^+\mu^-$
4. Bhabha scattering
5. Discovery of the Tau-Lepton
6. $e^+e^- \to$ hadrons
7. Hadronic resonances

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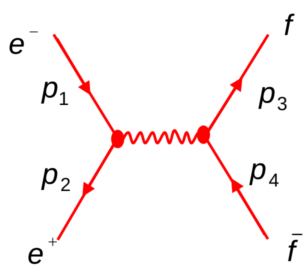
4. Fermion pair production in e^+e^- annhihilation

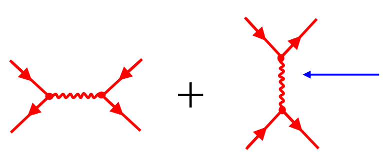
$$e^+e^- \to f\bar{f}$$



$$f\bar{f} = e^+e^-, \mu^+\mu^-, \tau^+\tau^-$$

$$= \underbrace{u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}}_{\text{Observed as hadrons}}$$

In case of electron pair production there are two interfering amplitudes:



Nothing really new:
$1/q^4$ term (Rutherford+spin)

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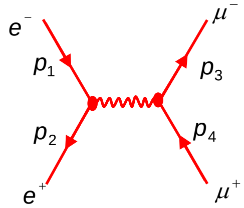
4.1 QED prediction

$$e^+e^- \to \mu^+\mu^-$$



$$d\sigma = \frac{|M_{fi}|^2}{F} \cdot (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \cdot \frac{d^3p_3}{2E_3(2\pi)^3} \cdot \frac{d^3p_4}{2E_4(2\pi)^3}$$

flux

$$F = 2E_1 2E_2 \frac{|\vec{p}_1|}{E_1}$$

$$T_{fi} = \langle \mu^+(p_4)\mu^-(p_3) | \mathbf{S} | e^+(p_2)e^-(p_1) \rangle = -i \cdot (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) M_{fi}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot |M_{fi}|^2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \underbrace{\frac{|\vec{p}_3|}{|\vec{p}_1|}}_{\approx 1} \cdot |M_{fi}|^2$$

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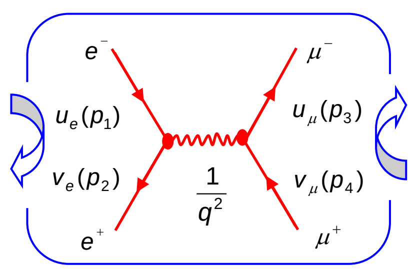
Lorentz invariant transition amplitude M_{fi}

Reminder:

$$J^\nu = -ie\overline{\psi}\gamma^\nu\psi$$

electron current

$$J_e^\nu = -ie\overline{v}_e\gamma^\nu u_e$$



muon current

$$J_{muon}^\nu = -ie\overline{u}_\mu\gamma^\nu v_\mu$$

$$M_{fi} = (ie)^2 \underbrace{\overline{u}_\mu(p_3)\gamma^\nu v_\mu(p_4)}_{J_{muon}^\nu} \frac{1}{q^2} \underbrace{\overline{v}_e(p_3)\gamma_\nu u_e(p_4)}_{J_{e\,\nu}}$$

See next lecture by J.Pawlowski.

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Spin states

$$M_{fi} = (ie)^2\, \bar{u}_\mu(p_3)\gamma^\nu v_\mu(p_4)\frac{1}{q^2}\bar{v}_e(p_3)\gamma_\nu u_e(p_4)$$

This result is true only for one specific spin configuration.

For non-polarized ingoing particles and for non-observation of final state spin one observes unpolarized cross sections $\Rightarrow$ need to **average over possible initial spin states** and **sum over all final spin states**.

$$\overline{|M_{fi}|^2} = \frac{1}{(2s_e+1)(2s_e'+1)} \cdot \sum_{s_e,s_e'}\sum_{r_\mu,r_\mu'} |M_{fi}|^2$$

See next lecture by J.Pawlowski.

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See next lecture by J.Pawlowski.

Averaging and summing over spins of initial/final state:

$$\overline{|M_{fi}|^2} = \frac{1}{4}\frac{e^4}{s^2} \sum_{s,s',r,r'} |\bar{u}_{\mu,s}(p_3)\gamma^\nu v_{\mu,s'}(p_4)\ \bar{v}_{e,r}(p_2)\gamma_\nu u_{e,r'}(p_1)|^2$$

neglect masses

$$= 4\frac{e^4}{s^2}\left[(p_1p_4)(p_2p_3) + (p_2p_4)(p_1p_3)\right]$$

Lorentz invariant!

