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Modelling QCD

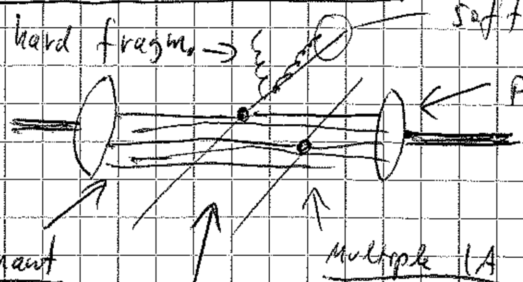
Overview

1. Motivation
2. Factorisation
3. Parton Evolution / Fragmentation
4. Hadronisation
5. All Order Monte Carlo

Non pert. QCD has to be correctly modelled to test pert. QCD!

1. Motivation

reaction $pp \to X$



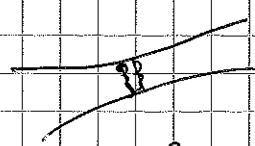
hard process (perturbative)

$Q^2 \gg 1$ GeV

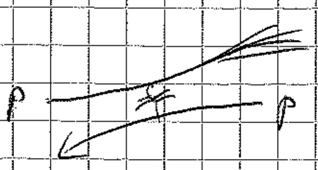
⇒ Implementation in Monte Carlo
- Initial State Processes
- hard process
- final state Processes

Soft Interactions: (non-pert) ⇒ dominant cross section @ LHC

$pp \to pp$ diffractive scattering



$pp \to p N^*(X)$ single dissociative



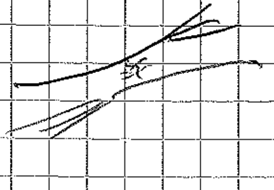
- Regge Model (π, ρ, ω)
- Pomeron Model (gluon ladder)

⇒ effective model

$$\sigma_{tot} = \frac{1}{M_i^2}\left(\frac{s}{M_i}\right)^{\alpha_i(0)-1}$$

with $\alpha_{IP}(0) \simeq 1.08$

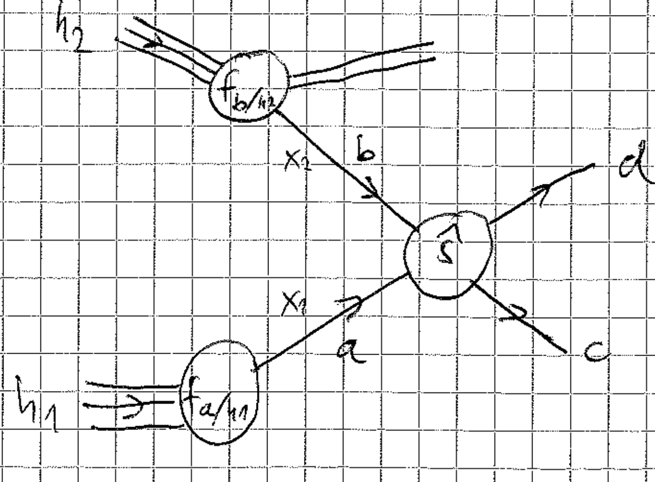
$pp \to N^* N^*(X)$ double dissociative



⇒ different models used to describe non-pert. QCD

2. Factorisation

hard process : $ab \to cd$



$$d\sigma_{h_1 h_2 \to cd} = \int_0^1 dx_1 dx_2 \sum_{a,b} f_{a/h_1}(x_1, \mu_F^2)\, f_{b/h_2}(x_2, \mu_F^2) \times d\hat{\sigma}(ab \to cd)\,(Q^2, \mu_F)$$

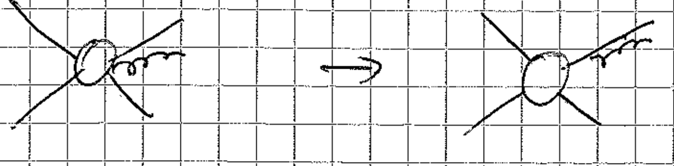
Assumption: no interference between subprocesses

Hard process: n = number of final state particles

$$\hat{\sigma} = C_{LO}\,\alpha_s^{n} + C_{NLO}\,\alpha_s^{n+1} + C_{NNLO}\,\alpha_s^{n+2}$$

Note: higher order processes have singularities which have to be absorbed in parton density functions / fragmentation functions

cut value μ_F : factorisation scale



(part of parton shower)

Higher order processes can also be model by parton shower
⇒ but no interferences effects !

Example 2-jet production:

- jet can be quark or gluon
- processes:

	color factor
$gg \to gg$	$\propto C_A^2$
$gq \to gq$	$\propto C_A C_F$
$qq \to qq$	$\propto C_F^2$
$gg \to qq$	$\propto C_A T_F$
$qq \to gg$	$\propto C_A T_F$

PDFs for gluon + quarks needed !

