IV Electroweak Theory
(Quantum Flavourdynamics, QFD)

1930 Pauli, suggestion of neutrino ν
discovery Reines 53-59

1933-34 Fermi, Theory of β-decay

$$n \to p + e^- + \bar{\nu}_e$$

Fermi interaction:

$$H = G \int d^3x \, [\bar{p}(x) \gamma^\mu n(x)] [\bar{e}(x) \gamma_\mu \nu(x)] + h.c.$$

Fermi constant $\simeq 1.1 \cdot 10^{-5}$ GeV^{-2}

Important: parity violation in β-decay!

$$H = G_\beta/\sqrt{2} \, [\bar{p}(x) \gamma^\mu (1 - g_A/g_V \gamma_5) n(x)] [\bar{e}(x) \gamma_\mu (1-\gamma_5) \nu(x)] + h.c.$$

$$G_\beta = 1.147 \cdot 10^{-5} \text{ GeV}^{-2}$$

$$g_A/g_V = 1.255$$

- weak interaction distinguishes between left- and right-handed particles

- Universality of weak interaction

γ_5 and handedness
Fermions revisited:

$$u(p) = \sqrt{p^0+m}\begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{p_0+m}\chi_s \end{pmatrix} \quad \text{with} \quad \chi_{1/2} = \begin{pmatrix}1\\0\end{pmatrix}, \quad \chi_{-1/2} = \begin{pmatrix}0\\1\end{pmatrix}$$

with standard rep. $\gamma^0 = \begin{pmatrix}1\!\!1_2 & 0\\ 0 & -1\!\!1_2\end{pmatrix}$, $\gamma^i = \begin{pmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix}$, $\gamma_5 = \begin{pmatrix}0 & 1\!\!1_2\\ 1\!\!1_2 & 0\end{pmatrix}$

$m = 0$
$$= \sqrt{p}\begin{pmatrix}\chi_s \\ \vec{\sigma}\cdot\hat{p}\,\chi_s\end{pmatrix}$$
$|\vec{p}| = p_0 = p$

$\hat{p} = \vec{p}/p$

Spin-orientation: $\vec{\sigma}\cdot\hat{p}\,\chi_\pm = \pm\chi_\pm$ ← Helicity
Helicity

Define: $u_\pm(p) = \sqrt{p}\begin{pmatrix}\chi_\pm \\ \pm\chi_\pm\end{pmatrix}$

$m=0$: with $\gamma_5 u_\pm(p) = \pm u_\pm(p)$

We define left- and right-handed spinors

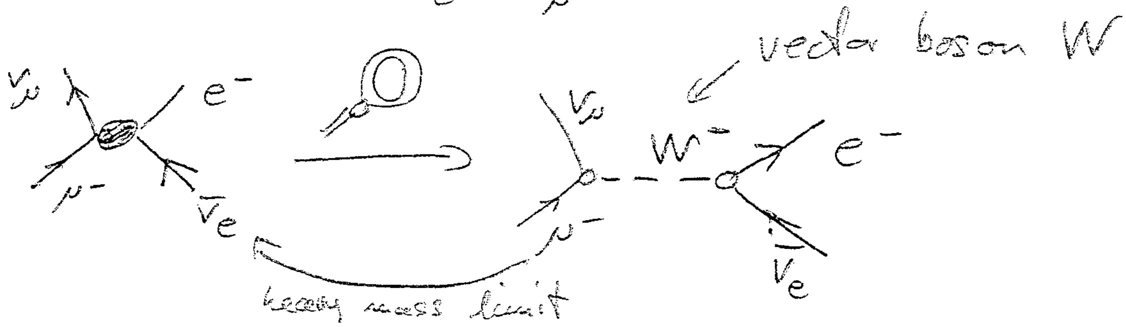
$$\psi_{L/R} = \frac{1\mp\gamma_5}{2}\psi \overset{m=0}{=} \psi_\mp$$

with $\gamma_5\psi_{L/R} = \mp\psi_{L/R}$
↳ chirality

Lagrange density of electroweak theory

Fermi interaction via gauge theory:

Consider $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$



Gauge principle: (for the time being we consider massless fermions)

