Lecture:

Standard Model of Particle Physics

Heidelberg SS 2016

Beauty Physics at B-factories

Contents

CKM Matrix

Definition:

$$V_{\mathrm{CKM}} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

Experimental values: fit to data! no theory prediction!

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix},$$

(Particle Data Group 2012)

Standard Parameterisation (Euler Angles):

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Unitary matrix has N x N parameters: 3 families $\rightarrow$ 9 parameters

2 N -1 trivial phases: $\rightarrow$ 5 phases

N (N-1)/2 rotation angles $\rightarrow$ 3 angles

N N - N (N-1)/2 - (2 N -1) = N (N-3)/2 +1 $\rightarrow$ **1 CPV phase**

$\rightarrow$ **need 3 families for generating CP-violation!**

CKM Matrix

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \qquad s_{23} = A\lambda^2 = \lambda \left| \frac{V_{cb}}{V_{us}} \right|$$

$$s_{13}e^{i\delta} = V_{ub}^* = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\bar\rho + i\bar\eta)\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\bar\rho + i\bar\eta)]}$$

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\,.$$

CKM Matrix

Definition:

$$V_{\mathrm{CKM}} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

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(Particle Data Group 2012)

Wolfenstein Parameterisation:

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) .$$

fit =>

$$\lambda = 0.22535 \pm 0.00065 , \qquad A = 0.811^{+0.022}_{-0.012} ,$$

$$\bar{\rho} = 0.131^{+0.026}_{-0.013} , \qquad \boxed{\bar{\eta} = 0.345^{+0.013}_{-0.014} .}$$

Note: found CP violation is too small to explain observed matter-antimatter asymmetry in universe

Unitarity Triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$V_{\rm CKM} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

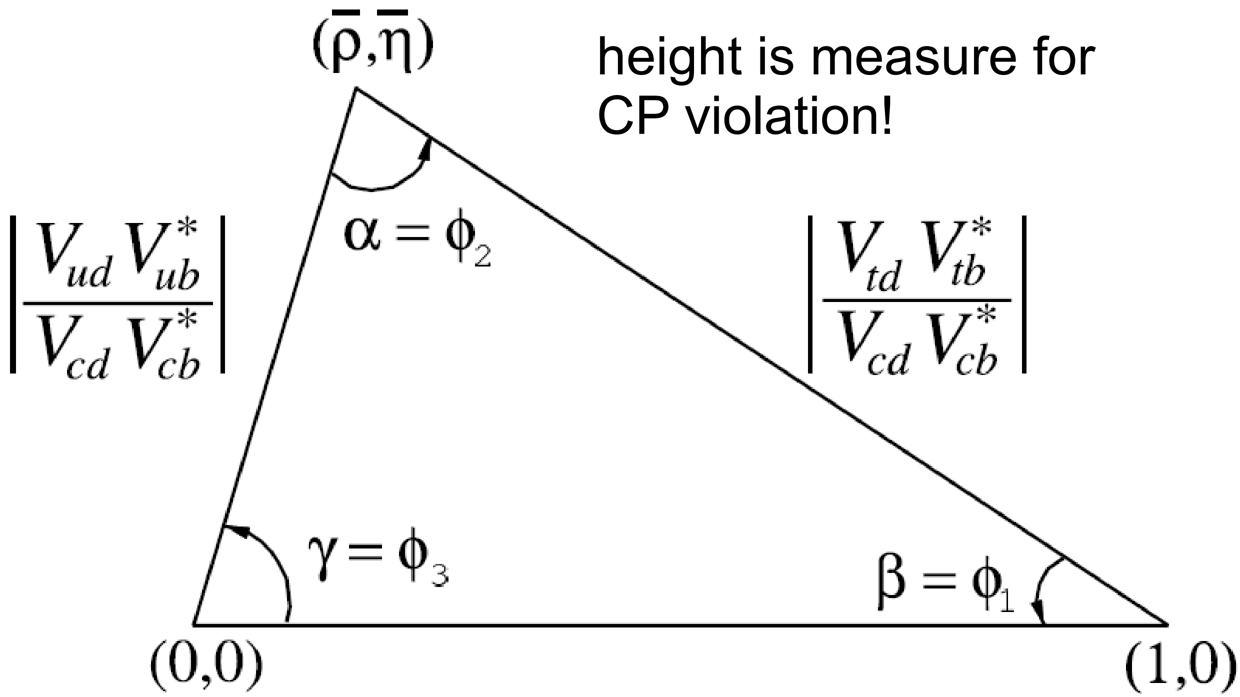


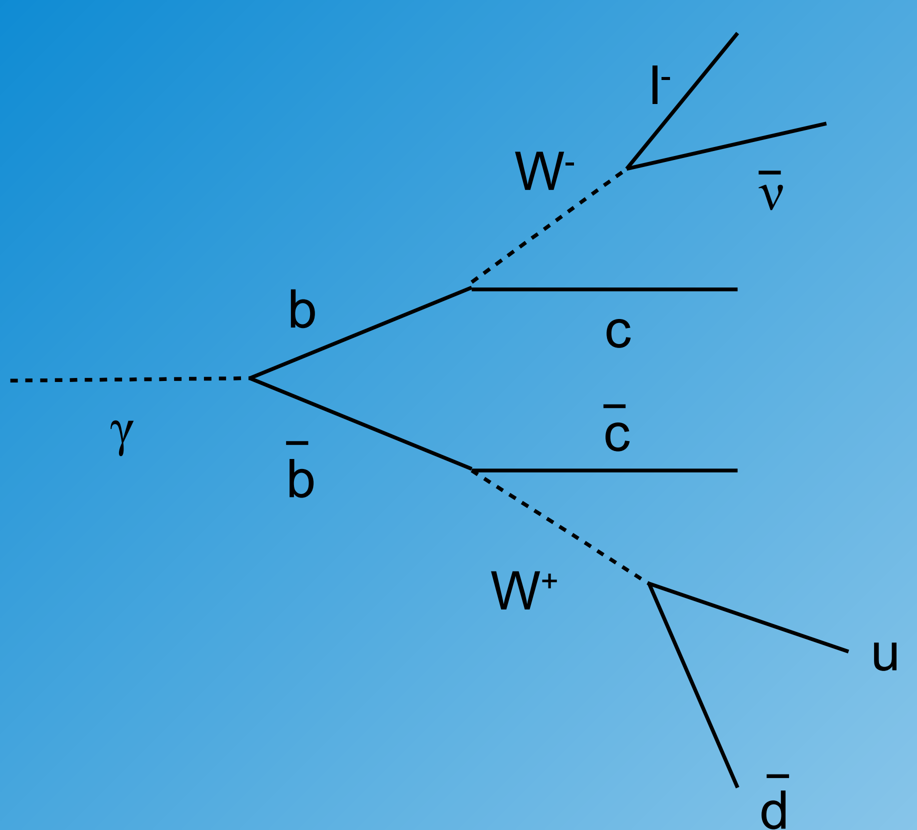
Figure 11.1: Sketch of the unitarity triangle.

test of unitarity triangle:
→ study of electroweak decays

Proof: $$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} = \frac{(1-\lambda^2/2)\,A\lambda^3(\rho-i\eta)^*}{(-\lambda)\,(A\lambda^2)^*} = -(1-\lambda^2/2)\,(\rho+i\eta) \approx (\bar\rho+i\bar\eta)$$

$O(\lambda^4)$ terms neglected

CP Violation and the Consequences

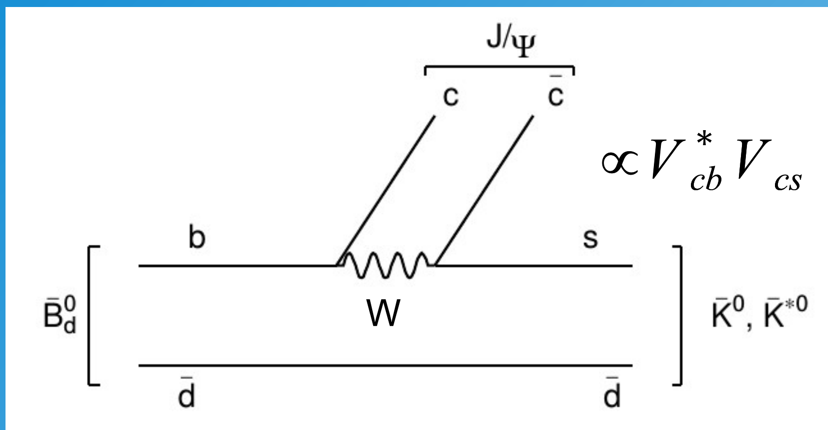


If CP-violated, the above final states do not occur with same rate!