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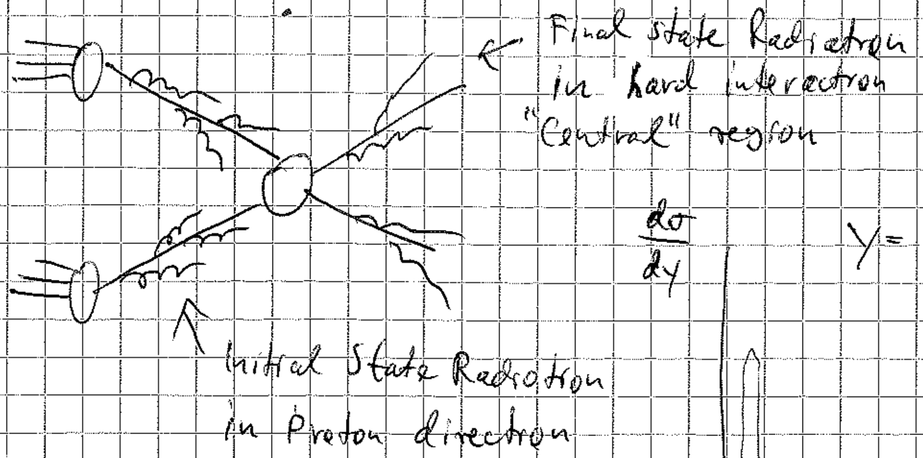
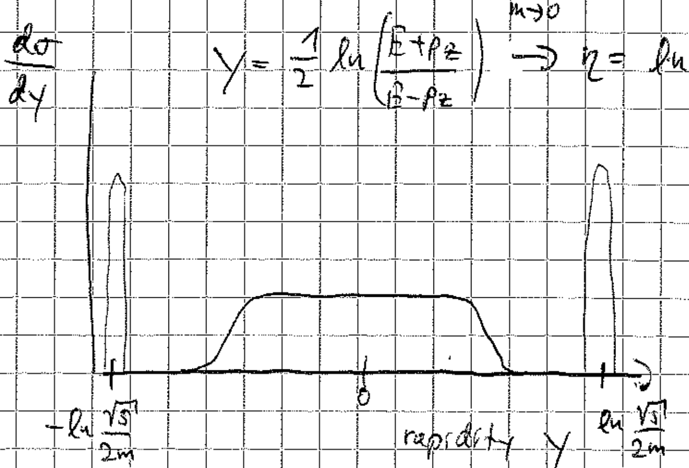
3. Parton Evolution / Fragmentation



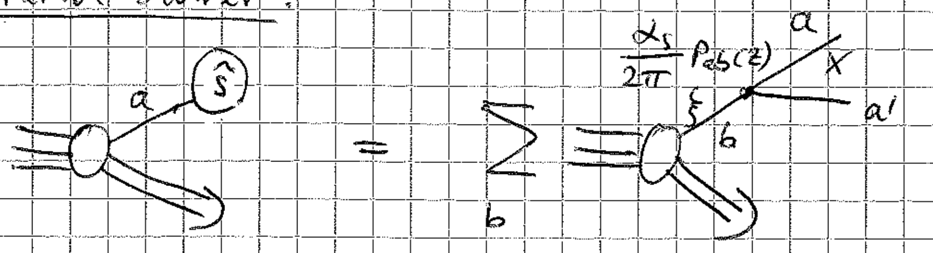
$$y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right) \xrightarrow{m\to 0} \eta = \ln\left(\cot\left(\frac{\theta}{2}\right)\right)$$



LHC $\sqrt{s} = 14\,\text{TeV}$, $m = 1\,\text{GeV}$

$\ln 7000 \approx 8.9$

3.1 Parton Shower:



Splitting (Kernel) Functions $P_{ab}(z)$

in collinear approximation (transverse momentum k_t is small)

$z = \frac{x}{\xi}$

$$P_{qq}(z) = C_F\left(\frac{1+z^2}{1-z}\right)$$

$$P_{qg}(z) = T_F\left(z^2 + (1-z)^2\right)$$

$$P_{gg}(z) = 2C_A\left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right)$$

$$P_{gq}(z) = C_F\,\frac{1+(1-z)^2}{z}$$

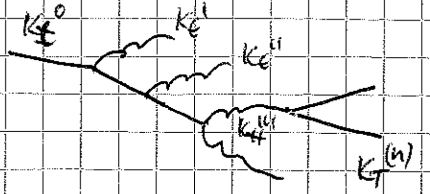
Monte Carlo:

difficult to implement $\frac{1}{z}$, $\frac{1}{1-z}$ poles !