Leptons: $\Psi_e = \begin{pmatrix} \psi_{\nu_e} \\ \psi_e \end{pmatrix}$, $\Psi_\mu = \begin{pmatrix} \psi_{\nu_\mu} \\ \psi_\mu \end{pmatrix}$, $\Psi_\tau = \begin{pmatrix} \psi_{\nu_\tau} \\ \psi_\tau \end{pmatrix}$

quarks: Ψ_q : $\begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$, $\begin{pmatrix} \psi_c \\ \psi_s \end{pmatrix}$, $\begin{pmatrix} \psi_t \\ \psi_b \end{pmatrix}$

free Lagrangian: (electron)

necessary because of QED

$$\mathcal{L}_0(x) = \bar{\Psi}_{eL}\, i\gamma^\mu \partial_\mu \Psi_{eL} + \bar{\psi}_{e_R}\, i\gamma^\mu \partial_\mu \psi_{e_R}(x)$$

$\mathcal{L}_0$ is invariant under global

$SU(2)$ rotations of Ψ_L:

$$\Psi_L \to U\,\Psi_L \qquad \text{with } U = e^{i\omega} \in SU(2)$$

$$\omega = \omega^a \sigma^a/2$$

singlet Ψ_R: $\Psi_R \to \Psi_R$

$\sigma^a/2$ are the generators of $SU(2)$ with Lie-algebra

$$[\sigma^a/2, \sigma^b/2] = i\varepsilon^{abc}\sigma^c/2 \quad , \varepsilon^{123} = 1$$

and Pauli-matrices (p. 2-6): $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Local symmetry (gauging) via minimal coupling

$$\mathcal{L}_0 \to \mathcal{L}(x) = \overline{\Psi_{e_L}}\, i\gamma^\mu D_\mu \Psi_{e_L} + \overline{\Psi}_{e_R}\, i\gamma^\mu \partial_\mu \Psi_{e_R}$$

with $$D_\mu = \partial_\mu + i g W_\mu$$

$$W_\mu = W_\mu^a \sigma^a/2$$

and gauge transformations

$$W_\nu(x) \to \mathcal{U}(x)\, W_\nu(x)\, \mathcal{U}^\dagger(x) - i/g\ \mathcal{U}(x)\, \partial_\nu\, \mathcal{U}^\dagger(x)$$

$$= -i/g\ \mathcal{U}(x)\, \mathcal{D}_\nu\, \mathcal{U}^\dagger(x)$$

$$\Psi(x) \to e^{i\,\omega(x)(1-\gamma_5)/2}\ \Psi(x)$$

$$= \begin{pmatrix} \mathcal{U}(x)\ \psi_L \\ \psi_R \end{pmatrix}$$

with $\Psi(x) = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$. Within this notation

$\mathcal{L}$ reads

$$\mathcal{L}(x) = \overline{\Psi}\ i\gamma^\nu \mathcal{D}_\nu\, \Psi$$

with

$$\mathcal{D}_\nu = \partial_\nu + i g\, W_\nu \quad , \quad W_\nu = W_\nu{}^a \cdot T^a$$

$$T^a = \begin{pmatrix} \sigma^a/2 & 0 \\ 0 & 0 \end{pmatrix}$$

Coupling via

$$W_\nu{}^a \sigma^a = \begin{pmatrix} W_\nu^3 & W_\nu^1 - i W_\nu^2 \\ W_\nu^1 + i W_\nu^2 & -W_\nu^3 \end{pmatrix}$$

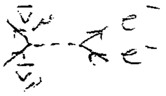
charged

neutral

neutral gauge boson W_μ^3 :

- is not the photon: no L-R symmetry
  $\Rightarrow$ add. U(1)-gauge boson $\sim Z^0$ (triumph of theory)
- is not Z^0 : no coupling to right-handed fermions

(existence of neutral currents)

e.g. $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$ 

or $\nu_\mu + n \to \nu_\mu + n$

Consider

$$W_\mu^3 = \cos\Theta_W \, Z_\mu^0 + \sin\Theta_W \, A_\mu$$

photon

with Weinberg angle (weak mixing angle)

$$\sin^2\Theta_W = 0.23117(6)$$

SLAC: 0.23098 ± 0.00028

Orthogonal combination

$$B_\mu = -\sin\Theta_W \, Z_\mu^0 + \cos\Theta_W \, A_\mu$$

with U(1)-gauge transformation

$$\left.\begin{array}{l} \Psi_L \to e^{i Y_L \omega} \\ \Psi_R \to e^{i Y_R \omega} \end{array}\right\} \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} \to e^{i Y \omega} \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix}$$

with $Y = \begin{pmatrix} Y_L & & \\ & Y_L & \\ & & Y_R \end{pmatrix}$

Hypercharge

B_μ commutes with W_μ !