- Direct CP-Violation: different partial decay widths for particles and antiparticles

- CPV can explain observed baryon asymmetry in universe if in addition **baryon-number violating** process exists

Example Decay $B_d \rightarrow J/Psi\ K_s$



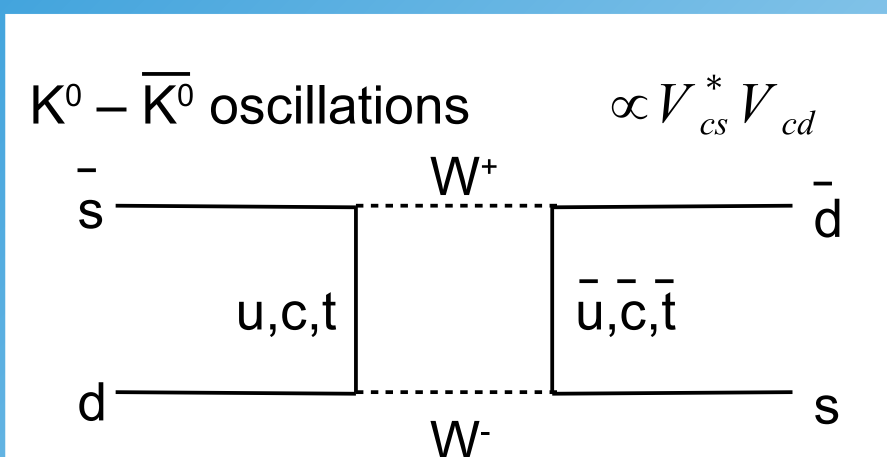
Cross section depends on CKM elements

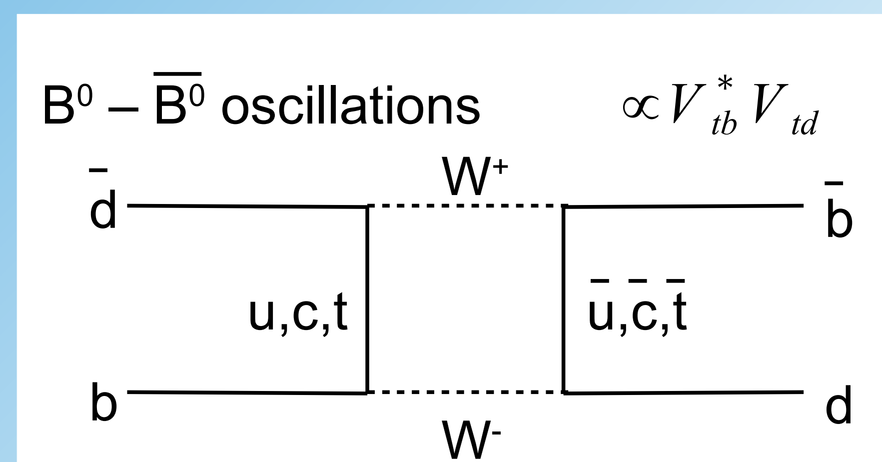
But large non-perturbative QCD corrections!

Precise measurement of CKM <u>elements</u> and <u>CP violation</u> from cross section not possible!

mixing decay mixing

Use trick: B and Kaon oscillate: $\overline{B}_d \rightarrow B_d \rightarrow J/Psi\ K^0 \rightarrow J/Psi\ \overline{K}^0$





can be used to measured ratio of CKM elements $\rightarrow$ CKM angles

How to measure CP violation?

General rule:

Complex phase can only be measured in interference processes:

Three possibilities for “direct” measurement:

- direct CP Violation
- CP violation in mixing
- interference between decays with and without mixing
  (for example $B_d \rightarrow J/Psi\ K_s$)
  → allows to search for additional (non-SM) CP sources!

Indirect measurement (model dependent!):

- measure all elements of CKM matrix V_{ij}
- fit of Wolfenstein parameterisation → η (or Euler angles → δ)

CP transformation

<u>Consider CP violating (weak) process i → f:</u>

$$i \xrightarrow{\text{weak}} f$$

$$CP\,|a_i\rangle = e^{+i\varphi_i}\,|\bar{a}_i\rangle$$

$$CP\,|a_f\rangle = e^{+i\varphi_f}\,|\bar{a}_f\rangle$$

$$\bar{i} \xrightarrow{\text{weak}} \bar{f}$$

$$A_f = \langle a_f|O|a_i\rangle$$

transition amplitudes:

$$\bar{A}_{\bar{f}} = \langle \bar{a}_f|O|\bar{a}_i\rangle$$

from $[O, CP] = 0$ follows: $\quad \bar{A}_{\bar{f}} = e^{+i(\varphi_f - \varphi_i)}\,A_f$

CP transformation

Consider CP violating (weak) process i → f:

$$i \xrightarrow{\text{weak}} f \qquad\qquad \bar{i} \xrightarrow{\text{weak}} \bar{f}$$

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$$A_f = \langle a_f|O|a_i\rangle \qquad\qquad \bar{A}_{\bar{f}} = \langle \bar{a}_f|O|\bar{a}_i\rangle$$

from $[O, CP]=0$ follows: $\quad \bar{A}_{\bar{f}} = e^{+i(\varphi_f-\varphi_i)}\,A_f$

If quarks involved, there is an additional QCD phase shift:

$$i \xrightarrow{\text{QCD}} s \xrightarrow{\text{weak}} f \qquad\qquad \bar{i} \xrightarrow{\text{QCD}} \bar{s} \xrightarrow{\text{weak}} \bar{f}$$

$$A_f = |n|\,e^{+i(\theta+\varphi_i)} \qquad\qquad \bar{A}_{\bar{f}} = |n|\,e^{+i(\theta-\varphi_i)}$$

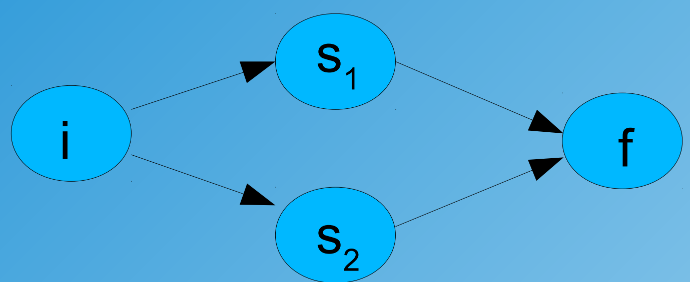
strong interaction (strong phase shift θ) is CP invariant!

I. Direct CP violation

Definition $|\bar{A}_{\bar{f}} / A_f| \neq 1$

not possible with single process as $|\exp(ix)|=1$ (see previous page)

Superposition of two processes:



like a classical double slit experiment

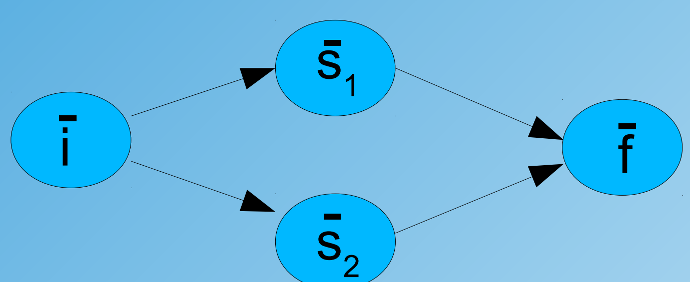
$$|A_f{}^2| = n_1^2 + n_2^2 + 2|n_1||n_2| \cos((\theta_1-\theta_2)+(\varphi_1-\varphi_2))$$

$$|\bar{A}_{\bar{f}}{}^2| = n_1^2 + n_2^2 + 2|n_1||n_2| \cos((\theta_1-\theta_2)-(\varphi_1-\varphi_2))$$

$$A_f = |n_1|\, e^{+i(\theta_1+\varphi_1)} + |n_2|\, e^{+i(\theta_2+\varphi_2)}$$

$$\bar{A}_{\bar{f}} = |n_1|\, e^{+i(\theta_1-\varphi_1)} + |n_2|\, e^{+i(\theta_2-\varphi_2)}$$

$|\bar{A}_{\bar{f}} / A_f| \neq 1$ if $\varphi_1 \neq \varphi_2$ and $\theta_1 \neq \theta_2$



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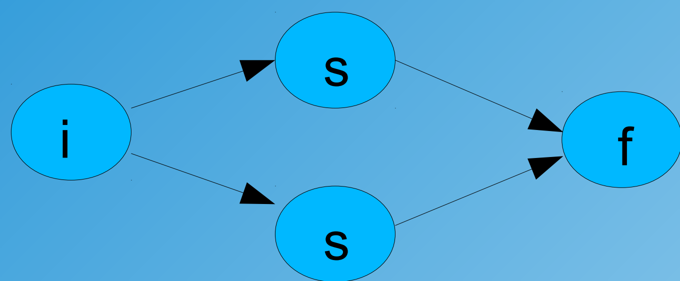


like a classical double slit experiment

$$\left|A_f{}^2\right| = n_1^2 + n_2^2 + 2|n_1||n_2|\cos\left((\theta_1-\theta_2)+(\varphi_1-\varphi_2)\right)$$
$$\left|\bar{A}_{\bar{f}}{}^2\right| = n_1^2 + n_2^2 + 2|n_1||n_2|\cos\left((\theta_1-\theta_2)-(\varphi_1-\varphi_2)\right)$$

$$A_f = |n_1|\, e^{+i(\theta_1+\varphi_1)} + |n_2|\, e^{+i(\theta_2+\varphi_2)}$$
$$\bar{A}_{\bar{f}} = |n_1|\, e^{+i(\theta_1-\varphi_1)} + |n_2|\, e^{+i(\theta_2-\varphi_2)}$$

$\left|\bar{A}_{\bar{f}} / A_f\right| \neq 1$ if $\varphi_1 \neq \varphi_2$ and $\theta_1 \neq \theta_2$

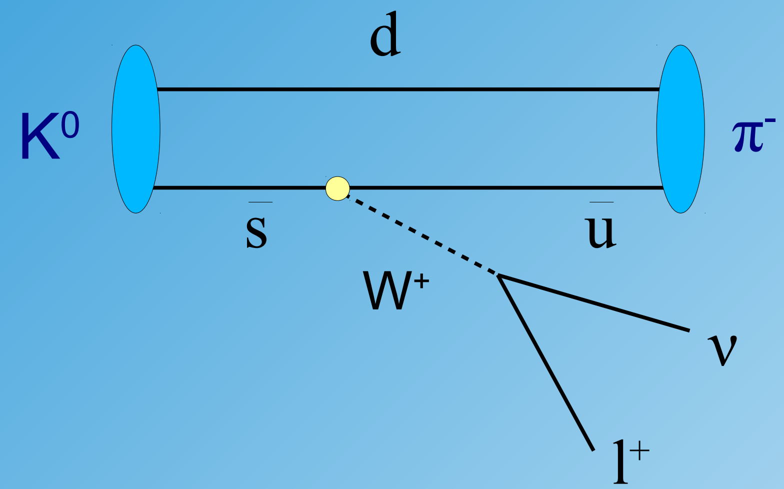
Can be measured in decays of charged and neutral particles!