By using the Mandelstam variables in the relativistic limit

$$s = (p_1+p_2)^2 = m^2 + m^2 + 2p_1p_2 \approx 2p_1p_2 \approx 2p_3p_4$$
$$t = (p_1-p_3)^2 = m^2 + M^2 - 2p_1p_3 \approx -2p_1p_3 \approx -2p_2p_4$$
$$u = (p_1-p_4)^2 = m^2 + M^2 - 2p_1p_4 \approx -2p_1p_4 \approx -2p_2p_3$$

if masses neglected

$$\overline{|M_{fi}|^2} = 2e^4\frac{t^2+u^2}{s^2}$$

Remember: matrix element squared can be expressed in s, u, t!

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$$\overline{|M|^2}_{e^+e^-\to\mu^+\mu^-}(s,t,u) = 2e^4\,\frac{t^2+u^2}{s^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\cdot\frac{1}{s}\cdot|M_{fi}|^2$$

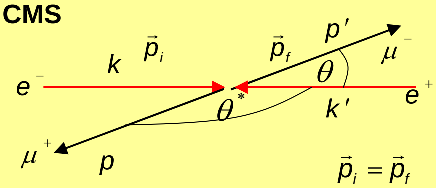
$$\frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2}\cdot\frac{1}{s}\cdot\frac{t^2+u^2}{s^2} = \frac{e^4}{64\pi^2}\cdot\frac{1}{s}\cdot(1+\cos^2\theta)$$

$$e^2 = 4\pi\alpha$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CMS} = \frac{\alpha^2}{4s}\cdot(1+\cos^2\theta)$$

Kinematics for high-relativistic particles

CMS



$\vec{p}_i = \vec{p}_f$

$$s = (k+k')^2 \approx 4E_i^2$$

$$t = (k-p)^2 \approx -2kp \approx -2E_i^2(1-\cos\theta^*) \approx -\frac{s}{2}(1+\cos\theta)$$

$$u = (k-p')^2 \approx -2kp' \approx -2E_i^2(1-\cos\theta) \approx -\frac{s}{2}(1-\cos\theta)$$

← 1/s dependence from flux factor

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$$\left.\frac{d\sigma}{d\Omega}\right|_{CMS} = \frac{\alpha^2}{4s}\cdot(1+\cos^2\theta)$$

$$\sigma_{tot} = \frac{4\pi\alpha^2}{3s} = \frac{86.86\,\text{nb}\,\text{GeV}^2}{s}$$

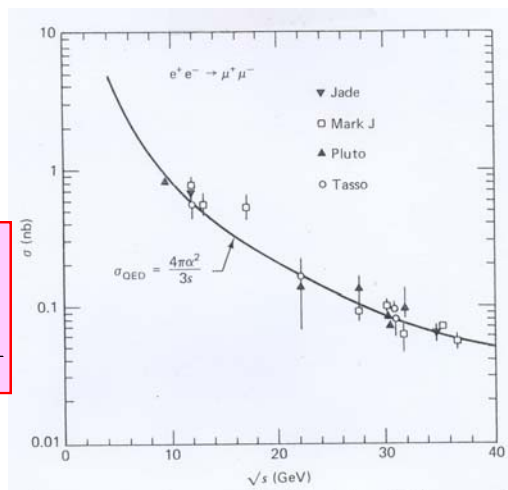

Fig. 6.6 The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

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4.2 Experimental methods

<u>e^+e^- accelerators</u> (personal selection)

Accelerator	Lab	$\sqrt{s}$	L_{int} / Exper.	
SPEAR	SLAC	2 – 8 GeV		
PEP	SLAC	→29 GeV	220 - 300 pb^{-1}	
PETRA	DESY	12 - 47 GeV	~20 pb^{-1}	today
TRISTAN	KEK	50 – 60 GeV	~20 pb^{-1}	
LEP	CERN	90 GeV	~200 pb^{-1}	Z physics

<u>Cross section</u> (experimental definition)

$$\sigma(e^+e^- \to f\,\bar{f}) = \frac{N_{ff}(1-b)}{\varepsilon\, L_{int}}$$

- N_{ff} number of detected $e^+e^- \to$ ff events
- b background fraction
- ε acceptance / efficiency
- L_{int} integrated luminosity of collider

