Lecture:

Standard Model of Particle Physics

Heidelberg SS 2016

Weak Interactions I
Low Energy

Spinors and Helicity States

momentum vectors in +z direction

$$\psi_R = |\vec{p}, \lambda = +1/2\rangle$$
$$\psi_L = |\vec{p}, \lambda = -1/2\rangle$$
$$\bar{\psi}_L = |\vec{p}, \lambda = -1/2\rangle$$
$$\bar{\psi}_R = |\vec{p}, \lambda = +1/2\rangle$$

fermions:

$$\psi = u\, e^{+i(pz - Et)} \qquad u_R = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{|\vec{p}|}{E+m} \\ 0 \end{pmatrix} \qquad u_L = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-|\vec{p}|}{E+m} \end{pmatrix}$$

anti-fermions:

$$\psi = v\, e^{-i(pz - Et)} \qquad v_L = \sqrt{E+m}\begin{pmatrix} \frac{|\vec{p}|}{E+m} \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad v_R = \sqrt{E+m}\begin{pmatrix} 0 \\ \frac{-|\vec{p}|}{E+m} \\ 0 \\ 1 \end{pmatrix}$$

limit $p \rightarrow \infty$ $\qquad u_R \rightarrow v_L \qquad u_L \rightarrow -v_R$

Chirality Operator

limit: $m \to 0, p \to \infty$

$$u_R \sim v_L \sim \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad u_L \sim v_R \sim \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

operator: $$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \underline{1} \\ \underline{1} & 0 \end{pmatrix}$$

right chiral states

$\gamma_5 u_R = u_R$

$\gamma_5 v_L = v_L$

left chiral states

$\gamma_5 u_L = -u_L$

$\gamma_5 v_R = -v_R$

eigenvalues ± 1

left-handed (chiral) particles: -1

right-handed (chiral) particles: +1

note: a right-handed chiral anti-particle has a left-handed helicity

Projection Operator

Definition: $\Pi^{\pm} = \frac{1 \pm \gamma_5}{2}$

in the limit of |E| → infinity

fermions		anti-fermions	
$\Pi^{+} \; u_R = u_R$	$\Pi^{+} \; u_L = 0$	$\Pi^{+} \; v_L = v_L$	$\Pi^{+} \; v_R = 0$
$\Pi^{-} \; u_L = u_L$	$\Pi^{-} \; u_R = 0$	$\Pi^{-} \; v_R = v_R$	$\Pi^{-} \; v_L = 0$

can reformulate Dirac Equation:

$$i\gamma^{\mu}\partial_{\mu} u_R = m\, u_L \qquad\qquad i\gamma^{\mu}\partial_{\mu} u_L = m\, u_R$$

note: massive fermions must have left-handed and right handed components

Recap

General Four Fermion Lagrangian

$$L = \frac{G}{\sqrt{2}} \left(\bar{f}\ \Gamma f\,' \right) \left(\overline{f\,''}\ \tilde{\Gamma} f\,''' \right)$$

Vector Current:

$$j_V^\mu = \bar{\psi} \gamma^\mu \psi$$

Axial-vector Current:

$$j_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$$

scalar coupling:

$$c_S = \bar{\psi} \psi$$

pseudoscalar coupling:

$$c_{PS} = \bar{\psi} \gamma^5 \psi$$

Tensor Coupling

$$\sigma_A^{\mu\nu} = \bar{\psi} \left(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu \right) \psi$$

Vector, Axial and Scalar Currents

Vector Current:

$$j_V^\mu = \bar{u}\gamma^\mu u$$

in QED: $\partial_\mu j_V^\mu = 0$

$$\bar{u}\gamma^\mu u = \bar{u}_L\gamma^\mu u_L + \bar{u}_R\gamma^\mu u_R$$

(no helicity flip)

Axial-vector Current:

$$j_A^\mu = \bar{u}\gamma^\mu\gamma^5 u$$

note: $\gamma^\mu\gamma^5 = -\gamma^5\gamma^\mu$

$$\bar{u}\gamma^\mu\gamma^5 u = \bar{u}_L\gamma^\mu\gamma^5 u_L + \bar{u}_R\gamma^\mu\gamma^5 u_R$$

(no helicity flip)

Scalar Coupling

$$\bar{u}u = \bar{u}_R u_L + \bar{u}_L u_R$$

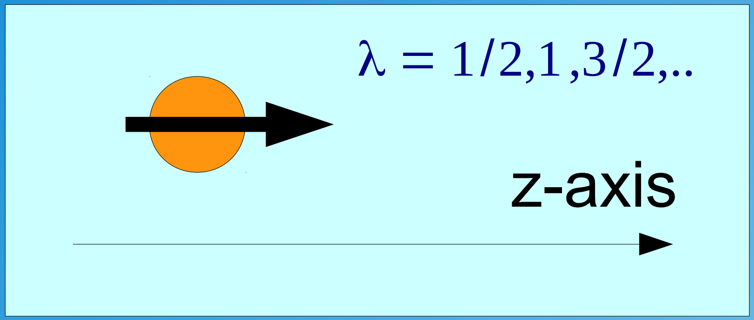
(helicity flip!)

Relations:

$$j_L^\mu = 1/2\,(j_V^\mu - j_A^\mu) \qquad j_R^\mu = 1/2\,(j_V^\mu + j_A^\mu)$$

Helicity Discussion I

Particle at rest with spin orientation in +z direction:

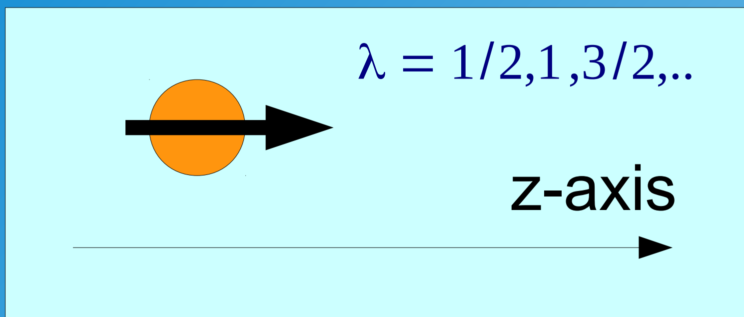


polarisation: $\lambda = \vec{j}\,\vec{z} = \frac{N_{+1/2} - N_{-1/2}}{N_{+1/2} + N_{-1/2}}$

helicity: $H = \frac{\vec{j}\,\vec{p}}{|\vec{p}|}$ (classical)

Helicity Discussion II

Particle at rest with spin orientation in +z direction:



polarisation: $\lambda = \vec{j}\,\vec{z} = \frac{N_{+1/2} - N_{-1/2}}{N_{+1/2} + N_{-1/2}}$

helicity: $H = \frac{\vec{j}\,\vec{p}}{|\vec{p}|}$ (classical)

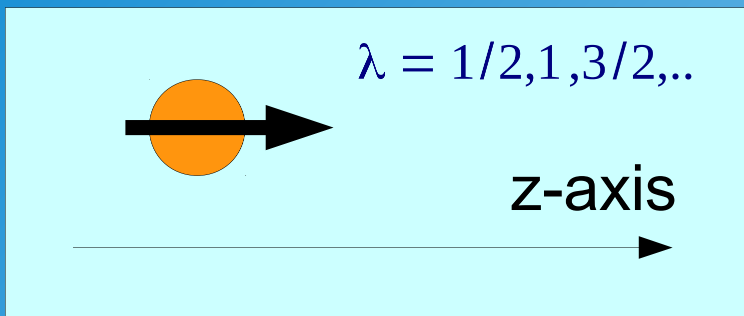
- Classical: states with defined helicities ($H = \pm 1$) can be prepared.
- Quantum mechanics: spin and momentum are replaced by operators

- For massive particles the helicity is not Lorentz invariant!

For $p \rightarrow p' = -p$ the helicity makes a flip: $H \rightarrow H' = -1$

Helicity Discussion III

Particle at rest with spin orientation in +z direction:



polarisation: $\lambda = \vec{j}\,\vec{z} = \dfrac{N_{+1/2} - N_{-1/2}}{N_{+1/2} + N_{-1/2}}$

helicity: $H = \dfrac{\vec{j}\,\vec{p}}{|\vec{p}|}$ (classical)

In a Lorentz invariant theory, particle interactions can not be described by non-Lorentz invariant quantities!

- Solution to this problem is provided by the Dirac equations

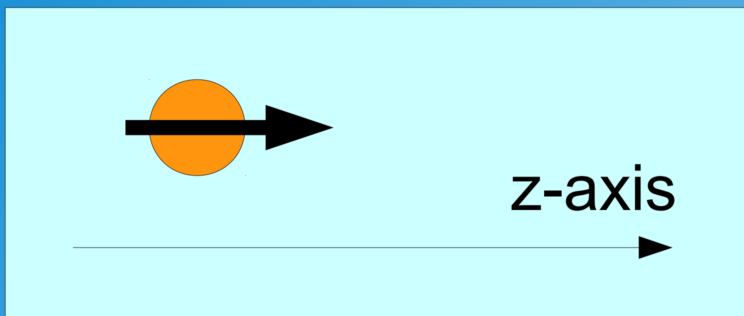
Dirac spinors: $u = u_L + u_R$

<u>Remark</u>:
only the chiral states
in the limit $p \rightarrow \infty$
are Lorentz invariant

$$u_R = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad u_L = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

Helicity Example

Particle at rest ($p=0$) with spin orientation in +z direction:



polarisation: $\lambda = \vec{j}\,\vec{z} = \dfrac{N_{+1/2} - N_{-1/2}}{N_{+1/2} + N_{-1/2}}$

$$u = N_{-1/2}\, u_L + N_{+1/2}\, u_R$$

$$u = N_{-1/2}\sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-|\vec{p}|}{E+m} \end{pmatrix} + N_{+1/2}\sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{|\vec{p}|}{E+m} \\ 0 \end{pmatrix} = N_{-1/2}\sqrt{2\mathrm{m}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + N_{+1/2}\sqrt{2\mathrm{m}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Projection of right/left helicity state

$$\Pi^{+} u = \frac{1+\gamma_5}{2} u = N_{-1/2}\frac{\sqrt{2m}}{2}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + N_{+1/2}\frac{\sqrt{2m}}{2}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

50% right chiral states

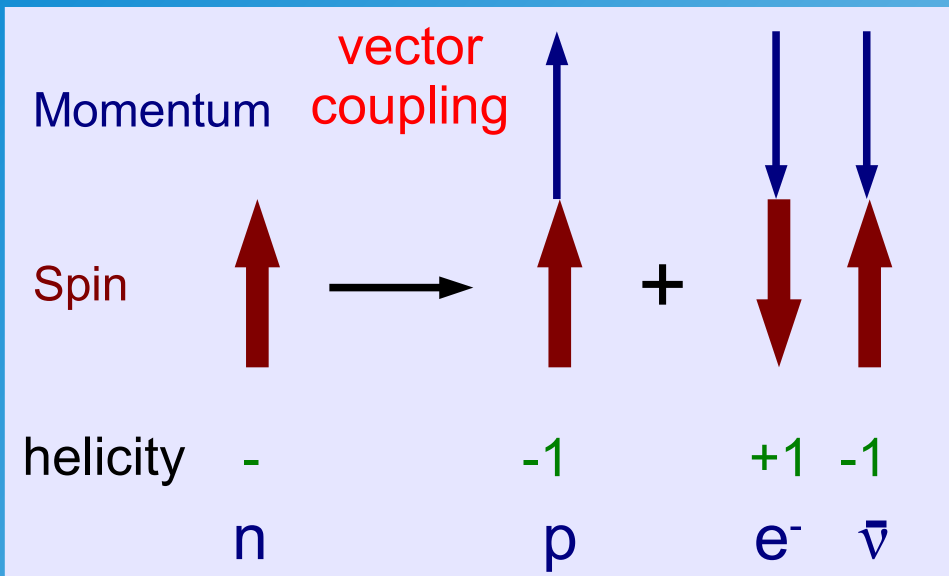
$$\Pi^{-} u = \frac{1-\gamma_5}{2} u = N_{-1/2}\frac{\sqrt{2m}}{2}\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} + N_{+1/2}\frac{\sqrt{2m}}{2}\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