example: $$\delta_L = \frac{\Gamma(K_L \to l^+ \nu_l \pi^-) - \Gamma(K_L \to l^- \bar{\nu}_l \pi^+)}{\Gamma(K_L \to l^+ \nu_l \pi^-) + \Gamma(K_L \to l^- \bar{\nu}_l \pi^+)}$$

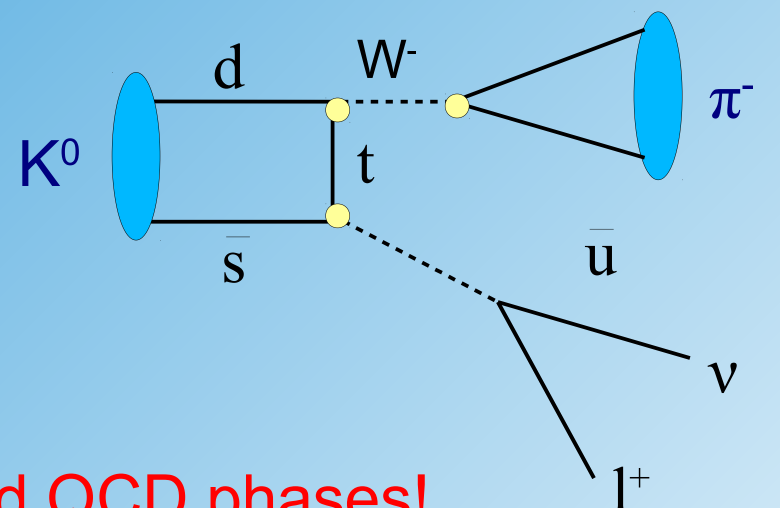
$\delta_L = (3.32 \pm 0.06) \times 10^{-3}$ (experiment)

Note: total decay width is not affect by CP violation (would violate CPT)

Example K_L Decays



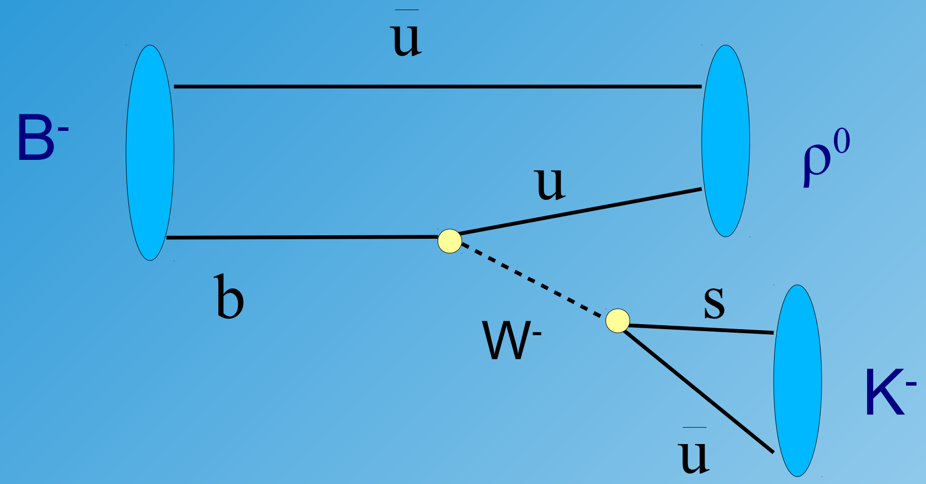
$$|K_L\rangle \approx \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$



different weak and QCD phases!

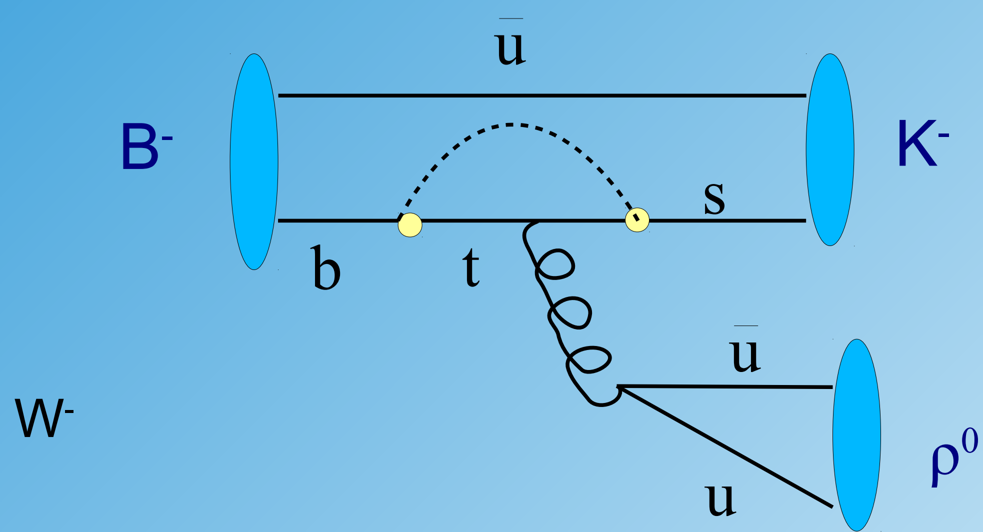
Direct CP Violation in Charged Meson Decay

$B^- \to K^- \rho^0$



V^*_{ub}

V_{us}



V^*_{tb} V_{ts}

different weak and QCD phases!

$$\mathcal{A}_{\rho^0 K^{\mp}} = +0.37 \pm 0.10.$$

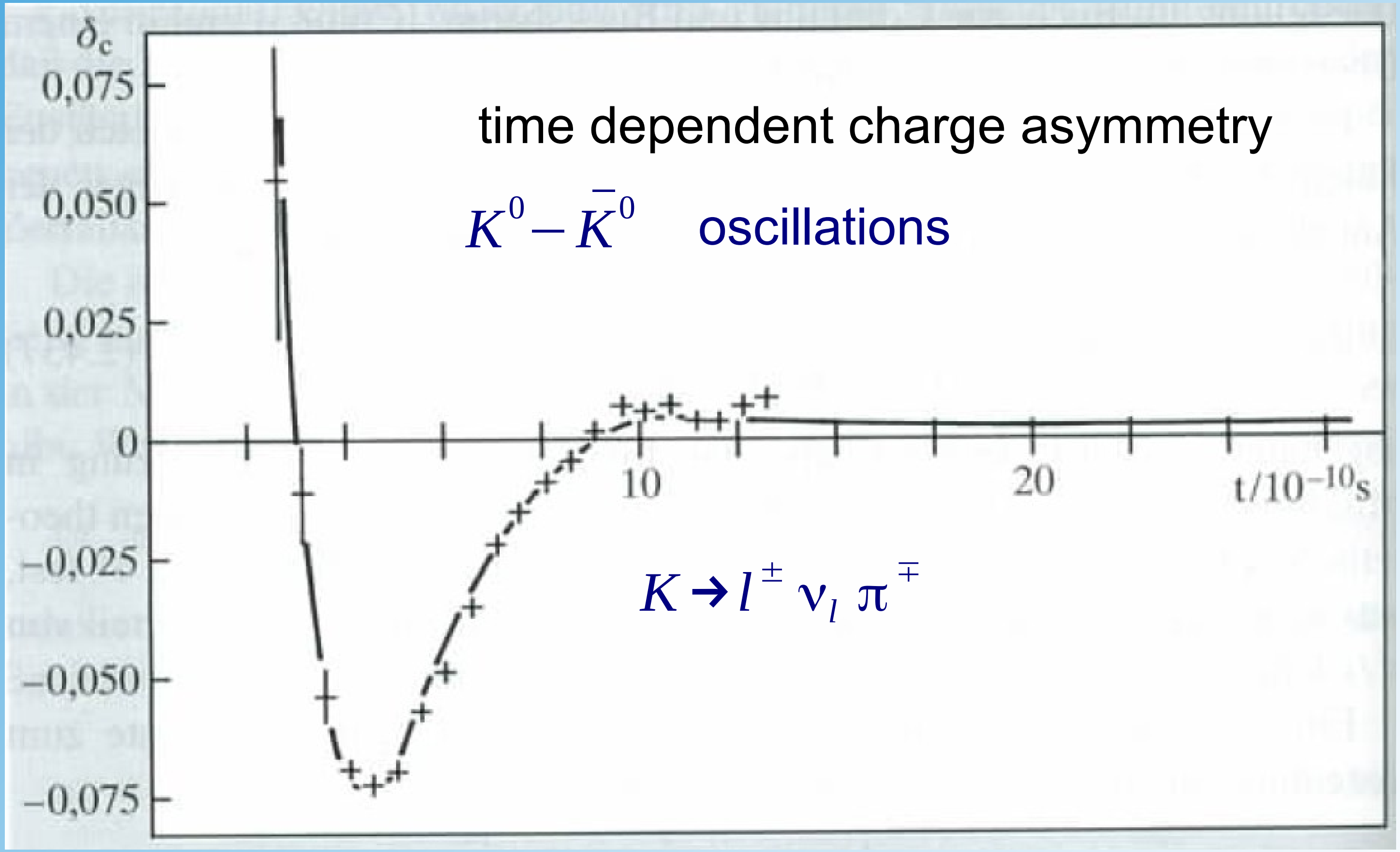
huge asymmetry!