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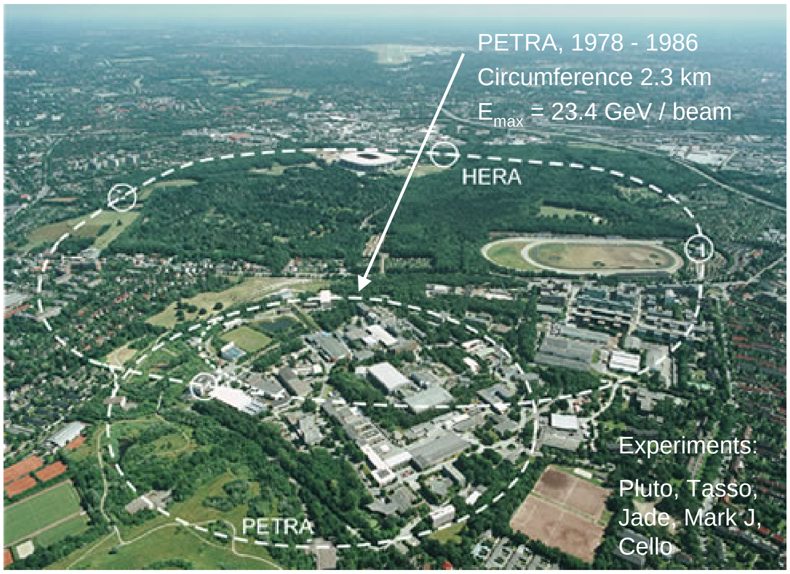

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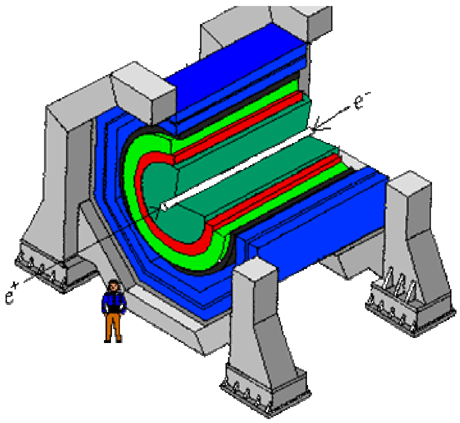
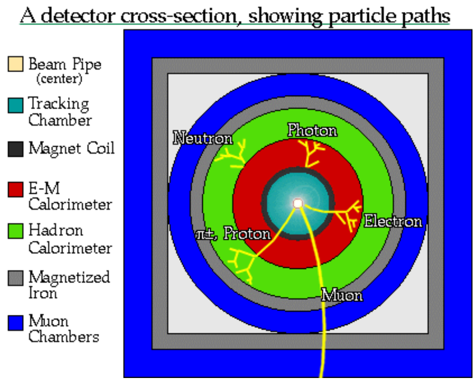
Particle detectors





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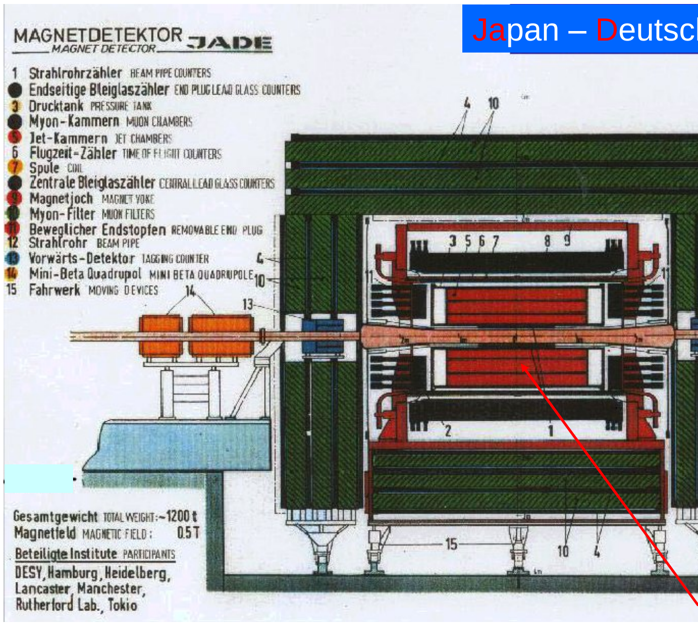
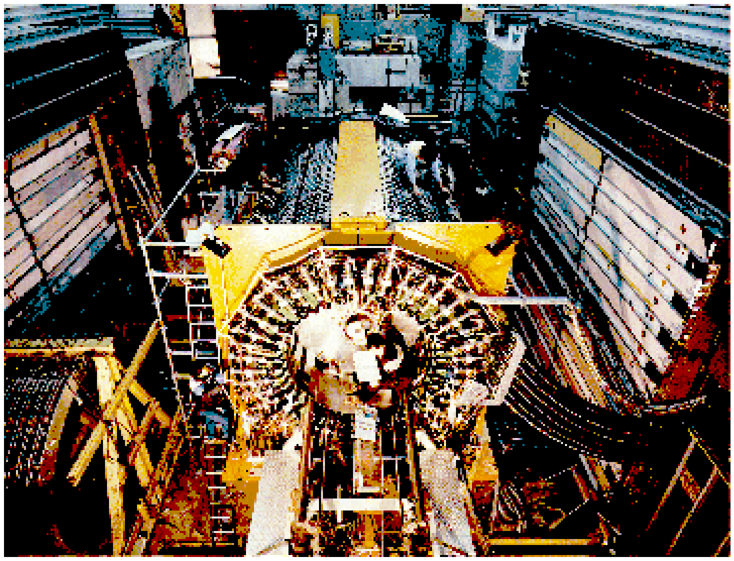
Japan – Deutschland – England