$$\boxed{SU(2) \times U(1)}$$

Hypercharge Y:

$$Y = Q - I_3$$

electric charge in |e| — 3rd isospin component

right-handed fermions: $Y = Q$

left-handed fermions: e.g. $\nu_L : Y = -1/2$ $\quad (Q\ \ I_3)$ $(0\ \ +1/2)$

$e_L : Y = -1/2$ $\quad (-1 - -1/2)$

$u_L : Y = 1/6$ $\quad (2/3 - +1/2)$

$d_L : Y = 1/6$ $\quad (-1/3 - -1/2)$

Interaction term: $\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$

$$\overline{\Psi}\, i\gamma^\mu D_\mu \Psi$$

with $D_\mu = \partial_\mu + ig W_\mu + i g' B_\mu Y$

where $W_\mu = W_\mu^a T^a$ $\quad , \quad T^a = \begin{pmatrix} \sigma^a/2 & 0 \\ 0 & 0 \end{pmatrix}$

$$Y = \begin{pmatrix} Y_L & & 0 \\ & Y_L & \\ 0 & & Y_R \end{pmatrix}$$

full Lagrangian: $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\varepsilon^{abc} W^b_\mu W^c_\nu$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\mathcal{L}_{EW} = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$+ \quad \overline{\Psi}\, i\gamma^\mu D_\mu \Psi$$

general gauge transformation:

$$\Psi \to e^{igw^a I^a + igwY} \Psi$$

Neutral gauge bosons:

$$-\overline{\Psi}\gamma^\mu \big[g(\cos\Theta_W Z^0_\mu + \sin\Theta_W A_\mu) T^3$$
$$+ g'(-\sin\Theta_W Z^0_\mu + \cos\Theta_W A_\mu) Y \big] \Psi$$

pl. to Photon: $\quad g\sin\Theta_W T^3 + g'\cos\Theta_W Y = eQ$
$$= eT^3 + eY$$

$$\Rightarrow g\sin\Theta_W = e$$
$$g'\cos\Theta_W = e$$

current: $eA^0_\mu j^\mu_{em}$

$$j^\mu_{em} = \overline{\Psi}\gamma^\mu\Psi = (\overline{\Psi_R}\gamma^\mu\Psi_R + \overline{\Psi_L}\gamma^\mu\Psi_L)$$

coupl. to Z^0:

$$g \cos\Theta_W T^3 - g' \sin\Theta_W Y$$

$$= \frac{e}{\sin\Theta_W \cos\Theta_W}\left(\cos^2\Theta_W T^3 - \sin^2\Theta_W Y\right)$$

$$= \boxed{\frac{2e}{\sin 2\Theta_W}}\left(T^3 - \sin^2\Theta_W Q\right)$$

current: $-Z^0_\mu j^\mu_{nc} \cdot \frac{2e}{\sin 2\Theta_W}$

$$j^\mu_{nc} = \overline{\Psi}_L \gamma^\mu \underbrace{(T^3 - \sin^2\Theta_W Q)}_{\sim\ g_L = \frac{1}{2}(g_V - g_R)} \Psi_L + \overline{\Psi}_R \gamma^\mu \underbrace{(-\sin^2\Theta_W Q)}_{g_R = \frac{1}{2}(g_V + g_A)} \Psi_R$$

$$\left\{ = \frac{1}{2} \overline{\Psi} \gamma^\mu \left(T^3_L(1-\gamma_5) - 2Q \sin^2\Theta_W\right) \Psi\right.$$

[electron: $= \frac{1}{2} \overline{\Psi}_{\nu_{eL}} \gamma^\mu \Psi_{\nu_{eL}} - \frac{1}{2} \overline{\Psi}_{e_L} \gamma^\mu \Psi_{e_L} - \sin^2\Theta_W j^\mu_{em}$]

Problems:

(i) Masses for $W^\pm, Z^0$: explicit mass-terms break gauge inv.!

(ii) Masses for matter fields: couple left- to right-handed fields → break weak gauge inv.!

(iii) Neutrino mixing

Resolution to (I), (ii): Higgs-mechanism: masses via spont. sym. break.