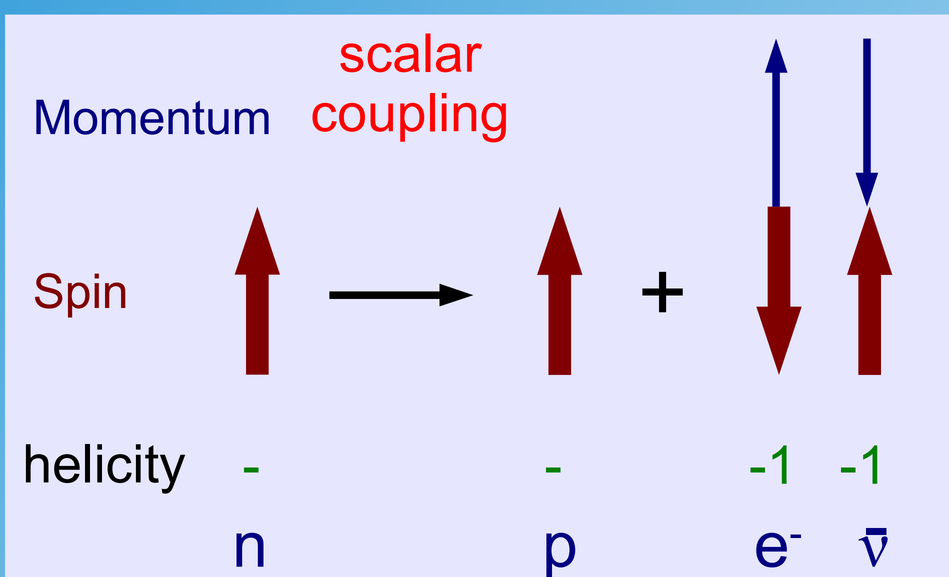
50% left chiral states

Test of Lorentz Structure in Beta Decays

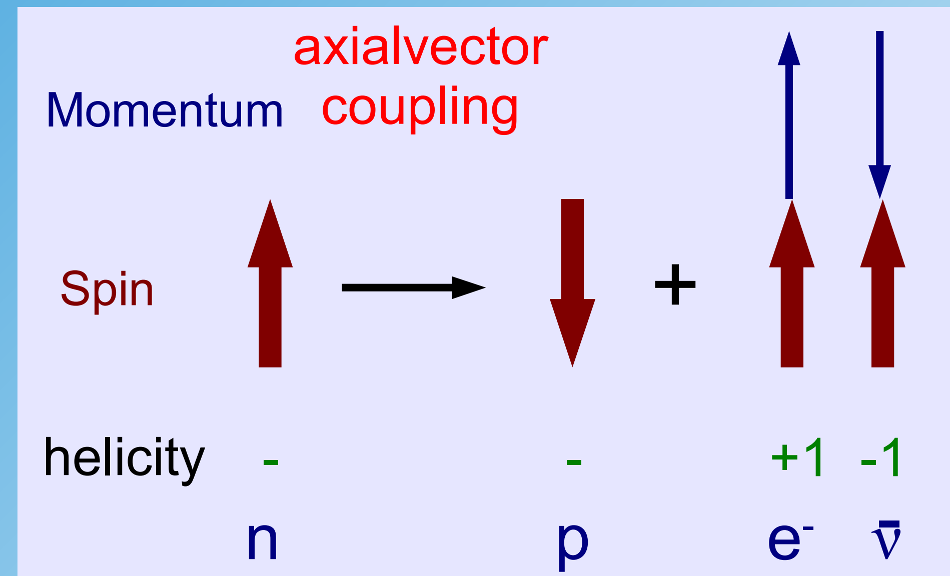
Fermi transition

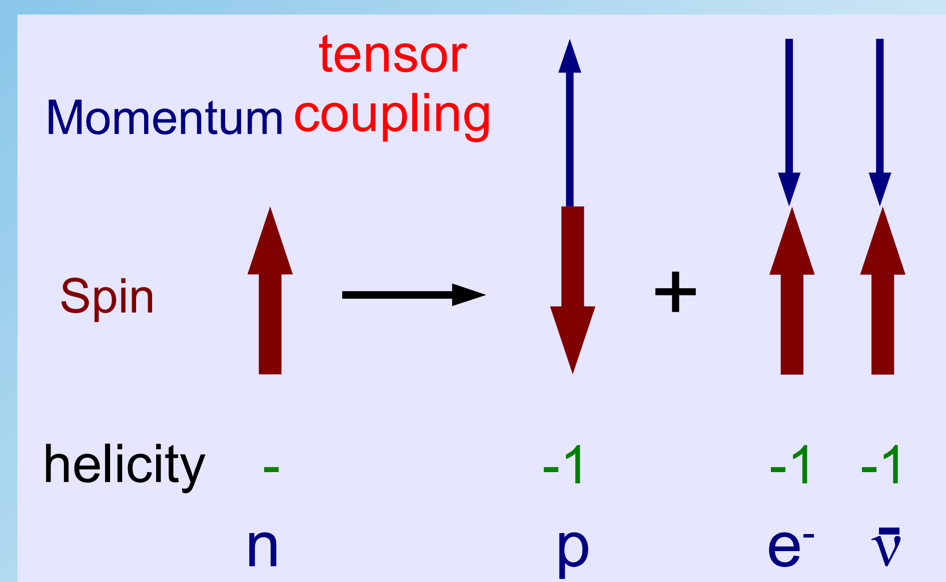
LH anti-neutrino helicity





Gamov Teller transition

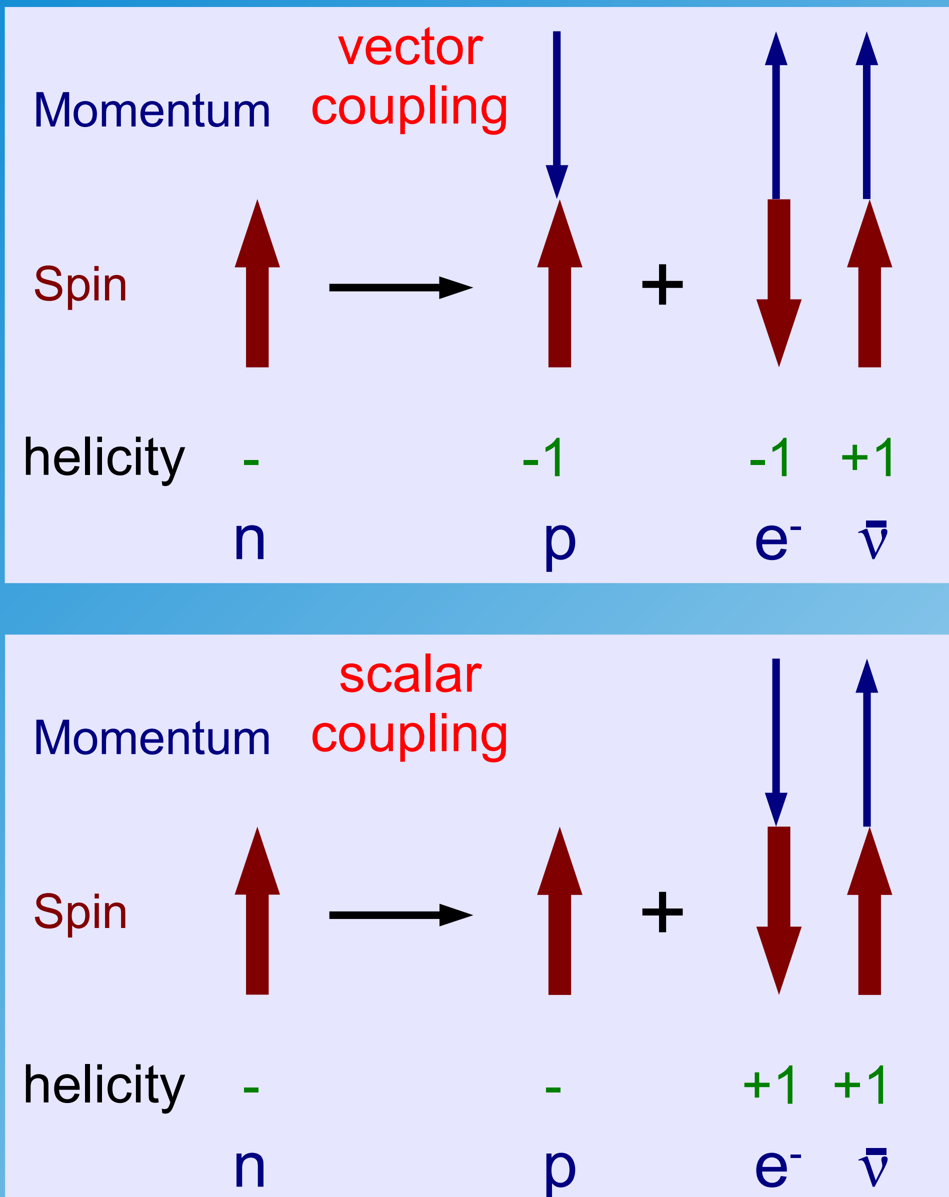




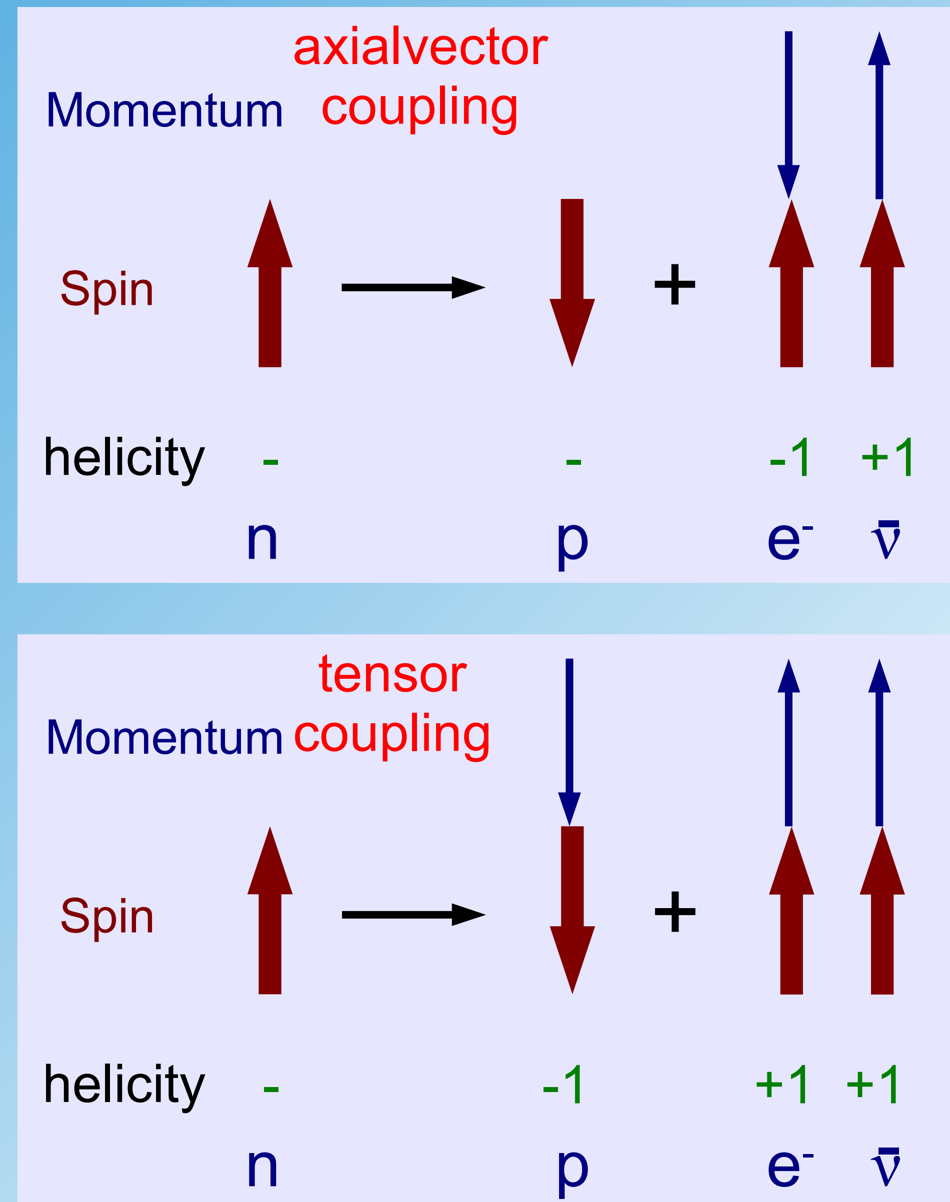
Test of Lorentz Structure in Beta Decays