II. CP violation in mixing

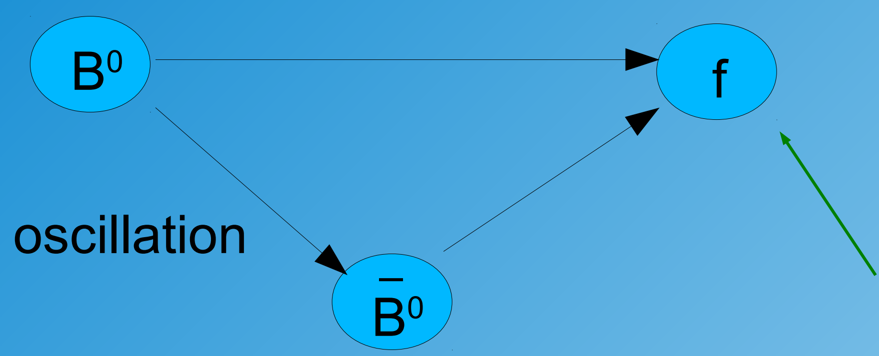
- oscillations of neutral mesons
- measure time dependent decay width

$$A_{osc} = \frac{d\Gamma/dt(\bar{M}^0 \to l^+ X) - d\Gamma/dt(M^0 \to l^- X)}{d\Gamma/dt(\bar{M}^0 \to l^+ X) + d\Gamma/dt(M^0 \to l^- X)}$$

$$\delta_C = A_{osc}$$



III. Interference Decay + Mixing



neutral final state

interference between decay without mixing and decay with mixing

$$A_\Gamma = \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)}$$

B-meson in two mass eigenstates

$$|B_L\rangle \; = \; p|B^0\rangle + q|\overline{B}^0\rangle$$
$$|B_H\rangle \; = \; p|B^0\rangle \Leftrightarrow q|\overline{B}^0\rangle$$

with $|q|^2 + |p|^2 = 1.$

Experimental measurement of ratio:

$$\lambda = \frac{q}{p}\frac{\overline{A}_f}{A_f}$$

related to

B-osc. decay K-osc. CKM-angle

$$\lambda(B \to \psi K_S) = \Leftrightarrow \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)\left(\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\right)\left(\frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*}\right) \Longrightarrow \mathcal{I}m\,\lambda_{\psi K_S} = \sin(2\beta)$$

Unitarity Triangle

one of six possible unitarity triangles

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

define angles α, β, γ

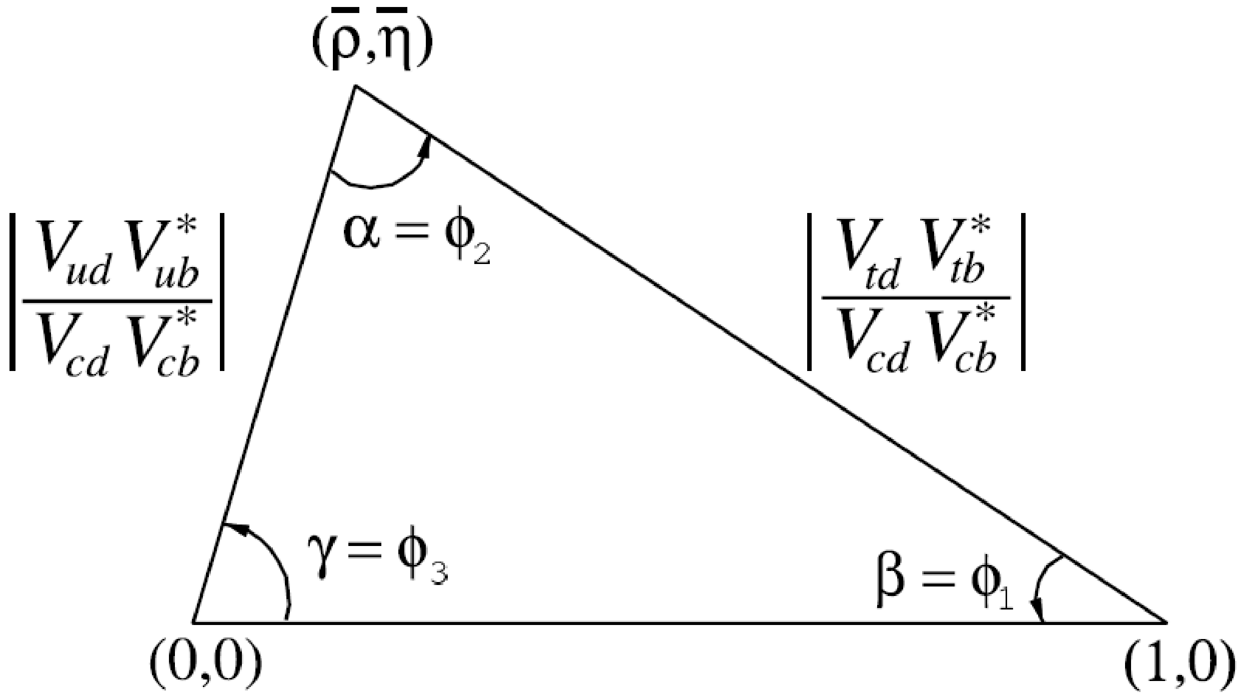


Figure 11.1: Sketch of the unitarity triangle.

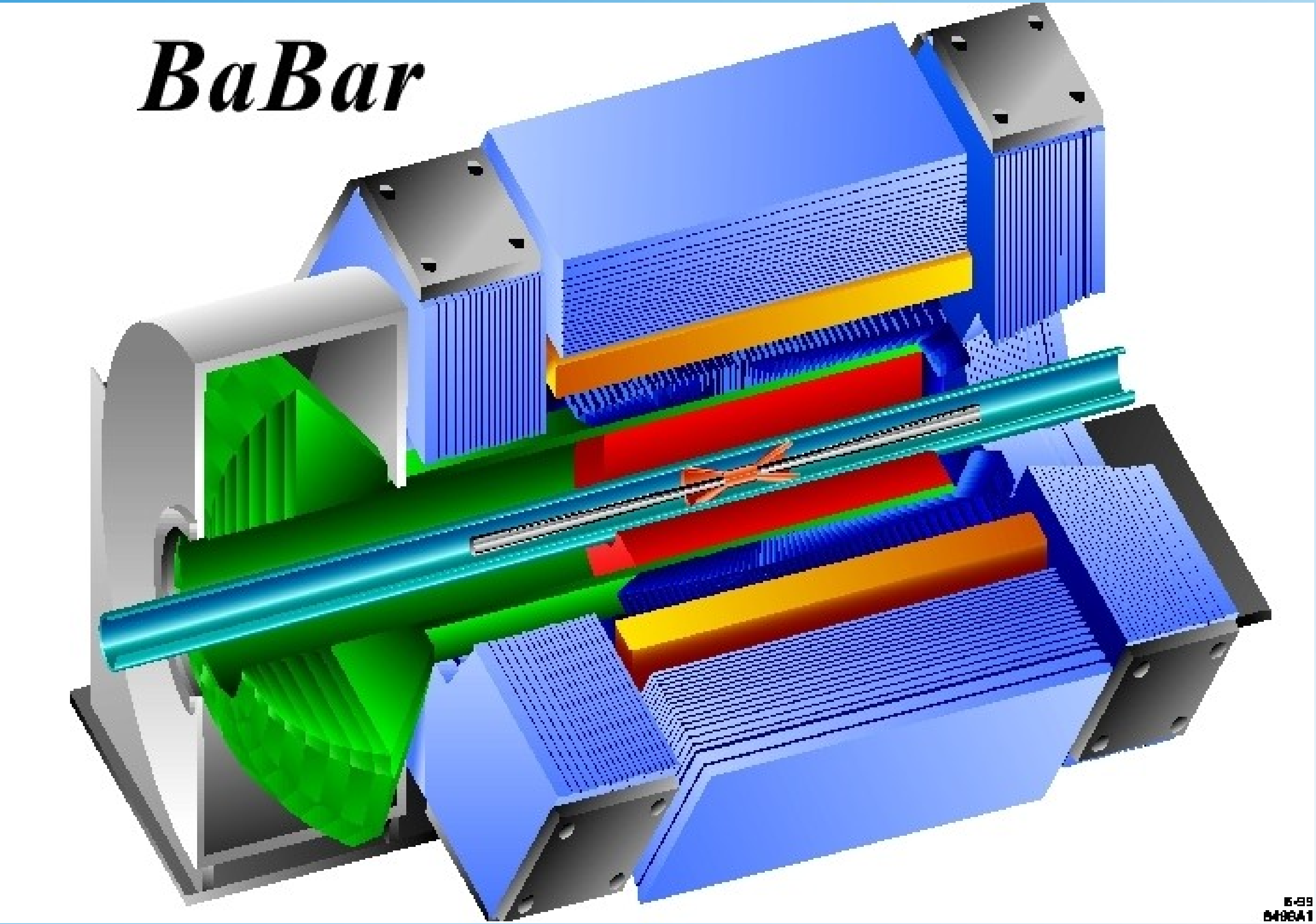
$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\alpha = \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