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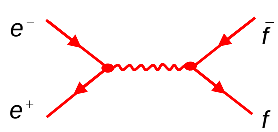
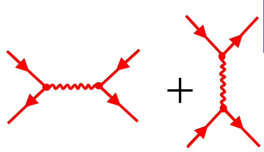
Experimental Signatures:

OPAL / LEP

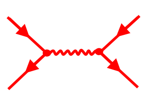


$f\,\bar{f} = e^- e^+$

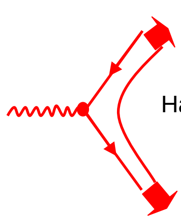


$\mu^- \mu^+$

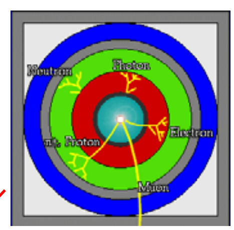
$\tau^- \tau^+$

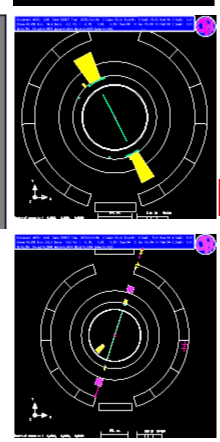


$q\,\bar{q}$ mit
$q = u,d,s,c,b,(t)$



Hadron jets

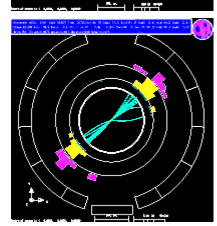




$e^- e^+$

$\mu^- \mu^+$

$q\,\bar{q}$



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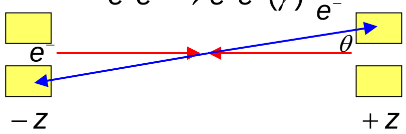
Determination of integrated luminosity

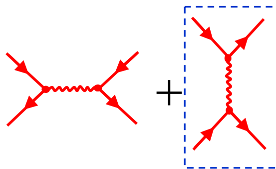
$$\sigma(e^+e^- \to f\,\bar{f}) = \frac{N_{ff}(1-b)}{\varepsilon\, L_{int}}$$

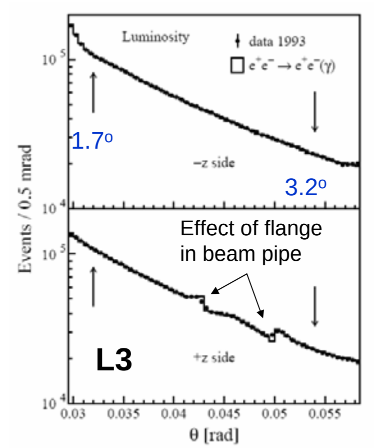
$$L_{int} = \int L_{ee}(t)dt$$

small angle Bhabha scattering
(low momentum transfer, QED works !!):

$$e^+e^- \to e^+e^-(\gamma)$$







Small angle Bhabha scattering is t channel dominated: theoretical cross section σ_{theo} well known.