RH anti-neutrino helicity

<u>Fermi transition</u>

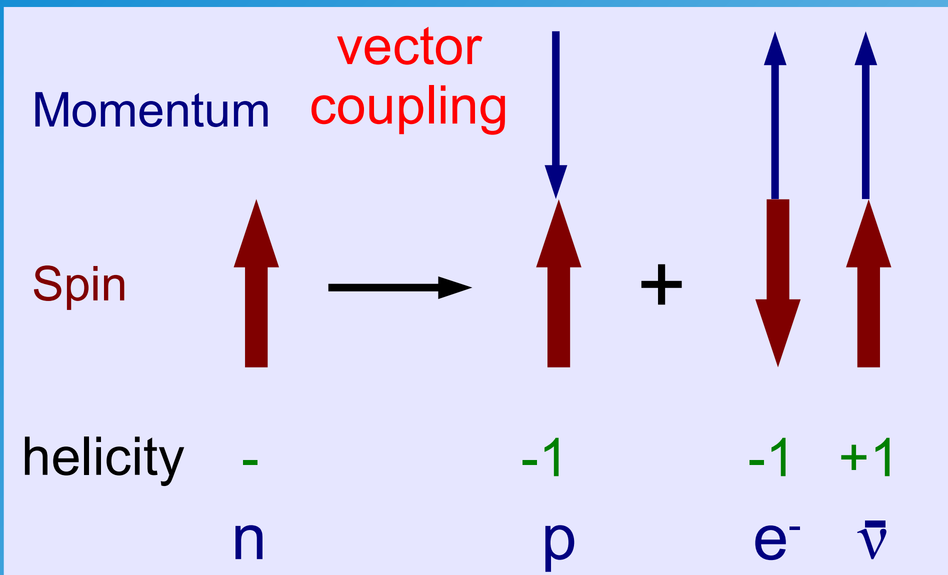


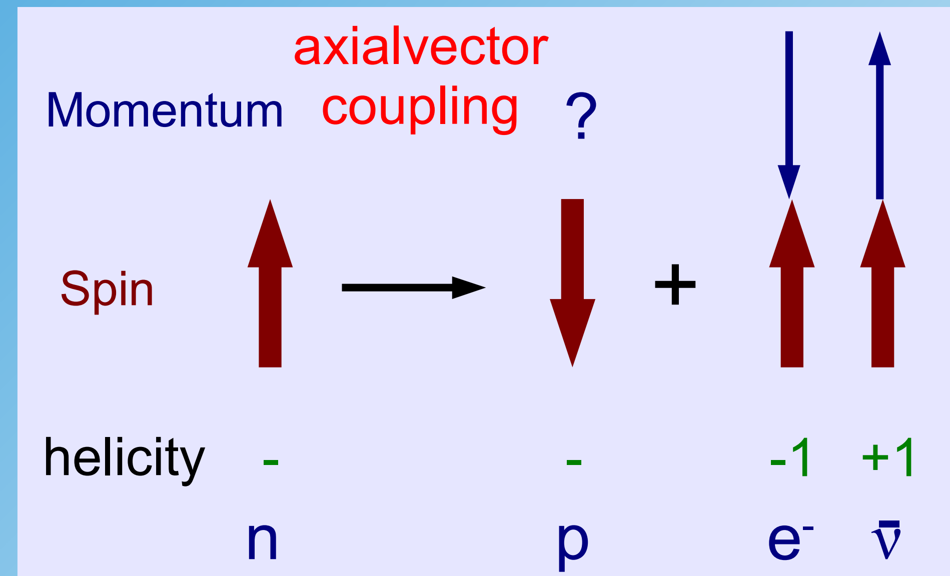
<u>Gamov Teller transition</u>

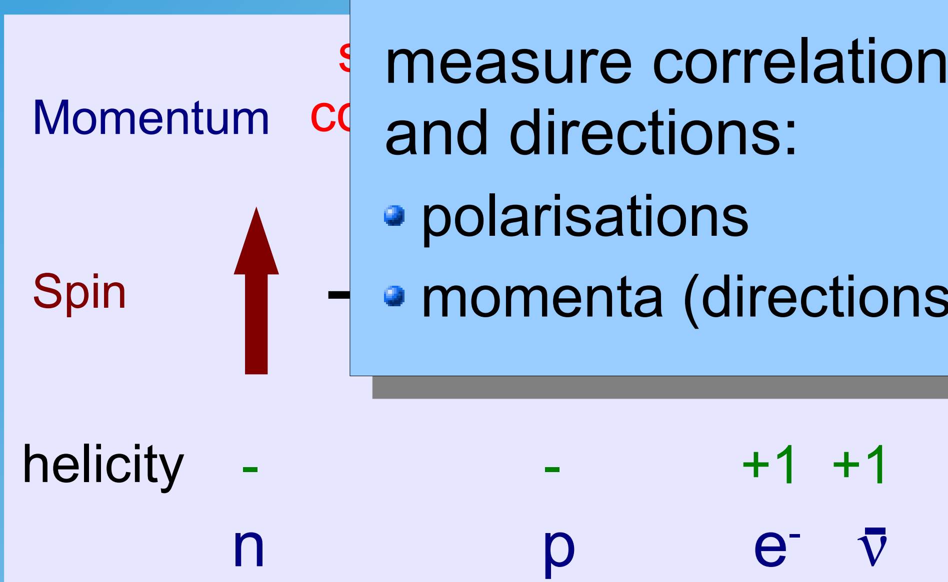


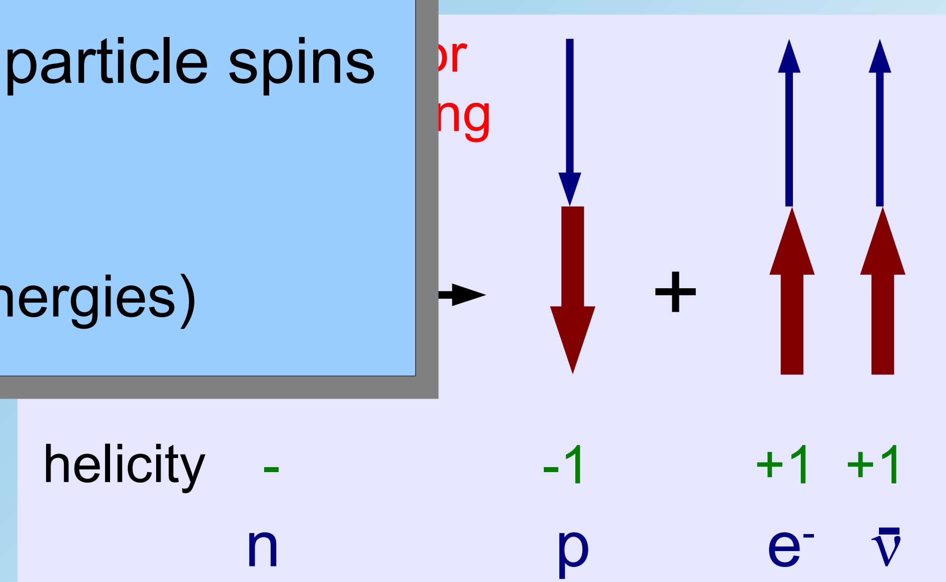
Test of Lorentz Structure in Beta Decays

<u>Fermi transition</u> RH anti-neutrino helicity <u>Gamov Teller transition</u>









measure correlations of particle spins and directions:

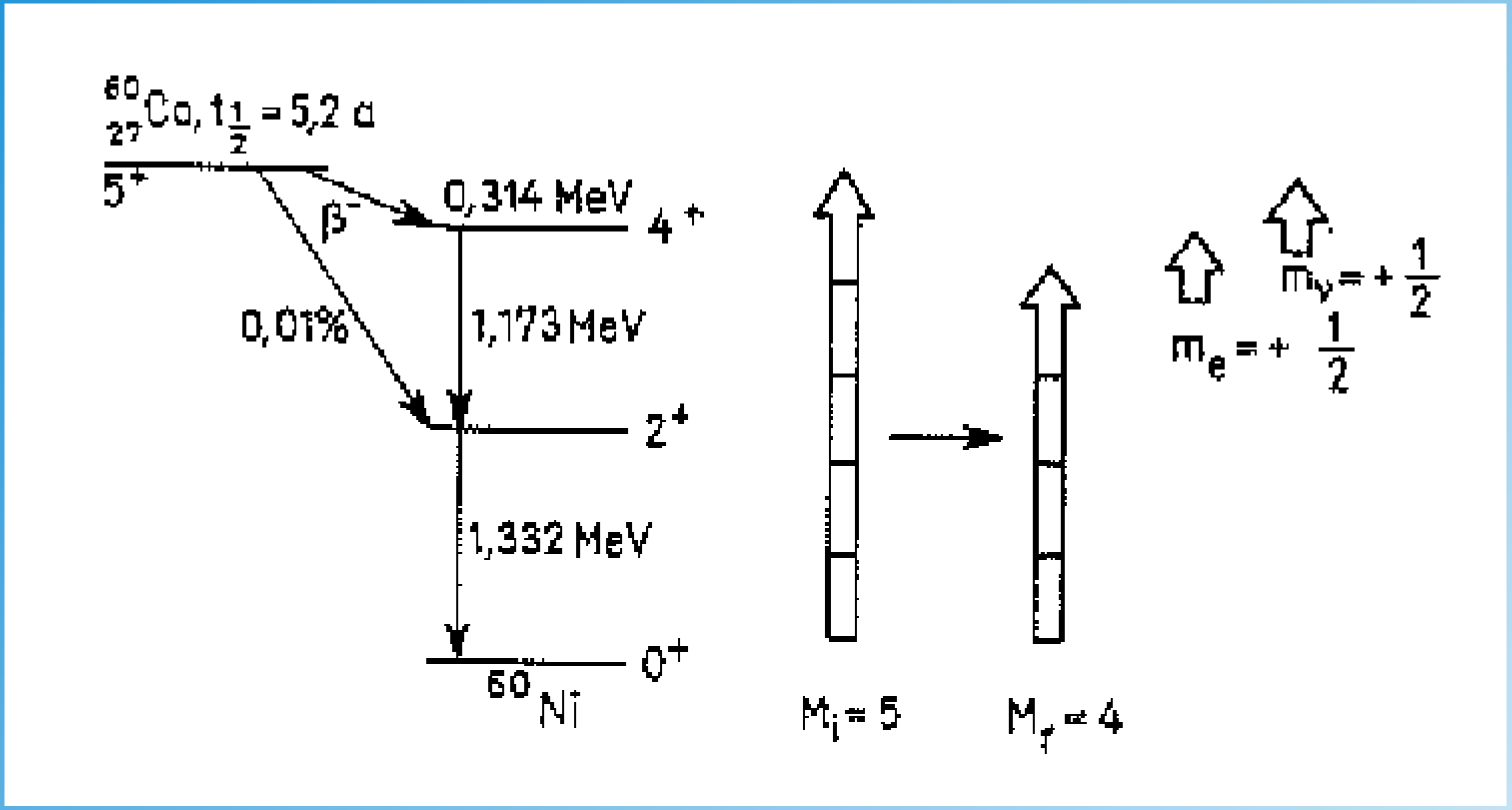
- polarisations
- momenta (directions + energies)

Important Experiments

- Wu-Experiment (1957): radioactive decay of Co^{60}
- Goldhaber-Experiment (1958): radioactive decay of Eu^{152}
- Muon Decay: Michel spectrum
- Nuclear Beta Decays
- Pion Decay: branching ratios
- Neutrino Nucleon Scattering: neutrino-antineutrino