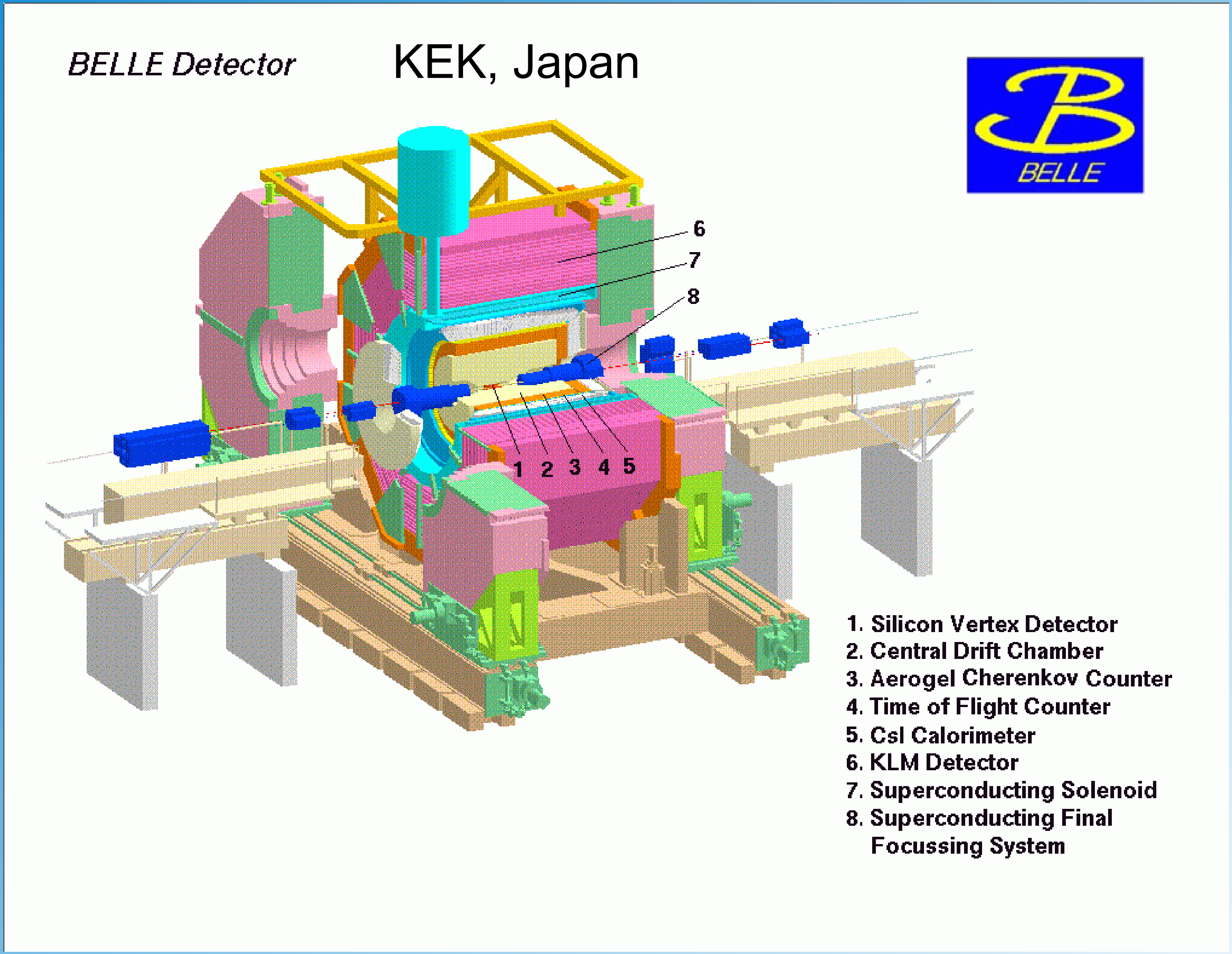
$$\gamma = \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