$$L_{int} = \frac{N_{ee}}{\sigma_{theo}\,\varepsilon}$$

At LEP:
typ. errors < 0.5%

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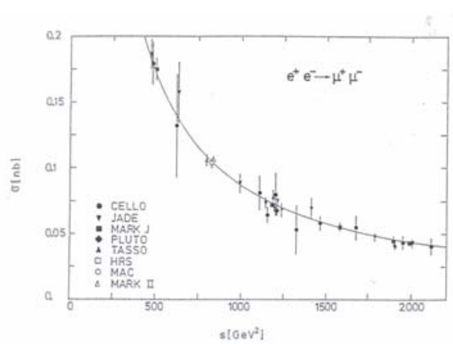
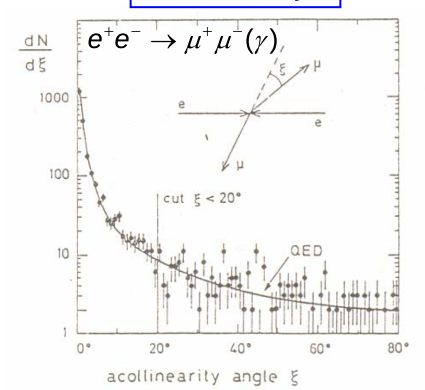
4.3 $e^+e^- \to \mu^+\mu^-$



Acollinearity

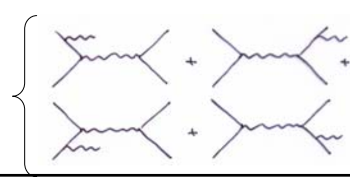


Good agreement with QED!

Quantitative limit for new physics ?

Effect of bremsstrahlung:

There will always be additional photons



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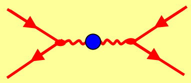
<u>Possible deviation from QED:</u>

- additonal heavy photon



Modifies propagator

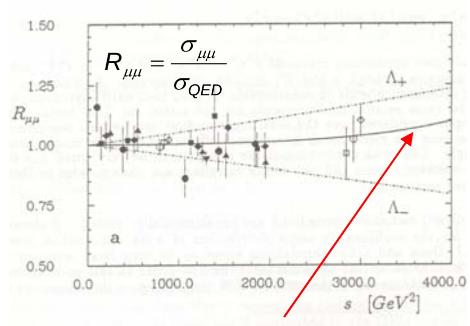
$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} - \frac{1}{q^2-\Lambda^2} = \frac{1}{q^2}\left(1-\frac{q^2}{q^2-\Lambda^2}\right) \approx \frac{1}{q^2}\left(1+\frac{q^2}{\Lambda^2}\right)$$

Λ corresponds to the mass of new photon

To also account for possible lower cross sections:

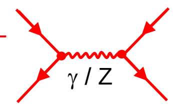
$$\frac{1}{q^2} \rightarrow \frac{1}{q^2}\left(1 \mp \frac{q^2}{q^2-\Lambda_\pm^2}\right)$$

Meaning of Λ_- not clear



$$R_{\mu\mu} = \frac{\sigma_{\mu\mu}}{\sigma_{QED}}$$

$$\frac{\sigma_{\mu\mu}^{\gamma+Z}}{\sigma_{QED}} \leftarrow$$



Additional heavy photon:

$$\sigma_{\mu\mu} = \frac{4\pi\alpha^2}{s}\left(1 \mp \frac{s}{s-\Lambda_\pm}\right)^2$$

$\rightarrow \Lambda_\pm > 200\,\text{GeV}$

Confirms "Coulomb law" down to 10^{-18} m

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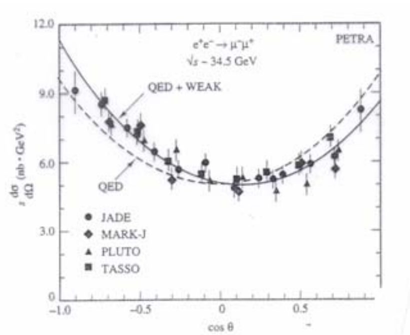
Effect of Z boson exchange

„heavy photon w/ different couplings"



$$\left.\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\cdot(1+\cos^2\theta)\right|_{QED}$$

$$\left.\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\cdot(1+\cos^2\theta + A\cos\theta)\right|_{\gamma+Z}$$

Clear deviation from QED:

$\Rightarrow$Effect of electro-weak γ/Z interference

The effect of the "heavy" Z boson is already seen at low energies!

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