Idea of the Wu-Experiment

$^{60}Co \rightarrow {}^{60}Ni + e^- + \bar{\nu}$

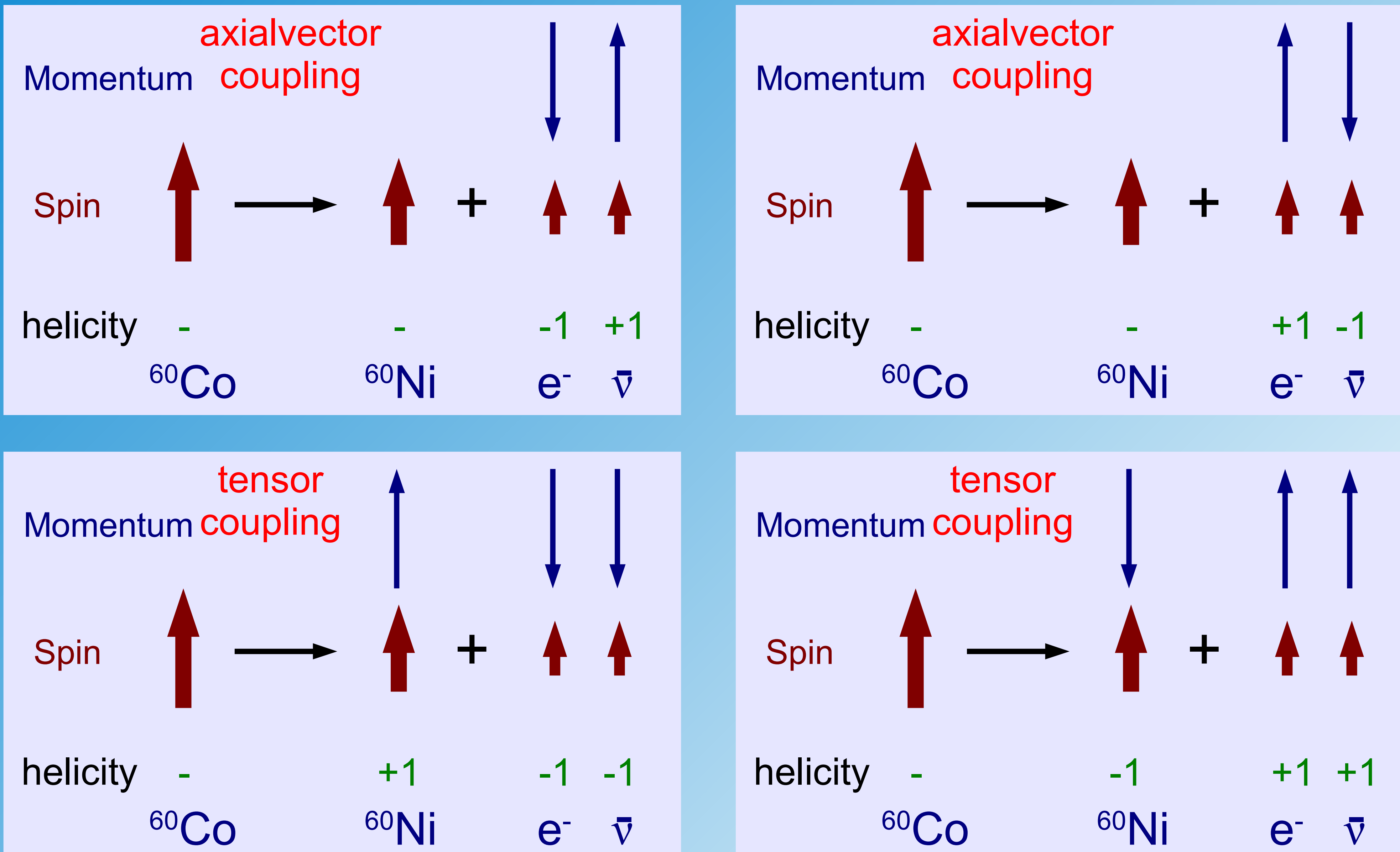


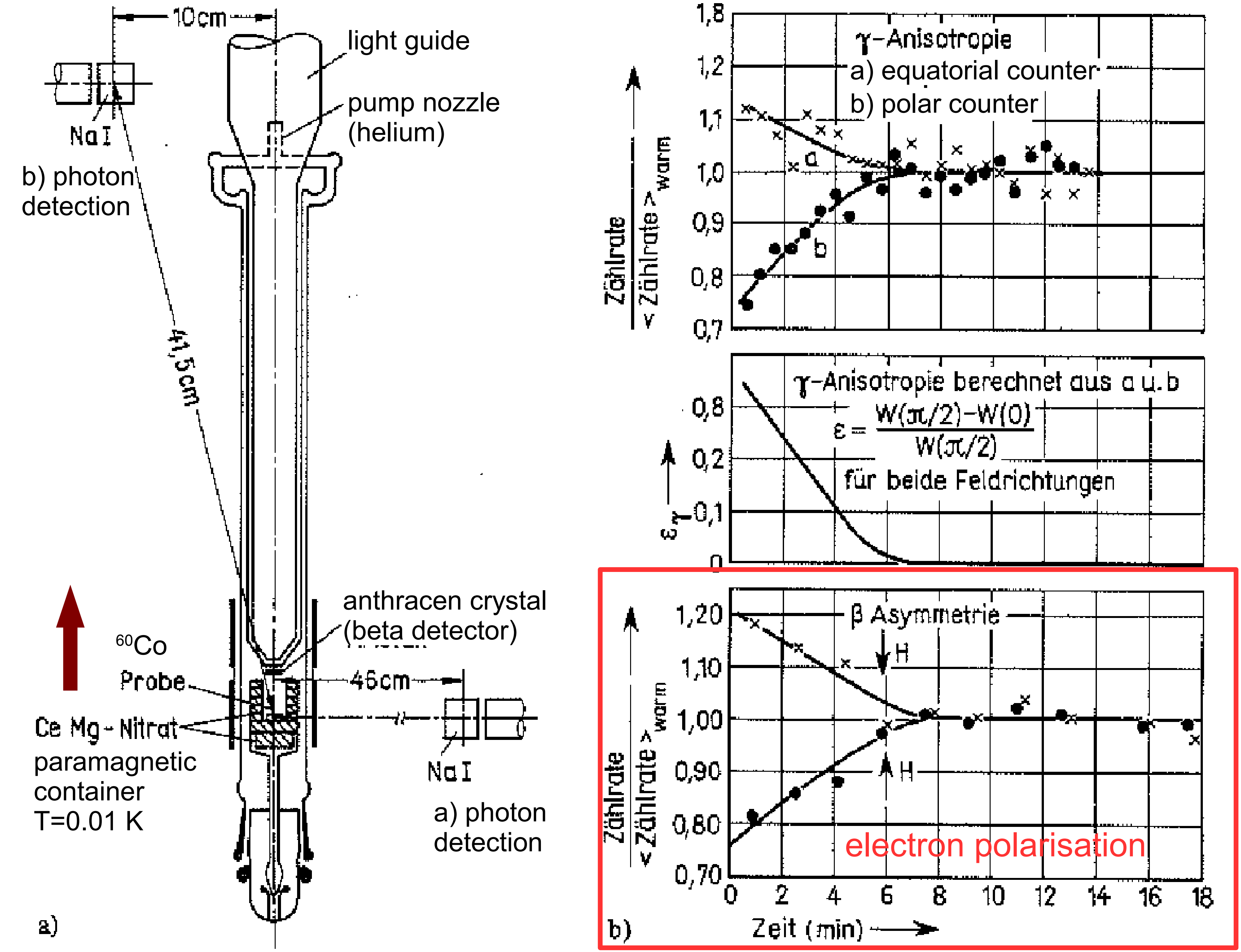
- High Spin of Cobalt leads to Gamow-Teller transition (S_{ev}=1)
- Polarisation of electron (neutrino)?

$$\lambda(e^-) = ?$$

Test of Lorentz Structure in ^{60}Co

Gamov Teller transitions



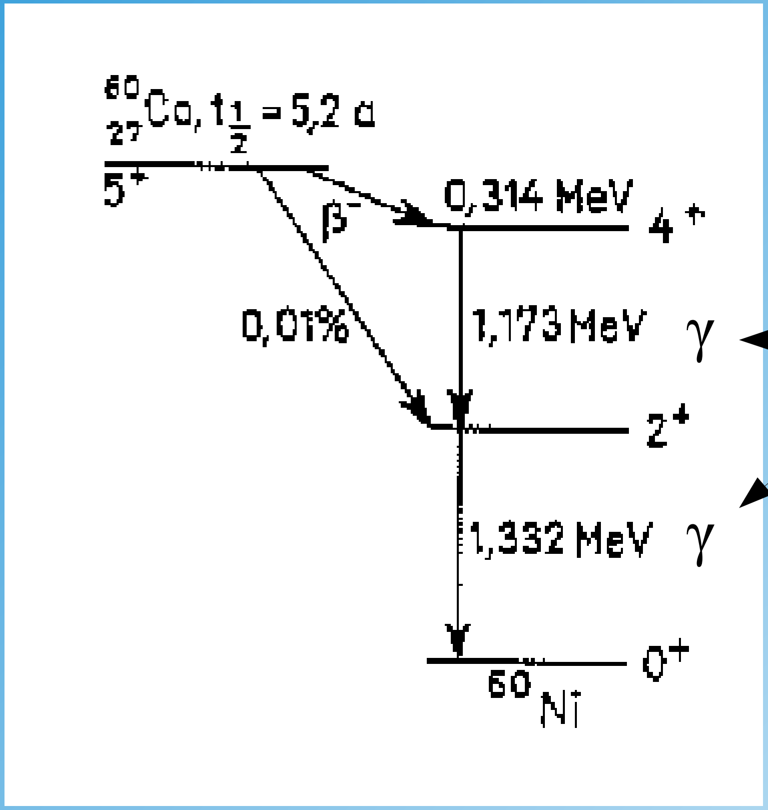

10cm
light guide
pump nozzle (helium)
NaI
b) photon detection
41,5cm
anthracen crystal (beta detector)
^{60}Co
Probe
46cm
Ce Mg - Nitrat
paramagnetic container
T=0.01 K
NaI
a) photon detection
a)
γ-Anisotropie
a) equatorial counter
b) polar counter
Zählrate / <Zählrate>warm
γ-Anisotropie berechnet aus a u. b
ε = W(π/2)−W(0) / W(π/2)
für beide Feldrichtungen
ε_γ
β Asymmetrie
H
electron polarisation
Zeit (min)
b)

Measurement of the Ni*-Polarisation

important cross check!

- ^{60}Ni*(J=4) is produced in an excited state!
- Beta Decay followed by photo-nuclear decay:

$$^{60}\mathrm{Ni}^* \longrightarrow {}^{60}\mathrm{Ni}^*\ \gamma$$



Photons are polarised and oriented (symmetrically) in direction of the Ni (Co) polarisation axis. Maximum is orthogonal to Ni (Co) polarisation

Conclusion Wu-Experiment

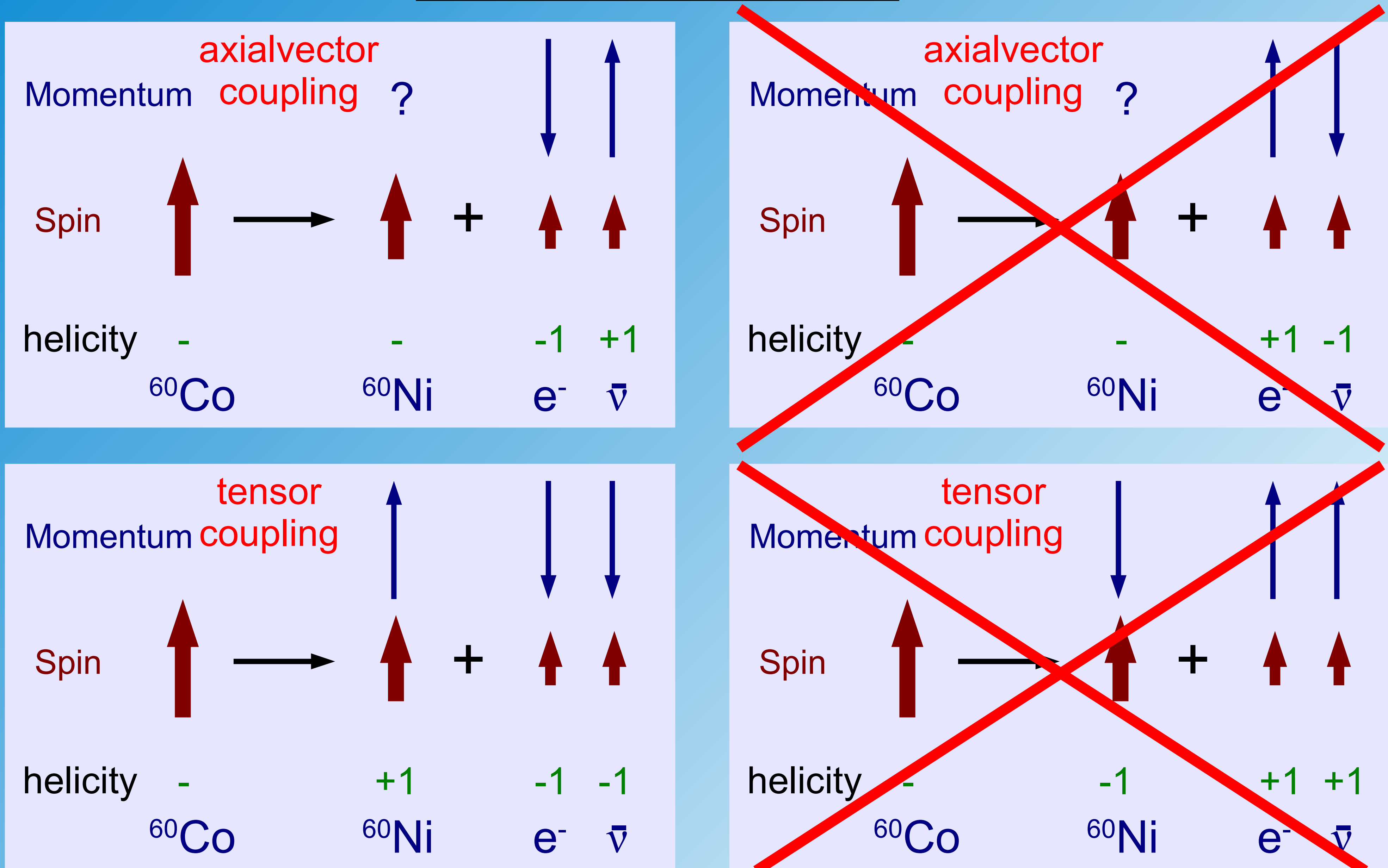
- Electrons are dominantly emitted opposite to Co spin direction
- Product **J · p / |p|** (helicity) is non-zero!
- Helicity is negative!
- Discrete-Parity symmetry is violated (initial state had H=0)