Babar Detector

PEP-C Accelerator, Stanford

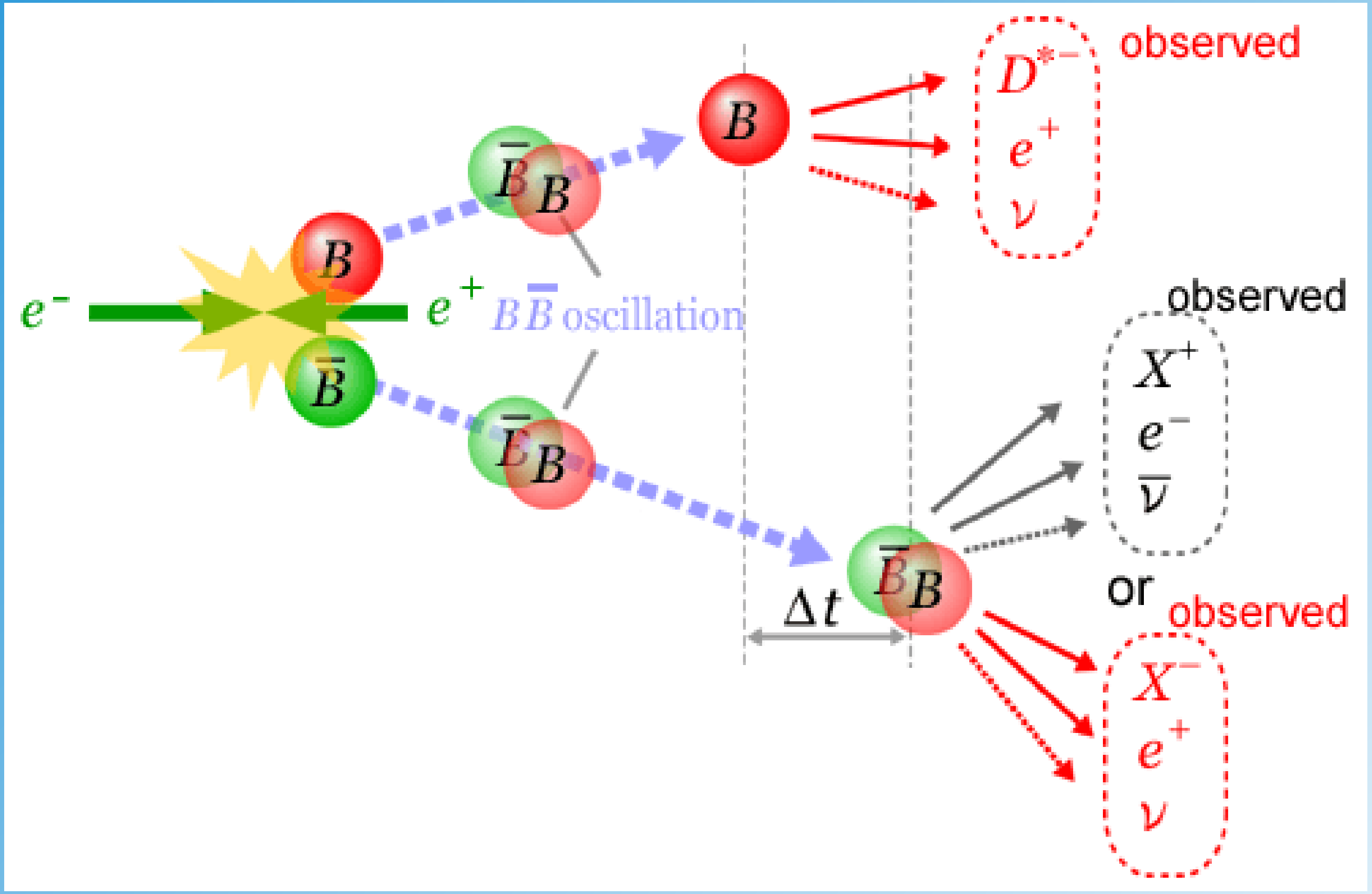


Belle Detector

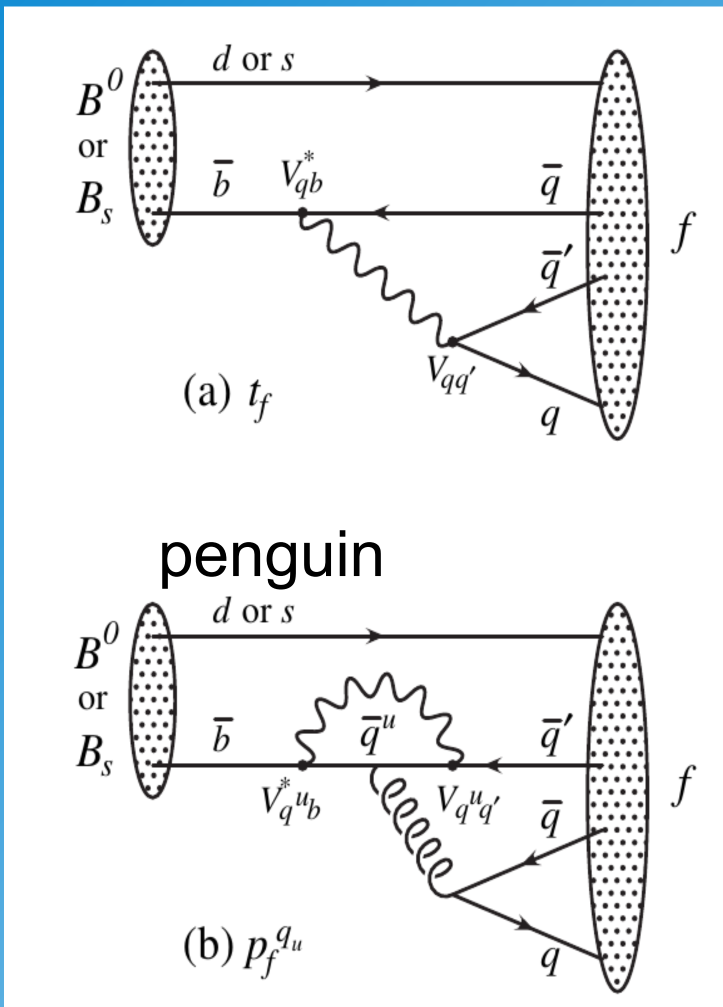


B-Oscillations at B-factories

Exploit different beam energies: $E(e^-) > E(e^+)$



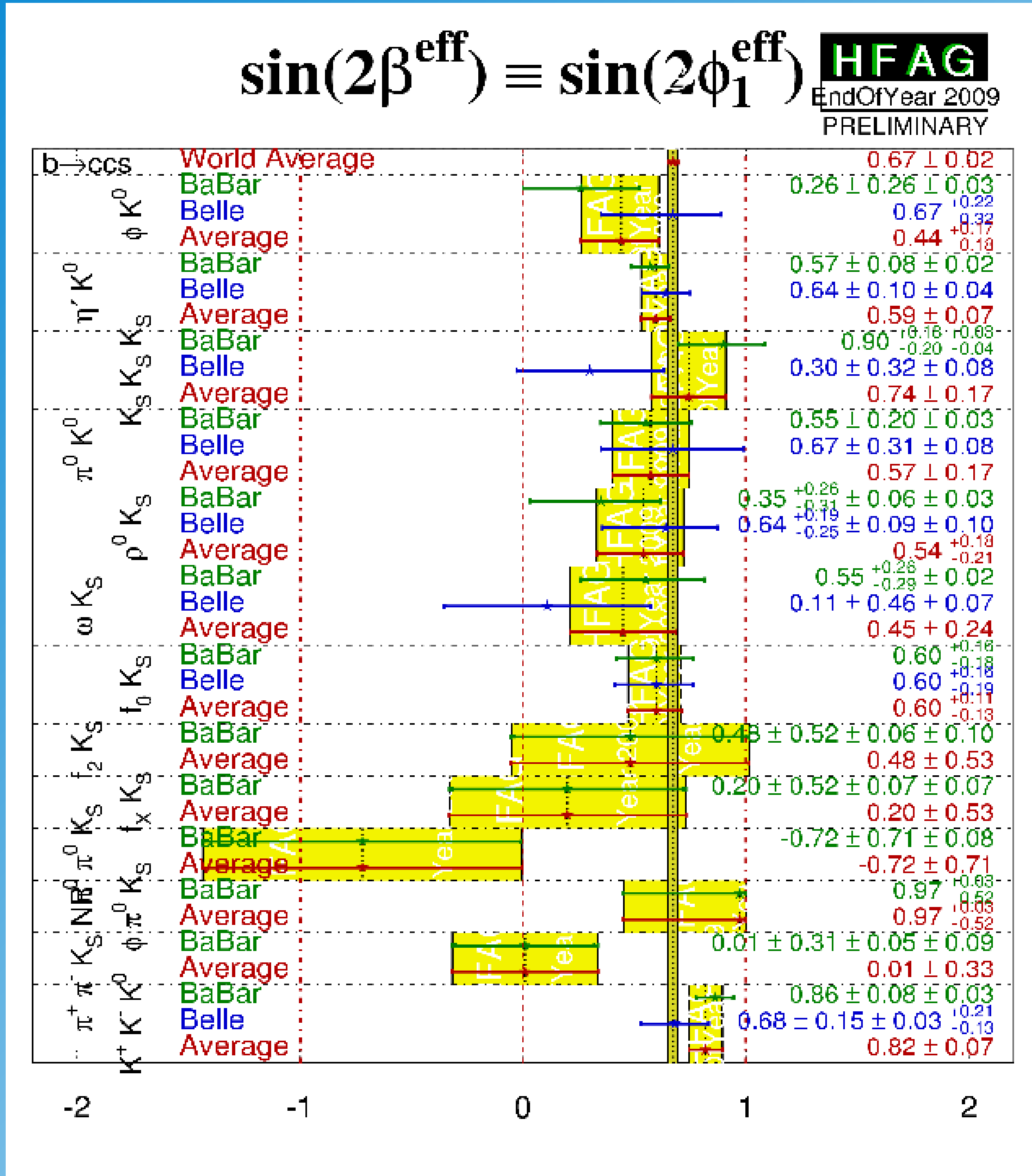
Classification of B-decays



$\bar{b} \to \bar{q}q\bar{q}'$	$B^0 \to f$	$B_s \to f$	CKM dependence of A_f	Suppression
$\bar{b} \to \bar{c}c\bar{s}$	ψK_S	$\psi\phi$	$(V^*_{cb}V_{cs})T + (V^*_{ub}V_{us})P^u$	loop $\times\ \lambda^2$
$\bar{b} \to \bar{s}s\bar{s}$	ϕK_S	$\phi\phi$	$(V^*_{cb}V_{cs})P^c + (V^*_{ub}V_{us})P^u$	λ^2
$\bar{b} \to \bar{u}u\bar{s}$	$\pi^0 K_S$	K^+K^-	$(V^*_{cb}V_{cs})P^c + (V^*_{ub}V_{us})T$	λ^2/loop
$\bar{b} \to \bar{c}c\bar{d}$	D^+D^-	ψK_S	$(V^*_{cb}V_{cd})T + (V^*_{tb}V_{td})P^t$	loop
$\bar{b} \to \bar{s}s\bar{d}$	$\phi\pi$	ϕK_S	$(V^*_{tb}V_{td})P^t + (V^*_{cb}V_{cd})P^c$	$\lesssim 1$
$\bar{b} \to \bar{u}u\bar{d}$	$\pi^+\pi^-$	$\pi^0 K_S$	$(V^*_{ub}V_{ud})T + (V^*_{tb}V_{td})P^t$	loop

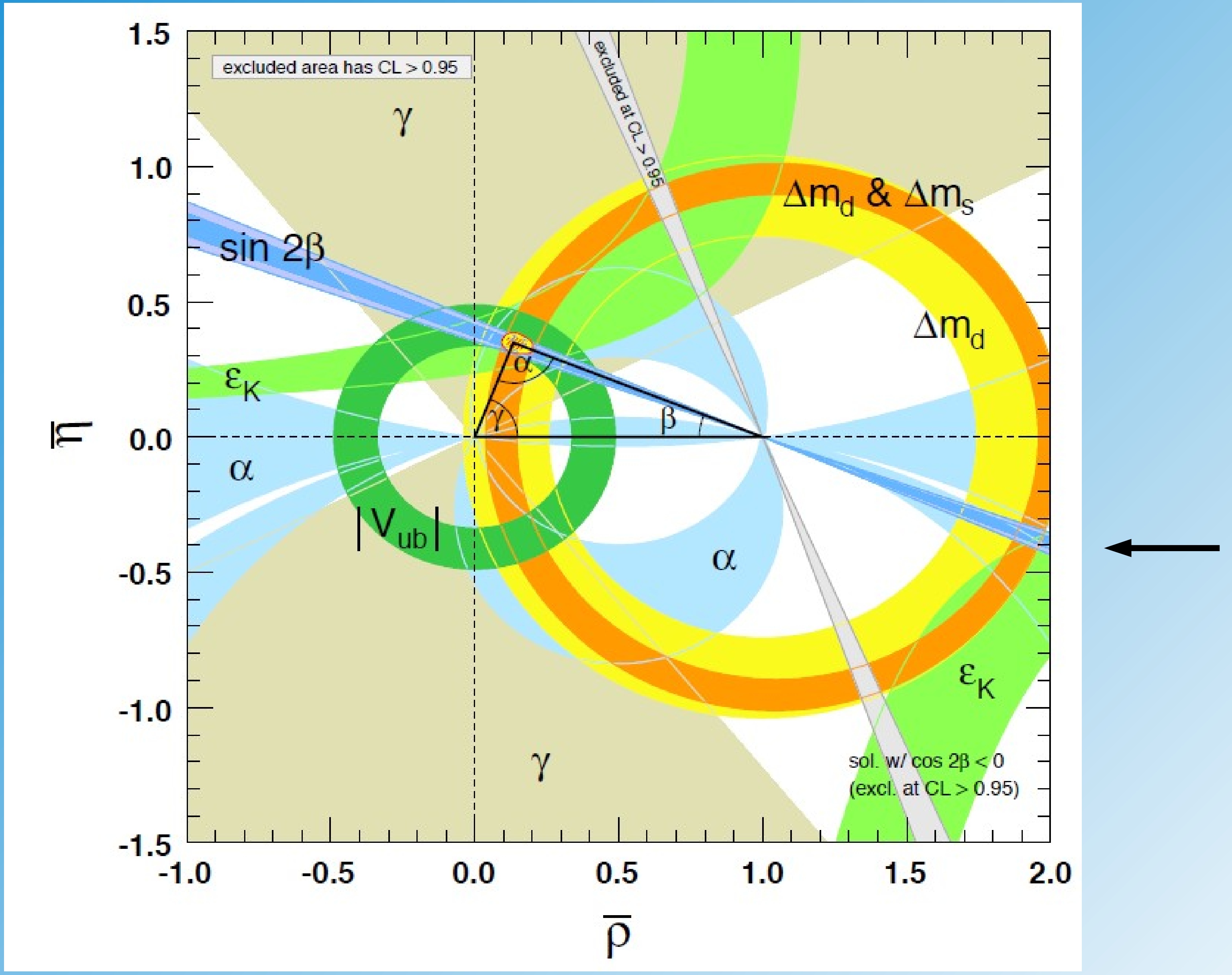
always two identical flavours

interfering diagrams



sin 2β clearly non zero!