⇒ Sudakov factors → soft part absorbed in PDF !
↳ scale dependence μ_F

Parton Evolution

- space-like evolution



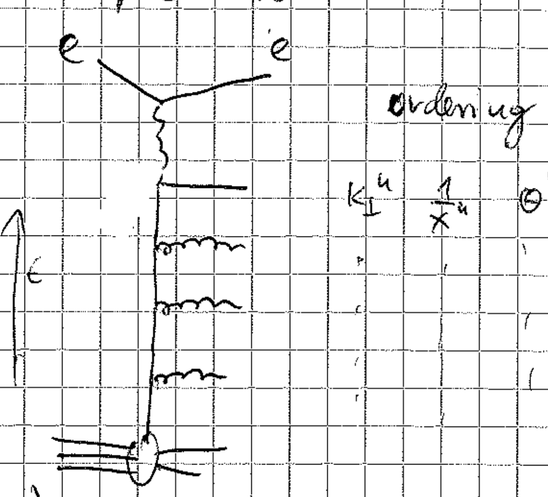
transverse momentum increases

$k_t^{(n)} > k_t^0$

bare k_t^0 given by proton Fermi motion

- unresolved part absorbed in PDF
- different evolution schemes
  - DGLAP ($k_\perp$ ordered)
  - BFKL (x - ordered) (Balitsky, Fadin, Kuraev, Lipatov)
  - CCFM (angular ordering) (Catani, Ciafaloni, Fiorani, Marchesini)

example DIS



PDF:

$$\mu \frac{\partial}{\partial \mu} f_{a/h}(x,\mu) = \int_x^{1-\epsilon(\mu)} \frac{dz}{z} \frac{\alpha_s(\mu,z)}{2\pi} P_{ab}(z) f_{b/h}\left(\frac{x}{z},\mu\right)$$

$$- f_{a/h}(x,\mu) \sum_{a\to cc'} \int_{\epsilon(\mu)}^{1-\epsilon(\mu)} dz \frac{\alpha_s(\mu,z)}{2\pi} P_{ca}(z)$$

- resolved part is simulated in <u>Monte Carlo</u>! → iteration → Markov process → probabilities

- time-like evolution (final state) [equivalent to space like]
  → fragmentation functions

h = parent / daughter

$$\mu \frac{\partial}{\partial \mu} D_a^h(x,\mu) = \int_x^{1-\epsilon(\mu)} \frac{dz}{z} \frac{\alpha_s(\mu,z)}{2\pi} P_{ba} D_b^h\left(\frac{x}{z},\epsilon\right)$$

$$- D_a^h(x,\mu) \sum_{a\to cc'} \int_{\epsilon(\mu)}^{1-\epsilon(\mu)} dz \frac{\alpha_s(\mu,z)}{2\pi} P_{ca}(z)$$

note: fragmentation function specific for heavy quarks (c, b)
⇒ mass effects

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4. Hadronisation

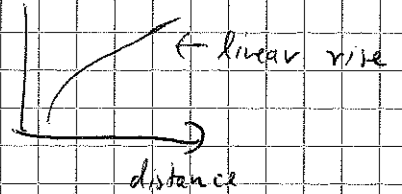
Full fragmentation function

$$D_a^h(x,\mu) = (\text{pQCD evolution } \mu \to \mu_0) \otimes (\text{model: } b \to H|_\mu) \otimes (\text{tables } H \to h_i h_i')$$

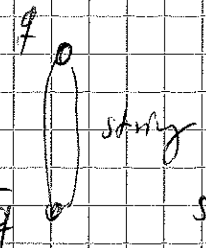
↑ hadronisation ↑ decays

- String hadronization (Lund model, VCLA model)

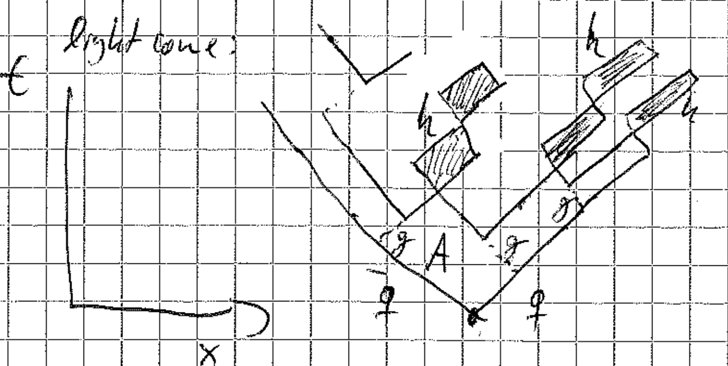
motivated by strong coupling α in confinement regime:



$\Rightarrow$ pot. energy $\propto$ distance



string breaks and creates new $q\bar{q}$ pairs until energy threshold is reached



probability for spring breaking

$$\frac{dP}{dA} = P_0 \, e^{-P_0 A}$$

A = area in $\Delta x \cdot \Delta t$

$\Rightarrow$ Lund fragmentation function

$$f(z) = \frac{N}{z}(1-z)^a \, e^{-b\frac{m_\perp^2}{z}} \quad \text{with } m_\perp^2 = m^2 + p_\perp^2$$

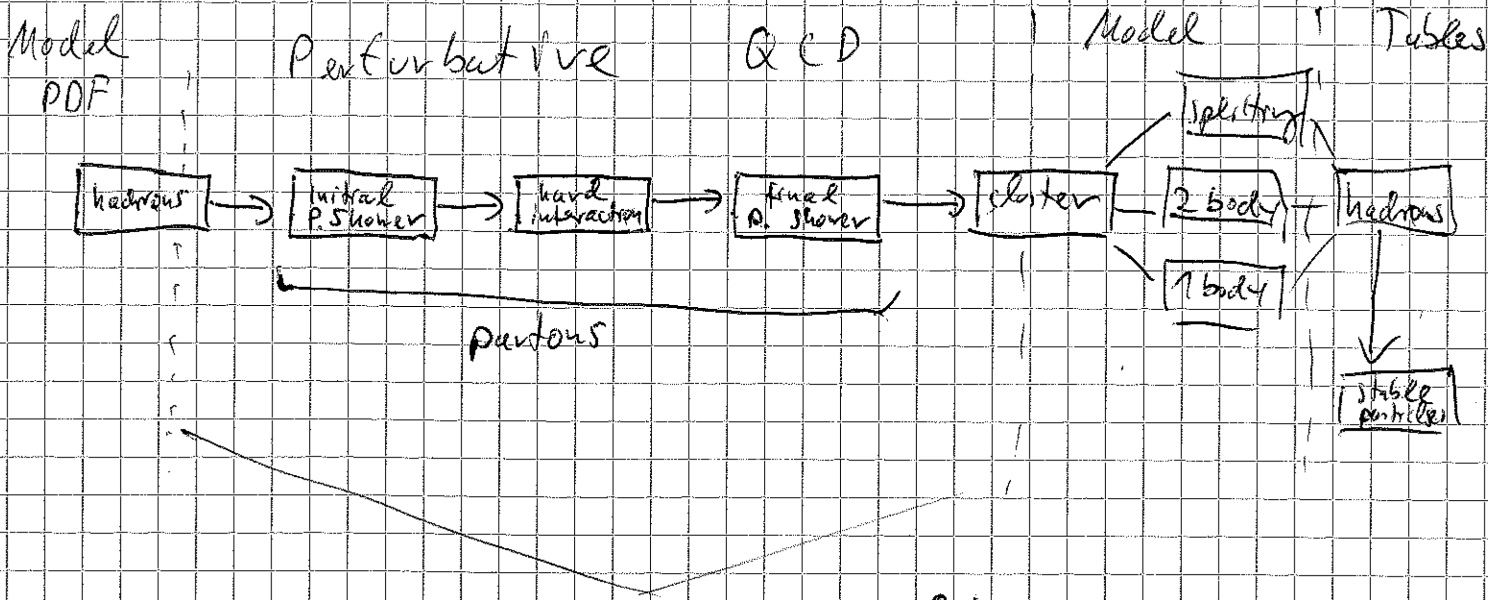
$p_\perp$ is transvers momentum relative to string plane.

- cluster hadronisation:
  - similar, but convert all gluons in $q\bar{q}$ pairs
  - form color singlet cluster combinations (more natural)

  Decays cluster $\Rightarrow h_1 h_2$, cluster $\to h$, cluster $\to$ cluster cluster

5. All Order Monte Carlos

• Summary



all Order Monte Carlo ?!

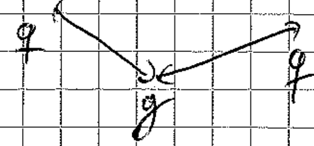
Note: Parton showers model higher orders!

Note!
LO → NLO : problem with negative cross section
(hard process) when merging pQCD + parton showers!

Issues: Soft gluon logarithms (not treated by collinear ansatz)
↳ angular ordering

• Color Dipole Model (CDM)

– treat QCD like e.m. dipoles → dipol radiation



2 dipoles

example: $gg \to q\bar{q}$
dipole between initial
+ final state ⇒ interference!