Note: the angular distribution of the electrons is given by:

$$\frac{dN}{d\cos\theta} \propto 1 + A\cos\theta$$

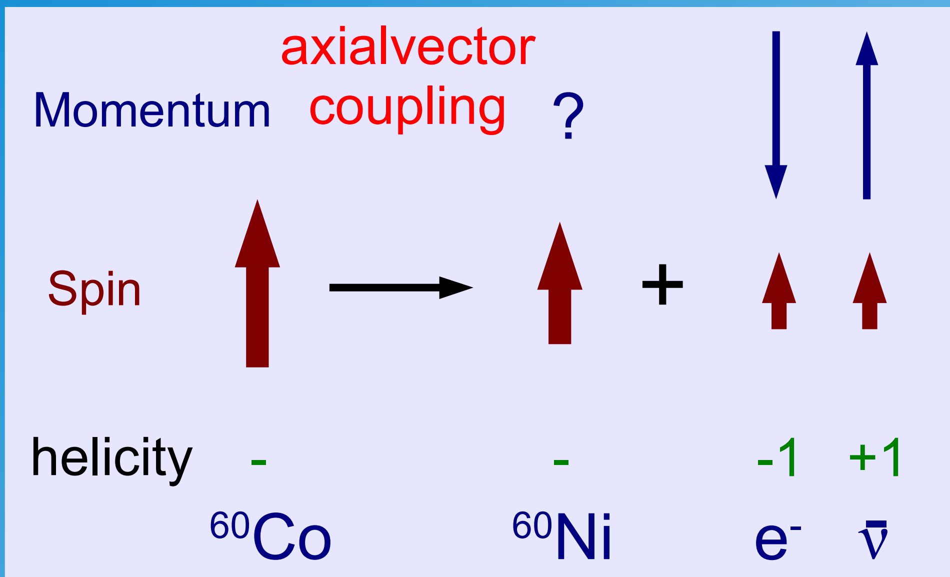
Test of Lorentz Structure in ^{60}Co

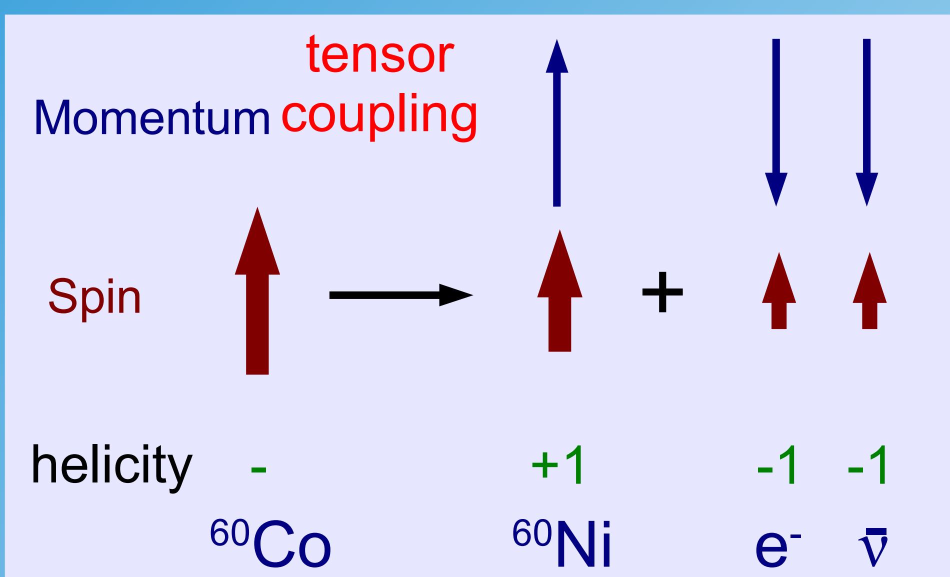
Gamov Teller transitions



Test of Lorentz Structure in ^{60}Co

Gamov Teller transitions

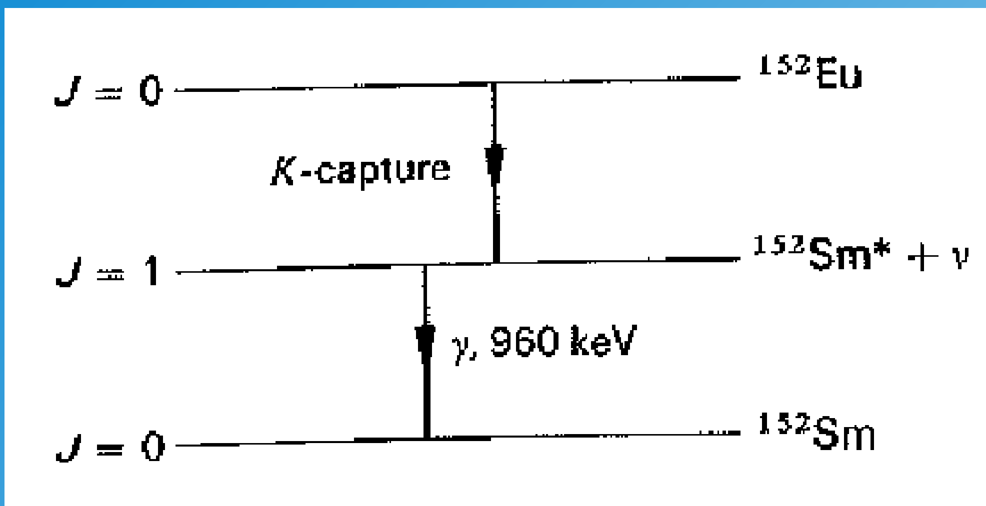


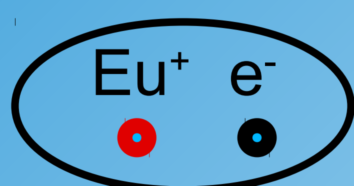


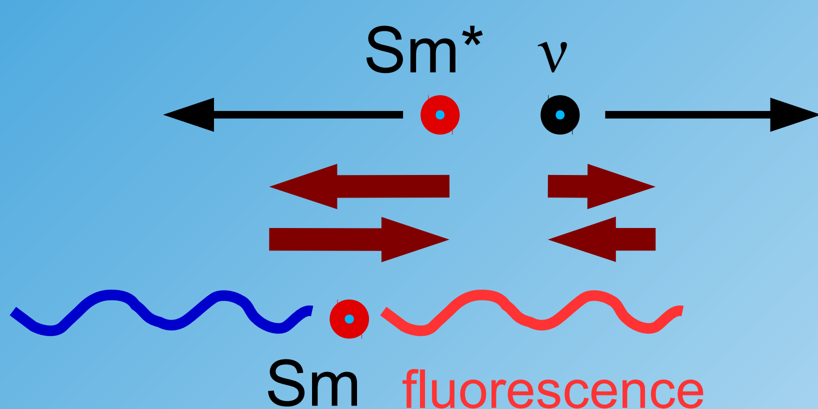
What about the neutrino helicity?

Europium Decay Chain

Lifetime ^{152}Eu = 13.5 a







$^{152}Eu + e^- \to {}^{152}Sm^* + \nu$

- Europium has no nuclear spin
- Sm* has nuclear spin
- Polarisation of neutrino and Sm* are opposite!
- The neutrino polarisation can be measured by determining the Sm* polarisation
- Luckily Sm* decays further:
  $Sm^* \to {}^{152}Sm + \gamma$
- photon is of low energy if emitted opposite to Sm* flight direction
  - fluorescence
- exploit Compton scattering to determine photon polarisation

Goldhaber Experiment

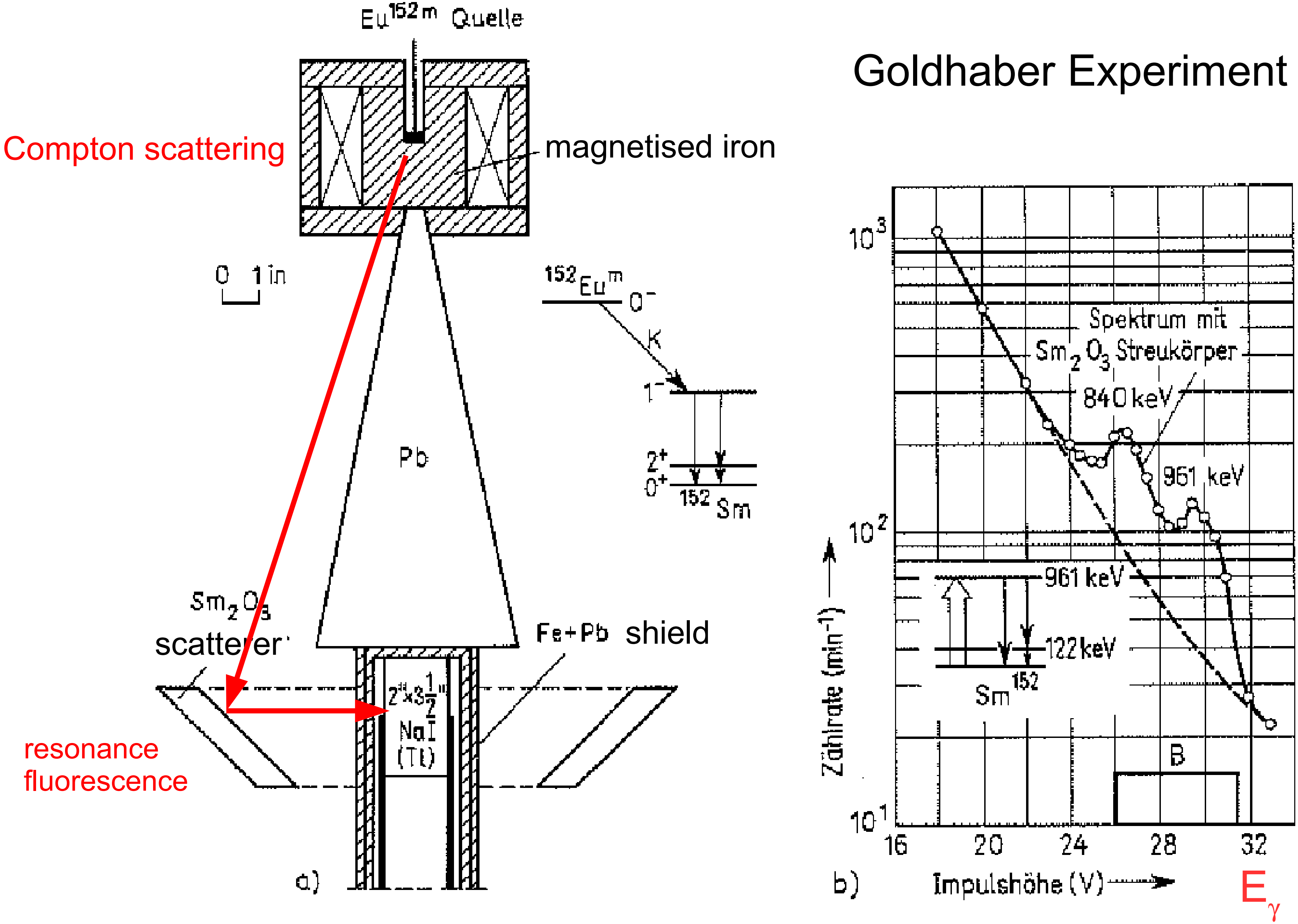


Fig. 144 a) Anordnung zur Messung der Helizität des Neutrinos (Goldhaber u. Mitarbeiter; b) Impulsverteilung für das γ-Streuspektrum. Gestrichelt: nicht-resonanter Untergrund; nach [Gol 58]

Goldhaber: Result + Conclusion

- Due to the ingenious construction of the experiment,
the polarisation of the photon corresponds to the polarisation of the neutrino

- The photon helicity was measured to be left-handed!