Summary of experimental Results



B-Mixing Formalism

Oscillations:

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\,.$$

$$\begin{pmatrix} |M^0(t)\rangle \\ |\bar{M}^0(t)\rangle \end{pmatrix} = \begin{pmatrix} M_{11} - i\Gamma_1/2 & M_{12} \\ M_{21} & M_{22} - i\Gamma_2/2 \end{pmatrix} \cdot \begin{pmatrix} |M^0(0)\rangle \\ |\bar{M}^0(0)\rangle \end{pmatrix}$$

Linear combination of low and high mass eigenstate

$$|M_L\rangle \propto p\sqrt{1-z}\,|M^0\rangle + q\sqrt{1+z}\,|\overline{M}^0\rangle$$
$$|M_H\rangle \propto p\sqrt{1+z}\,|M^0\rangle - q\sqrt{1-z}\,|\overline{M}^0\rangle$$

$$\left(\frac{q}{p}\right)^2 = \frac{\mathbf{M}_{12}^* - (i/2)\mathbf{\Gamma}_{12}^*}{\mathbf{M}_{12} - (i/2)\mathbf{\Gamma}_{12}}$$

$$z \equiv \frac{\delta m - (i/2)\delta\Gamma}{\Delta m - (i/2)\Delta\Gamma}\,,$$

$$\delta m \equiv \mathbf{M}_{11} - \mathbf{M}_{22} \quad , \quad \delta\Gamma \equiv \mathbf{\Gamma}_{11} - \mathbf{\Gamma}_{22}$$

$$\Delta m \equiv m_H - m_L = \mathcal{R}e(\omega_H - \omega_L) \quad ,$$
$$\Delta\Gamma \equiv \Gamma_H - \Gamma_L = -2\,\mathcal{I}m(\omega_H - \omega_L)\,.$$

both have been measured for B^0 and B_s

Classification of CP Violation

I. Direct CP-violation

$$|\bar{A}_{\bar{f}} / A_f| \neq 1$$

$$\mathcal{A}_{f^\pm} \equiv \frac{\Gamma(M^- \to f^-) - \Gamma(M^+ \to f^+)}{\Gamma(M^- \to f^-) + \Gamma(M^+ \to f^+)} = \frac{|\overline{A}_{f^-}/A_{f^+}|^2 - 1}{|\overline{A}_{f^-}/A_{f^+}|^2 + 1} .$$

II. CP-violation in mixing

$$|q/p| \neq 1$$

$$\mathcal{A}_{\rm SL}(t) \equiv \frac{d\Gamma/dt\left[\overline{M}^0_{\rm phys}(t) \to \ell^+ X\right] - d\Gamma/dt\left[M^0_{\rm phys}(t) \to \ell^- X\right]}{d\Gamma/dt\left[\overline{M}^0_{\rm phys}(t) \to \ell^+ X\right] + d\Gamma/dt\left[M^0_{\rm phys}(t) \to \ell^- X\right]} = \frac{1 - |q/p|^4}{1 + |q/p|^4} .$$

III. CP-violation in decay with and without mixing (neutral state)

$$\Im(\lambda_f) = \Im\left(\frac{q\,\bar{A}_f}{p\,A_f}\right) \neq 0$$

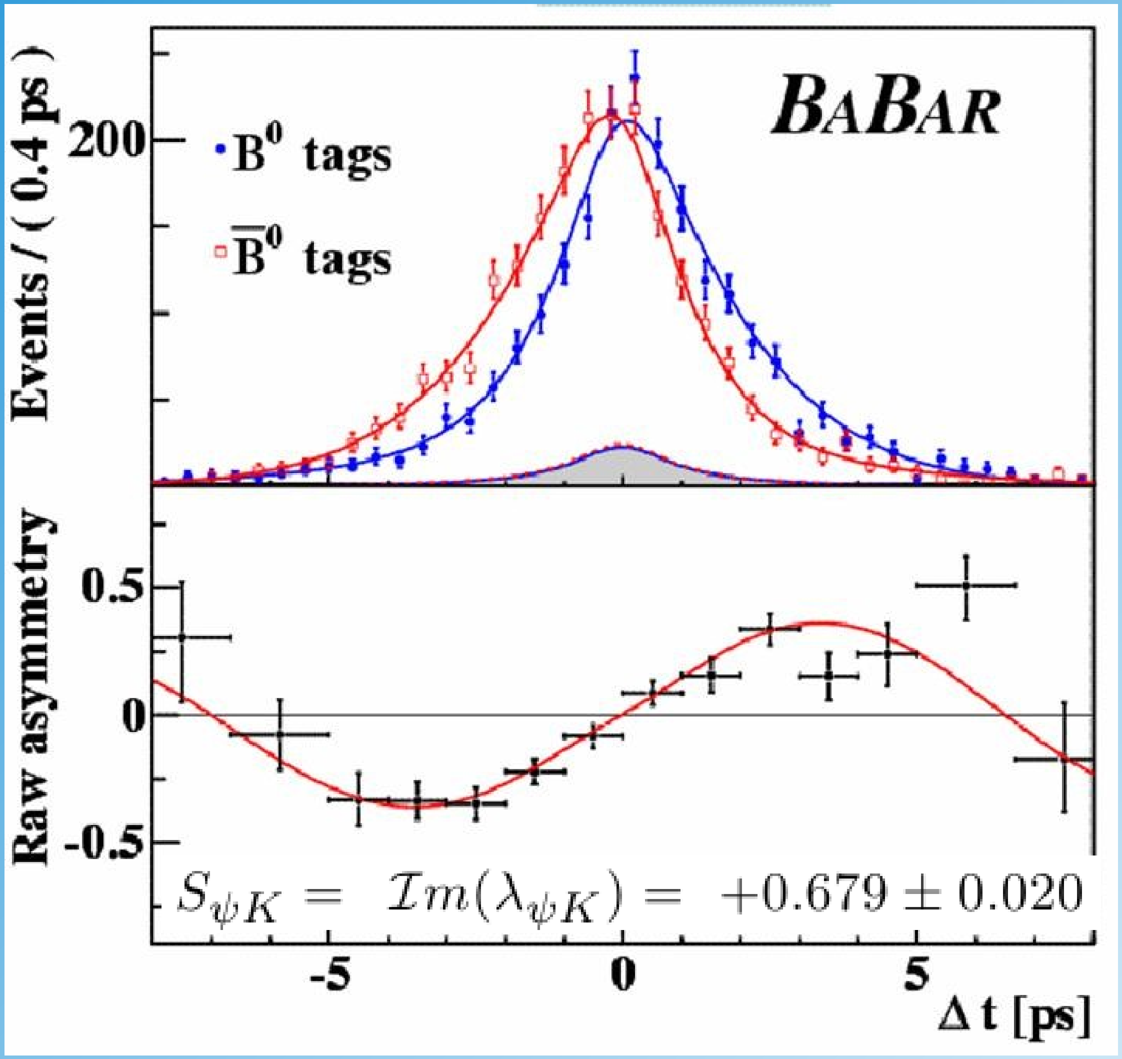
$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt\left[\overline{M}^0_{\rm phys}(t) \to f_{CP}\right] - d\Gamma/dt\left[M^0_{\rm phys}(t) \to f_{CP}\right]}{d\Gamma/dt\left[\overline{M}^0_{\rm phys}(t) \to f_{CP}\right] + d\Gamma/dt\left[M^0_{\rm phys}(t) \to f_{CP}\right]} .$$

asymmetry often called **S**

CP-eigenstate

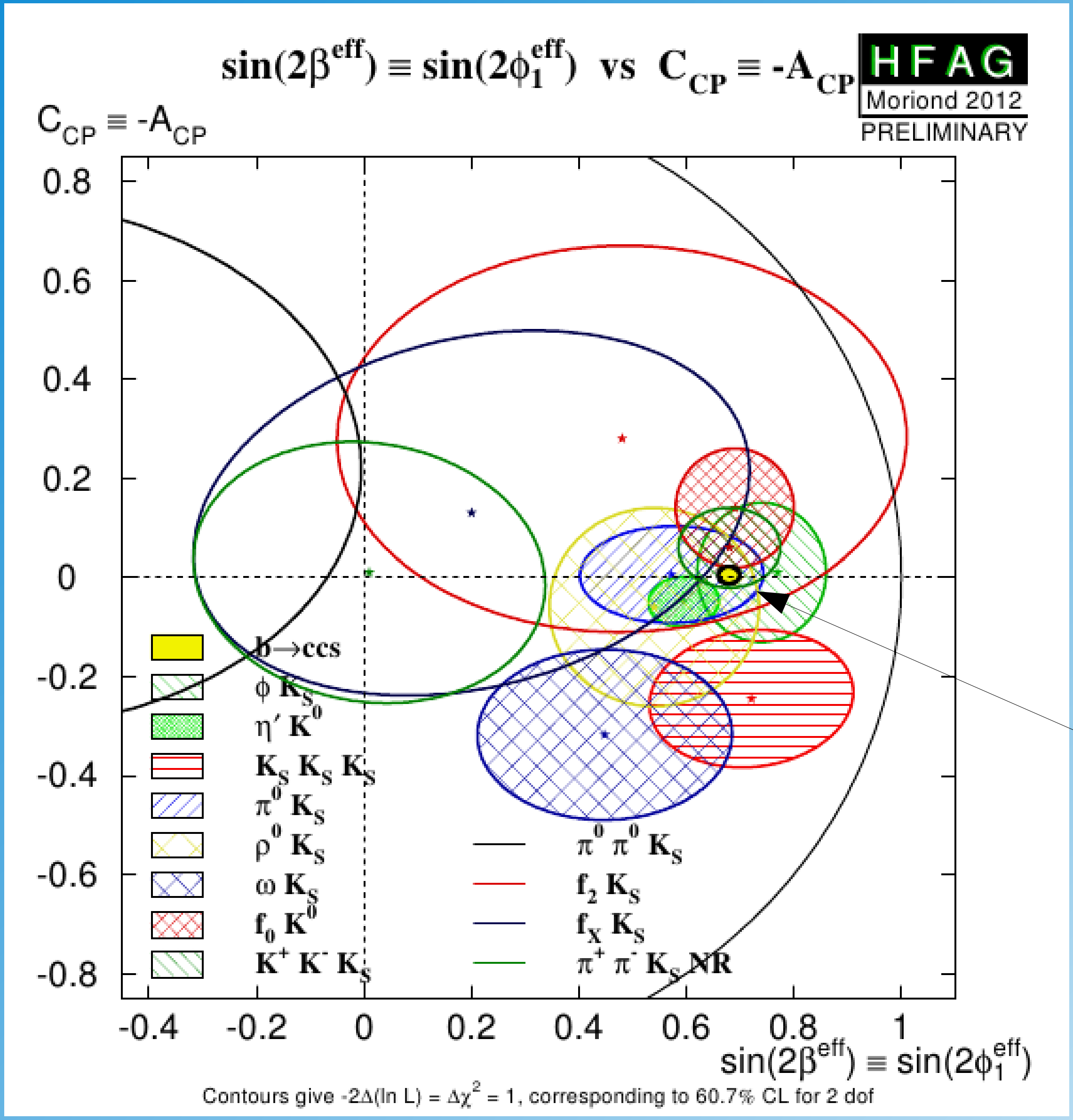
$$S_f = \frac{2\,{\rm Im}\lambda_f}{1 + |\lambda_f|^2}, \qquad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2},$$