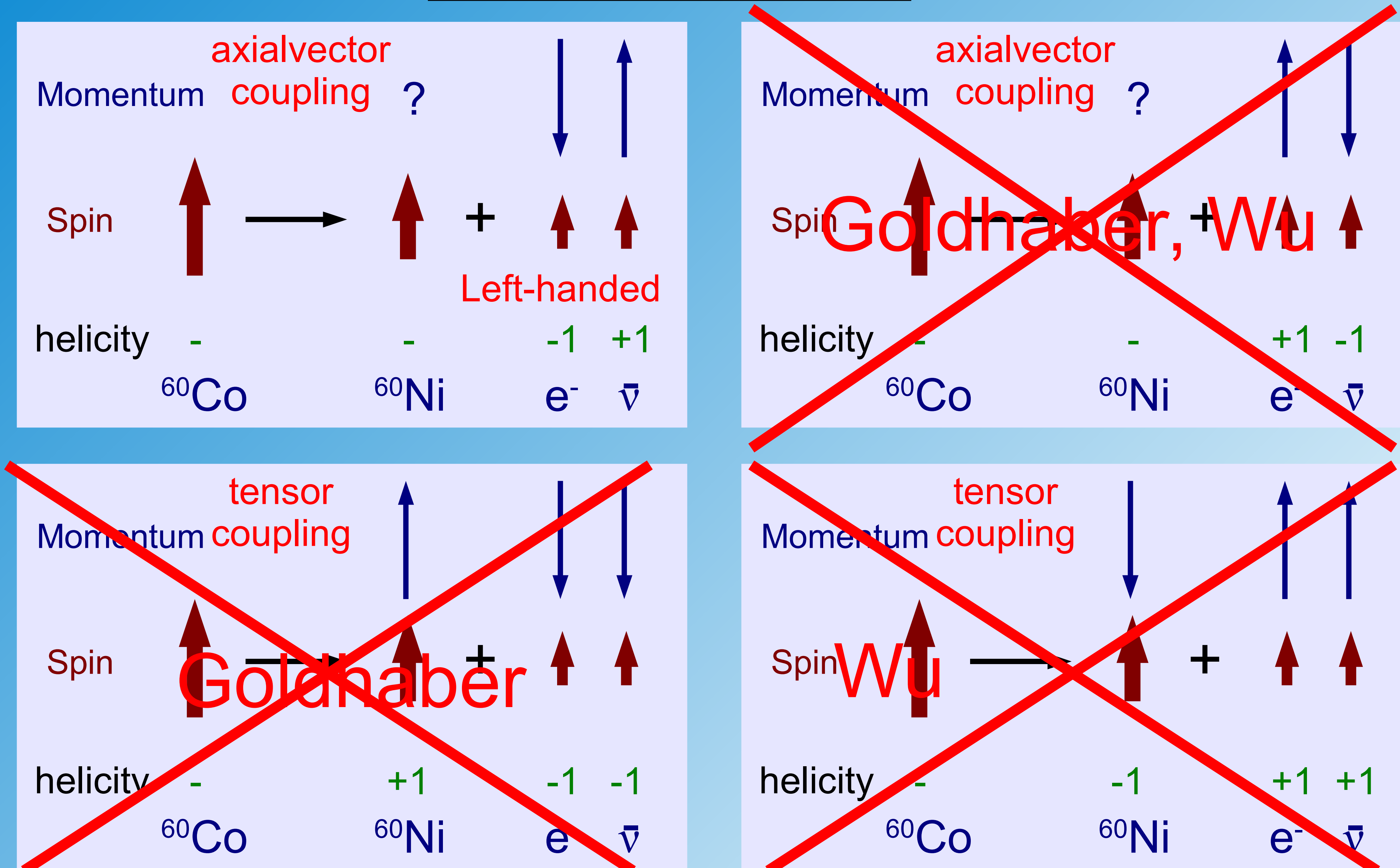
- As a result the helicity of the neutrino has to be left handed

- Left-handed chiral (neutrino!) fermion couplings can be described either by V-A coupling or T(P)-S couplings.

- T(P)-S couplings excluded by Wu- and Goldhaber-experiments

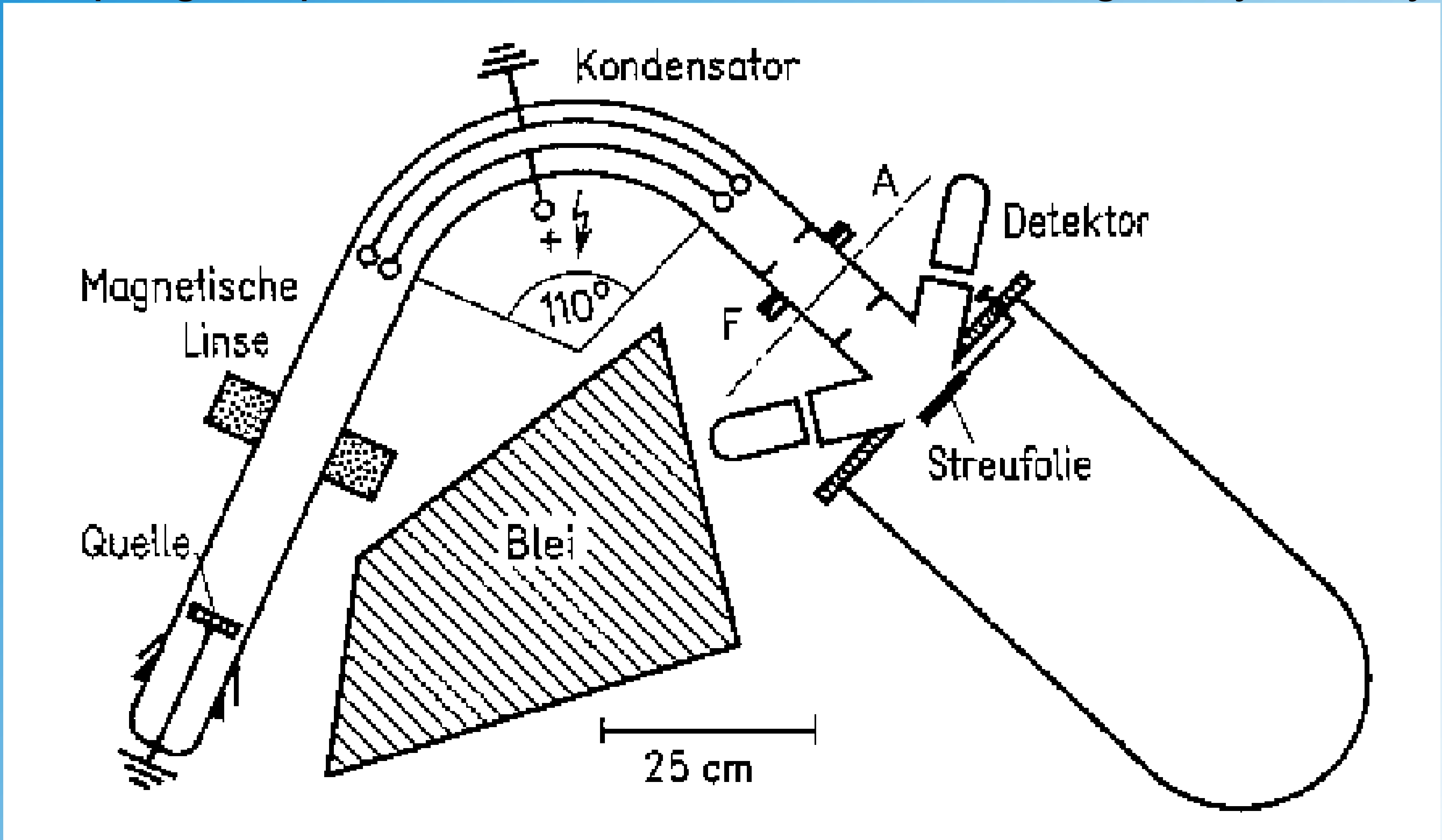
Test of Lorentz Structure in ^{60}Co

Gamov Teller transitions



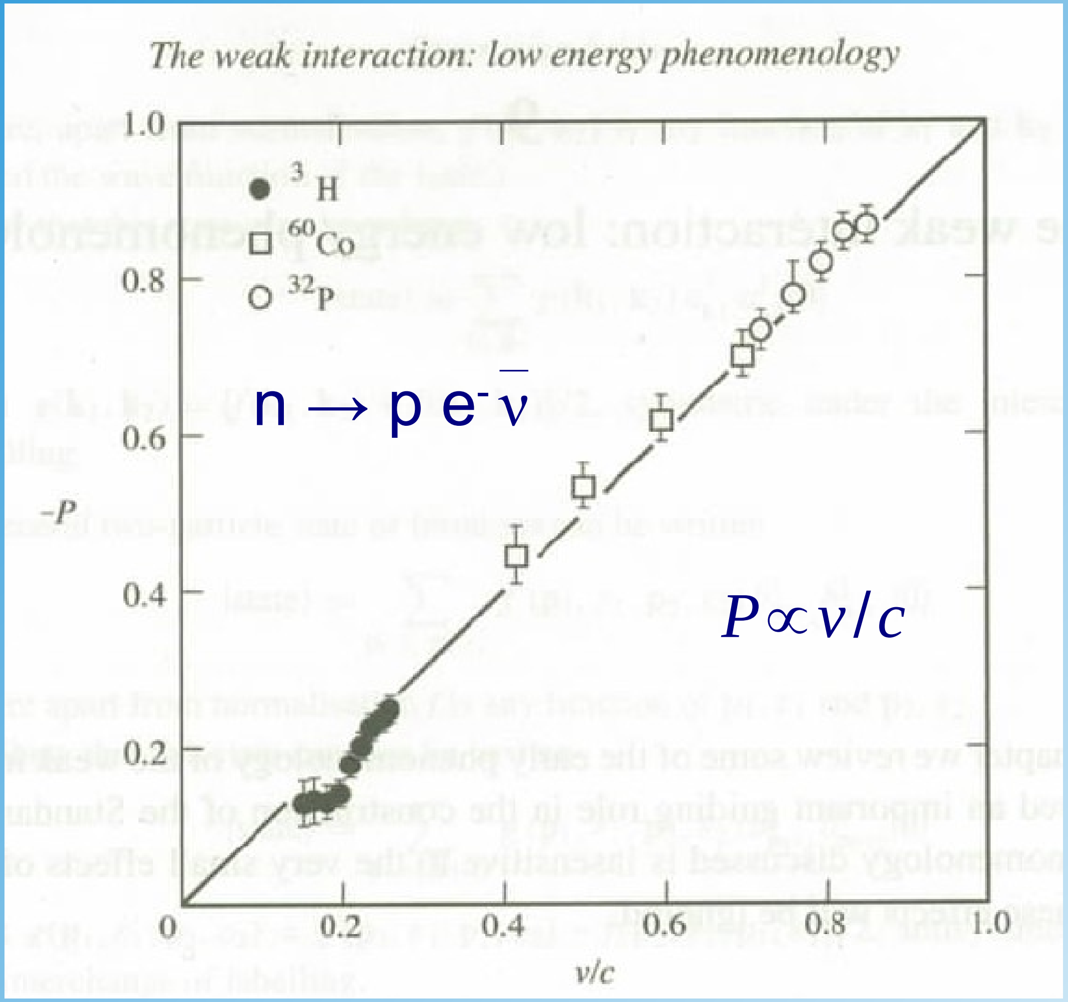
Mott Scattering for Electron-Polarisation Measurements

reaction: e N → e N is polarisation dependent:
coupling of spin and orbital momentum → left-right asymmetry



Polarisation of Electrons in Beta Decays

polarisation: $P = \frac{N(+\frac{1}{2}) - N(-\frac{1}{2})}{N(+\frac{1}{2}) + N(-\frac{1}{2})}$ in different radioactive decays:



only left-handed electrons!

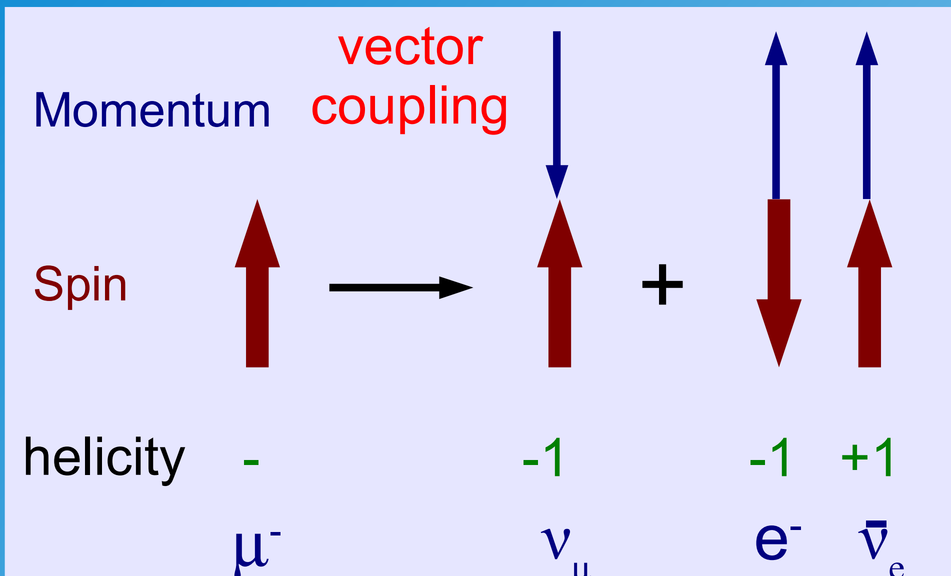
Myon Decay

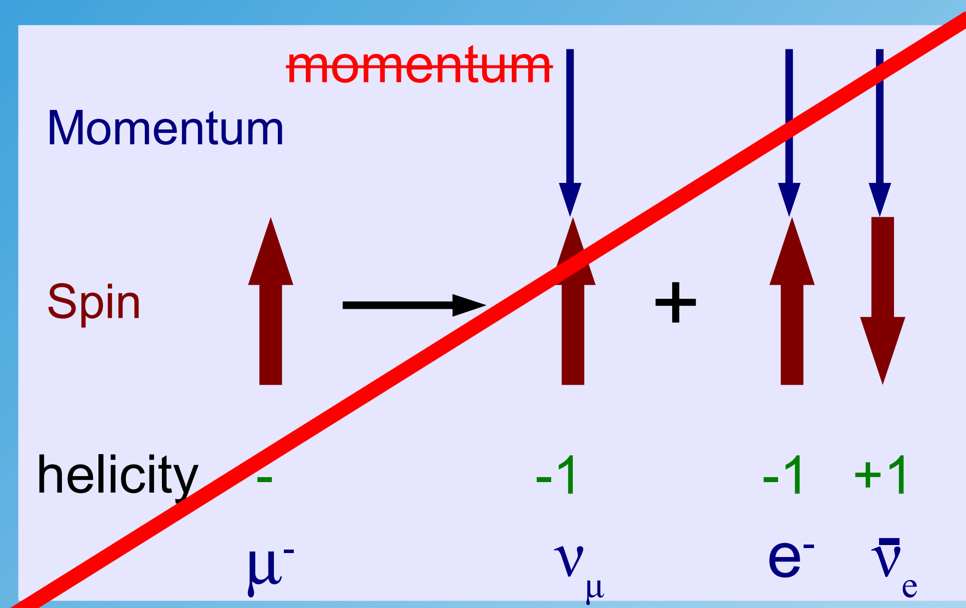
- classical 3-body decay $\mu \rightarrow e\,\bar{\nu}_e\,\nu_\mu$
- only left handed particles interact (V-A) coupling

$$L=\frac{G}{\sqrt{2}}\left(\bar{f}\;\Gamma\,f\,'\right)\left(\overline{f\,''}\;\tilde{\Gamma}\,f\,'''\right) \quad \text{with} \quad \Gamma_\mu=\gamma_\mu\left(1-\gamma_5\right)$$

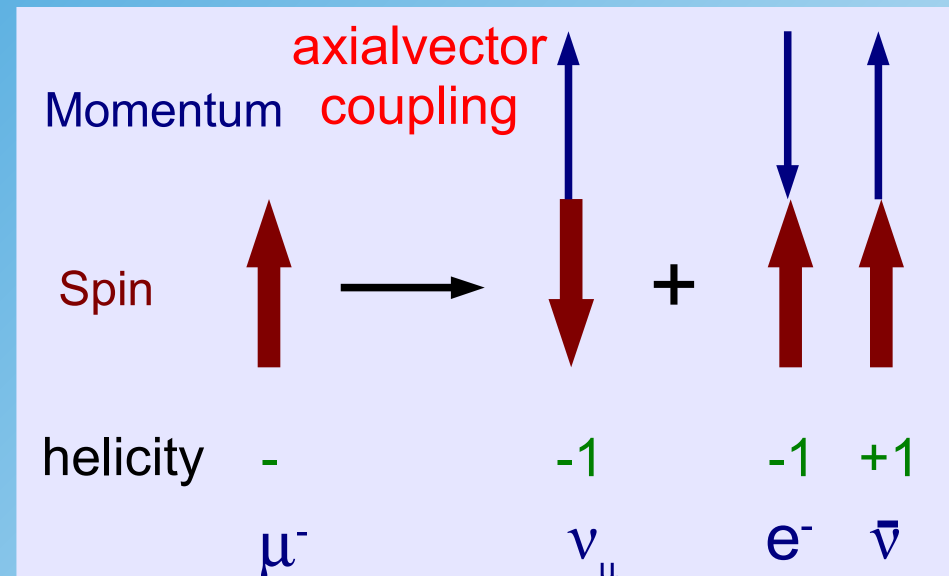
Lorentz Structure in Muon Decay

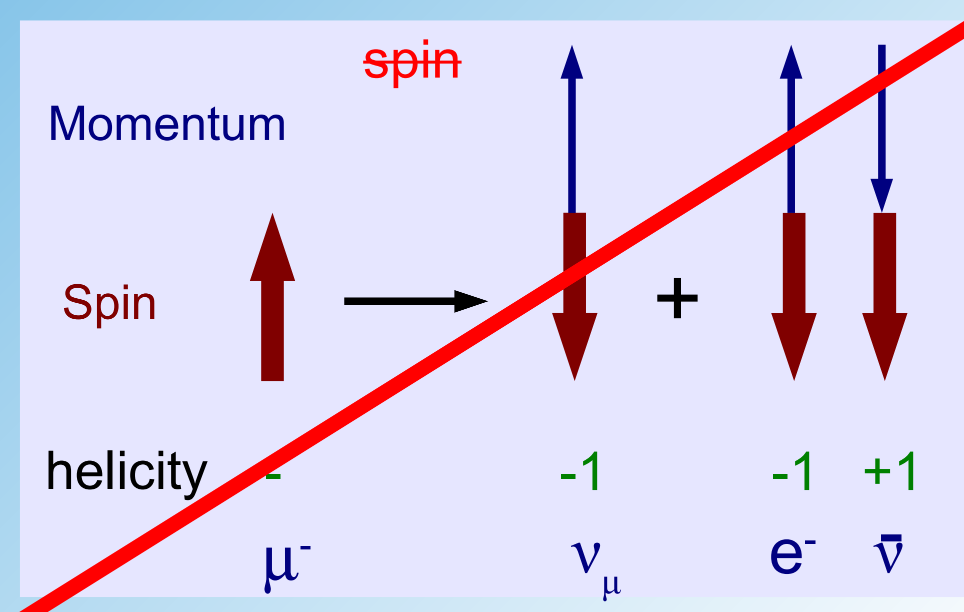
Fermi transition





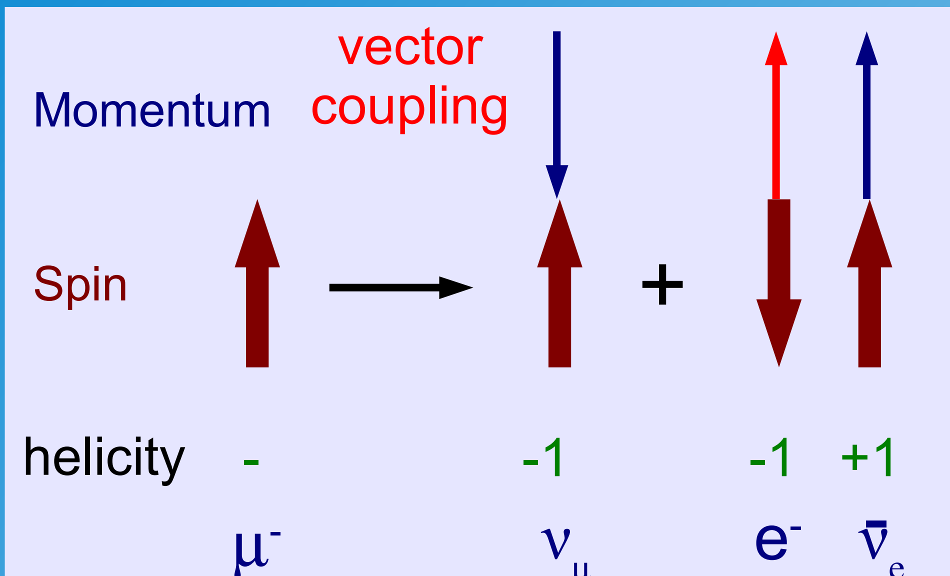
Gamov Teller transition





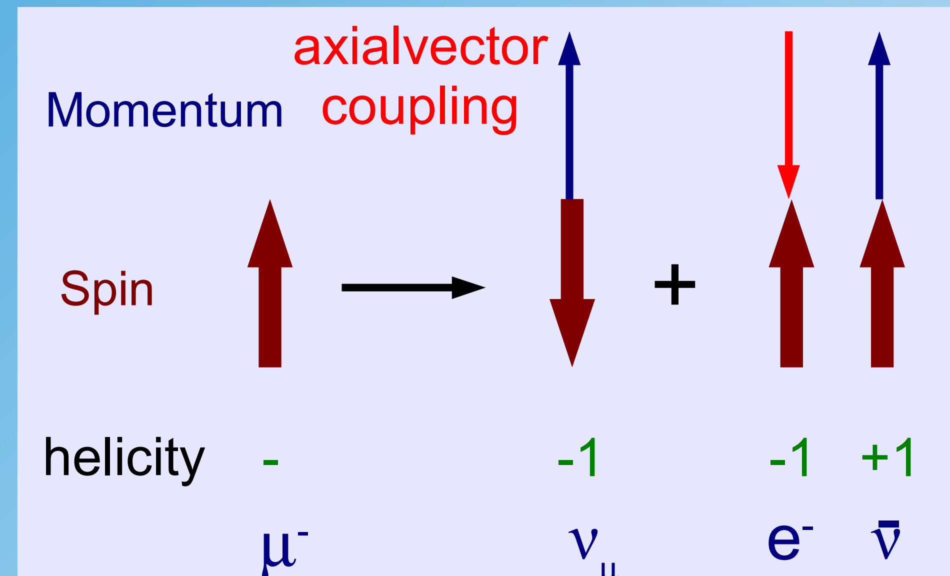
Lorentz Structure in Muon Decay

Fermi transition



myon neutrino has highest energy

Gamov Teller transition



electron has highest energy

Qualitative discussion:

From helicity considerations, the electron is expected to have in average an energy of 3/4 of half the muon mass →3/8 (exact: 7/10) in contrast to 1/3 naively expected from kinematics