CP Violation in B $\rightarrow$ J/ψ K



CP violating effect in B-sector is much larger than in K-sector!

Collection of CPV Measurements



large CP violation effects in B-meson decays

this measurement

Summary B-Asymmetry Parameters

I. Direct CP-violation is large

$$\mathcal{A}_{K^+\pi^-} = \frac{|\overline{A}_{K^-\pi^+}/A_{K^+\pi^-}|^2 - 1}{|\overline{A}_{K^-\pi^+}/A_{K^+\pi^-}|^2 + 1} = -0.087 \pm 0.008 \quad \text{(I)}$$

II. CP-violation in mixing is very small!

Sensitive to new BSM physics!

$$\mathcal{A}^d_{\mathrm{SL}} = (-3.3 \pm 3.3) \times 10^{-3} \implies |q/p| = 1.0017 \pm 0.0017.$$

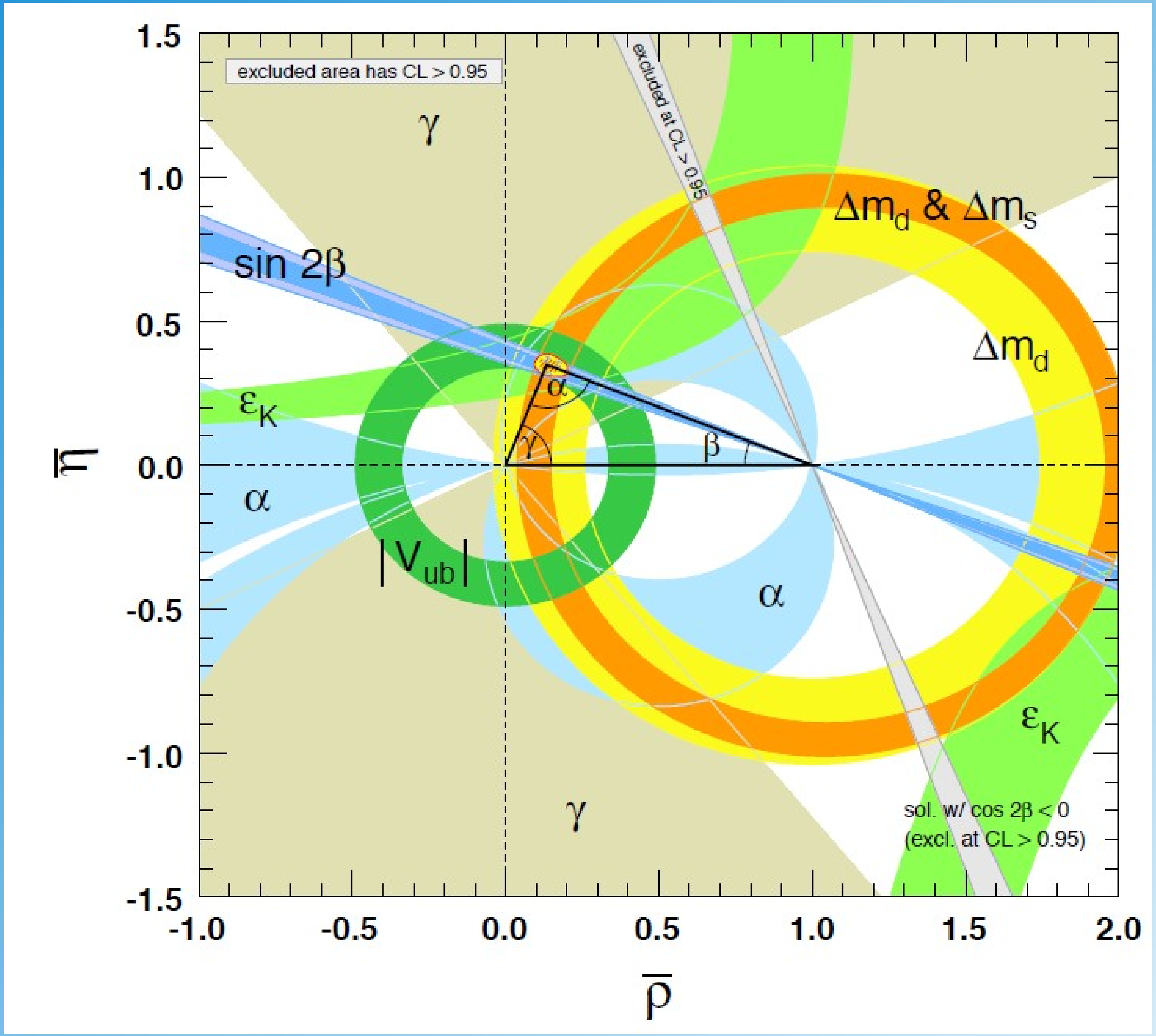
$$\mathcal{A}^d_{\mathrm{SL}} = \mathcal{O}\left[(m_c^2/m_t^2)\sin\beta\right] \lesssim 0.001.$$

top box dominates!

III. CP-violation in decay with and without mixing

$$S_{\psi K} = \mathcal{I}m(\lambda_{\psi K}) = +0.679 \pm 0.020\ . \qquad \text{(III)}$$

Summary of experimental Results

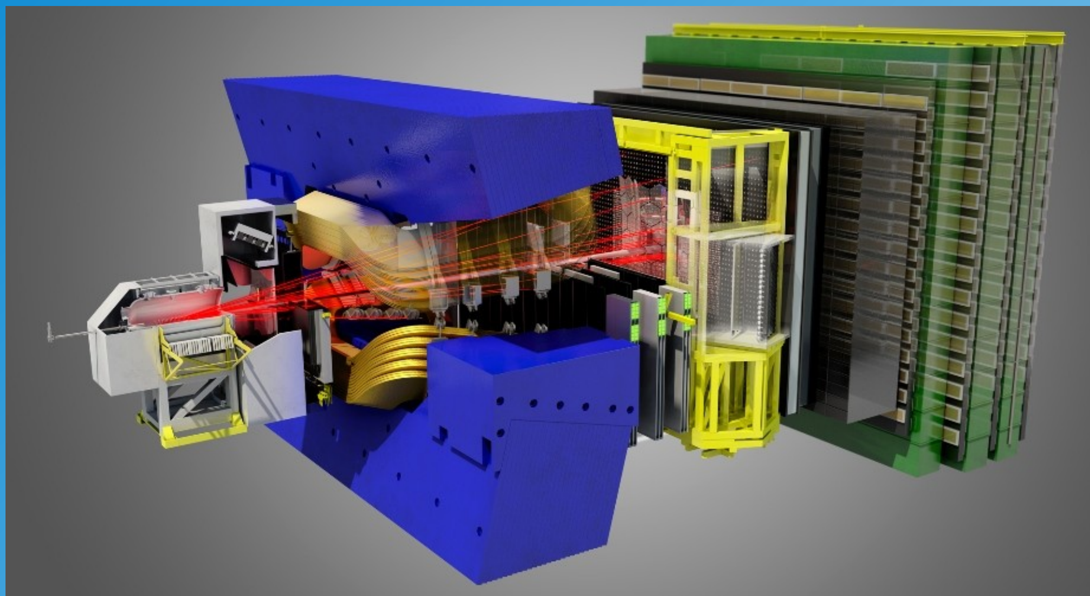


All measurements very consistent! No sign of non-CKM CP violation

Summary

- **The CKM Matrix elements are determined from a global fit to precision measurements**
- **The CKM matrix is measured to be unitary and has a non-zero CP violating phase**
- **CP violation can be measured in particle decays if at least two diagrams with different strong and weak phases interfere**
- **CP violation shows up in decays and mixing (oscillations)**
- **In Kaon system CP violation in mixing dominates**
- **In B-system CP violation in decays with and w/o mixing dominates**
- **CP violation in hadron sector fully explained by CKM matrix**

Running + Future Experiments



LHCb (CERN) Running!

B-physics in pp collisions

Belle2 (SuperKEKB, Japan)

$e^+ e^- \to B \, \overline{B}$

being commissioned!