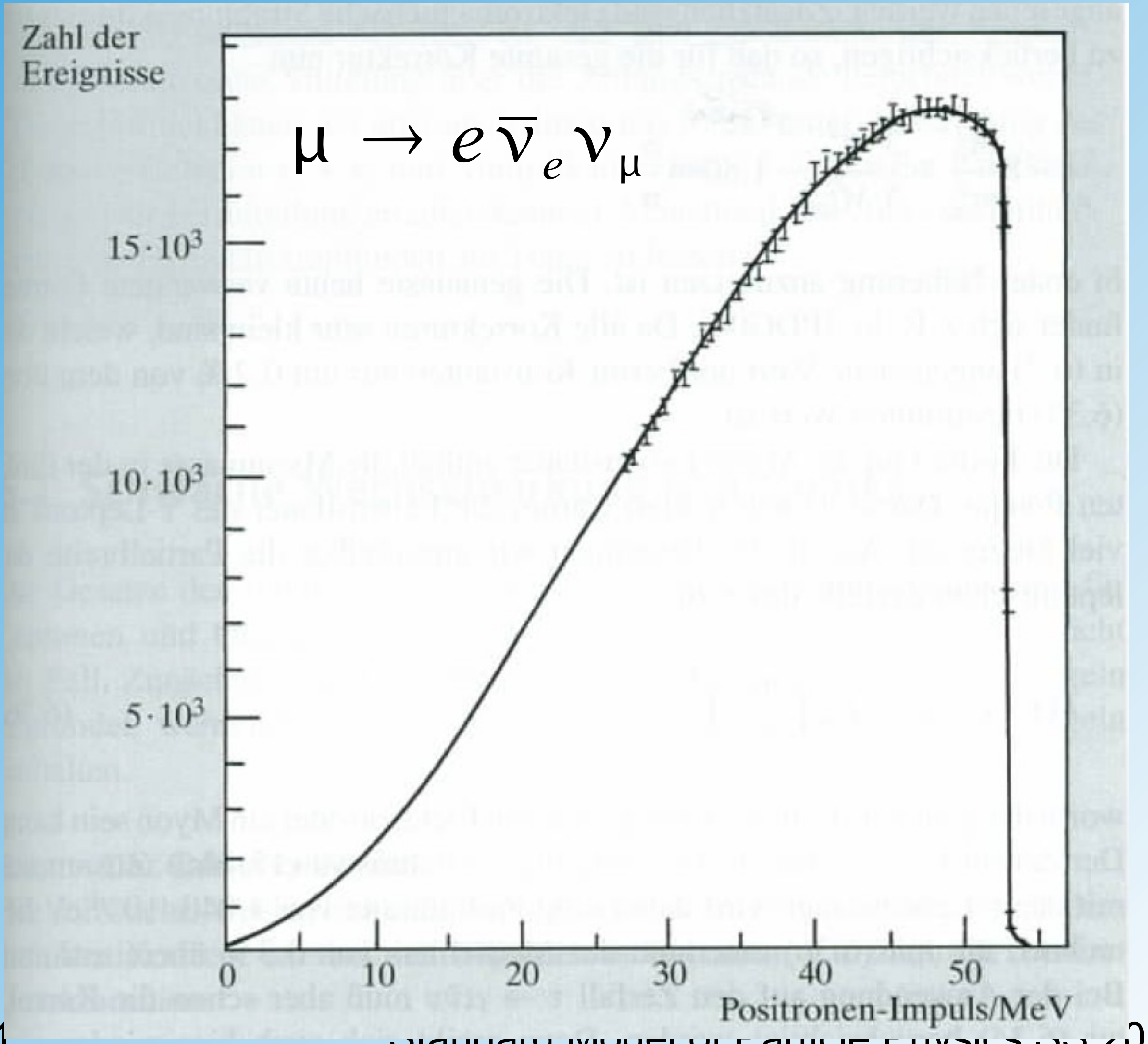
➔ Michel spectrum

Michel-Spectrum

$$\frac{d^2\Gamma}{dx\ \ d\cos\vartheta} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[3-2x\pm P_\mu\cos\vartheta(2x-1)\right]\ x^2$$

$$x \equiv 2E_e/m_\mu$$

Very good agreement between measurement and SM prediction (V-A) theory



Michel-Spectrum beyond the SM

The muon decay is one of the best SM testing grounds:

Extended Michel spectrum formula:

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}x\ \mathrm{d}\cos\vartheta} \sim x^2 \cdot \left\{ 3(1-x) + \frac{2\rho}{3}(4x-3) + 3\eta\ x_0(1-x)/x \right.$$
$$\left. \pm P_\mu \cdot \xi \cdot \cos\vartheta \left[1 - x + \frac{2\delta}{3}(4x-3) \right] \right\} . \qquad (2)$$

$$x_0 = \frac{2\, m_e m_\mu}{m_e^2 + m_\mu^2}$$

Inportant parameters: $\rho = \xi\delta = 3/4,\ \xi = 1,\ \eta = 0$

These parameters can be related to 4-fermion couplings

$$L = \frac{G}{\sqrt{2}} \left(\bar{f}\ \Gamma\ f\,' \right) \left(\overline{f\,''}\ \tilde{\Gamma}\ f\,''' \right)$$

Confirmation of V-A coupling

Experimental results:

$\lvert g^S_{RR}\rvert < 0.062$	$\lvert g^V_{RR}\rvert < 0.031$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.074$	$\lvert g^V_{LR}\rvert < 0.025$	$\lvert g^T_{LR}\rvert < 0.021$
$\lvert g^S_{RL}\rvert < 0.412$	$\lvert g^V_{RL}\rvert < 0.104$	$\lvert g^T_{RL}\rvert < 0.103$
$\lvert g^S_{LL}\rvert < 0.550$	$\lvert g^V_{LL}\rvert > 0.960$	$\lvert g^T_{LL}\rvert \equiv 0$
$\lvert g^S_{LR} + 6g^T_{LR}\rvert < 0.143$	$\lvert g^S_{RL} + 6g^T_{RL}\rvert < 0.418$	
$\lvert g^S_{LR} + 2g^T_{LR}\rvert < 0.108$	$\lvert g^S_{RL} + 2g^T_{RL}\rvert < 0.417$	
$\lvert g^S_{LR} - 2g^T_{LR}\rvert < 0.070$	$\lvert g^S_{RL} - 2g^T_{RL}\rvert < 0.418$	

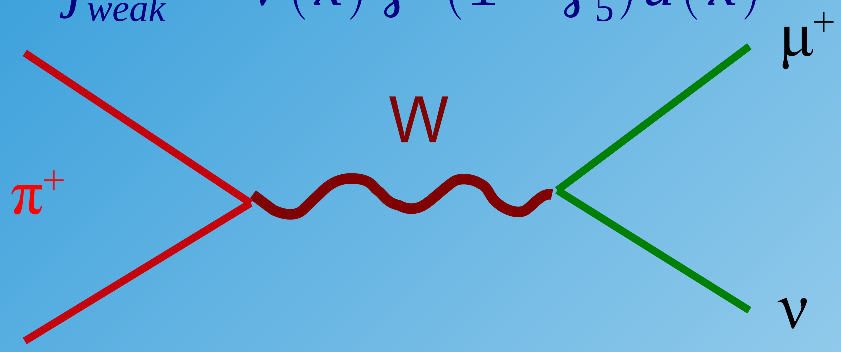
Charged Pion Branching Ratios

- dominant decay: $B(\pi^+ \to \mu^+ \nu) = 99.9877\ \%$
- suppressed decay: $B(\pi^+ \to e^+ \nu) = 1.23 \cdot 10^{-4}$

Similar to the neutral pion in QED:
→ the "more obvious" decay is suppressed!

<u>V-A Currents</u>:

$$j^{\mu}_{weak} = \bar{v}(x)\gamma^{\mu}(1-\gamma_5)u(x)$$



Polarisation of helicity state is given by fermion velocity:

$\langle \lambda \rangle = -\beta$ for left chiral states

- V-A currents conserve helicity.
- Resulting spin should be $J(\pi^+)=1$
- But pion is a Pseudo-scalar $J(\pi^0)=0$
  - ➜ helicity suppression

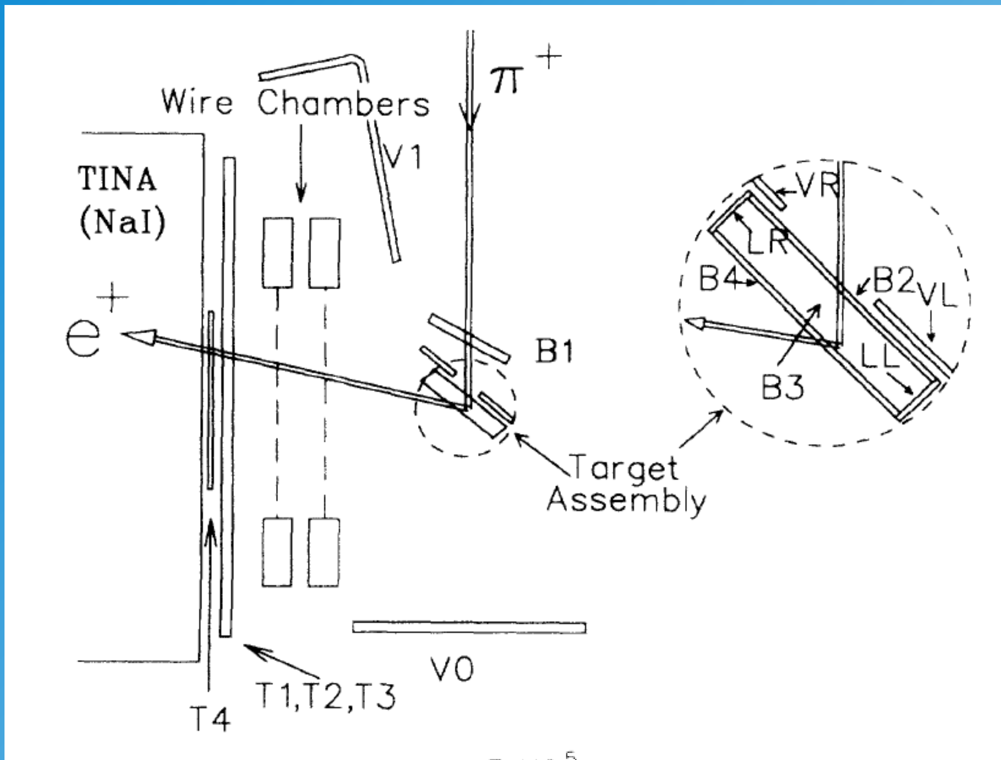
<u>Decay width:</u>

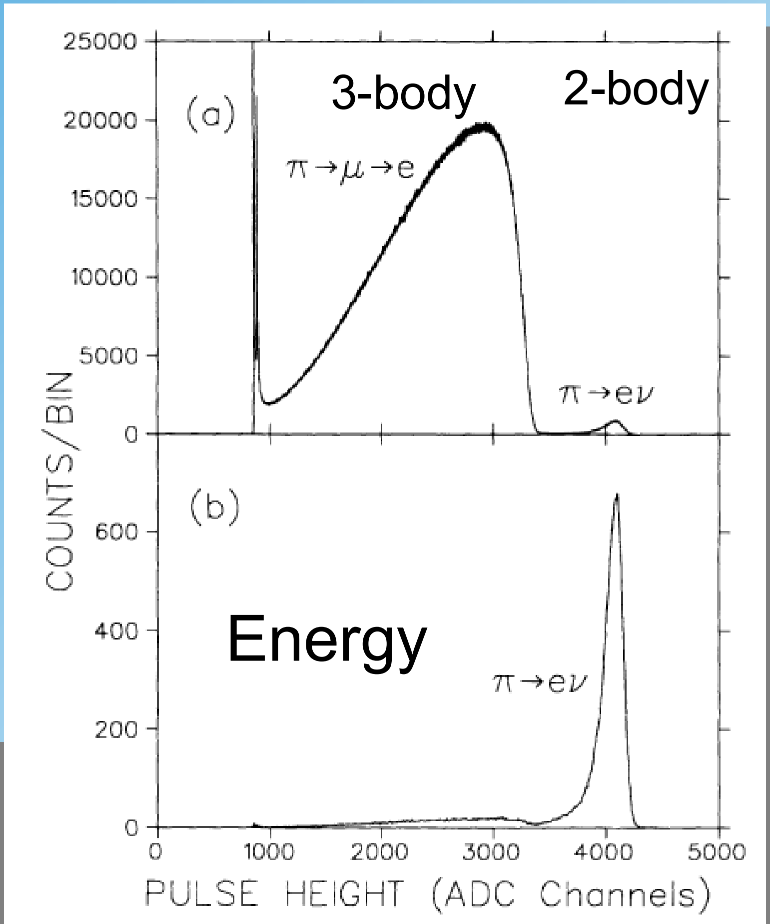
$$\Gamma(\pi^- \to \mu^-) = \frac{G_F^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)$$

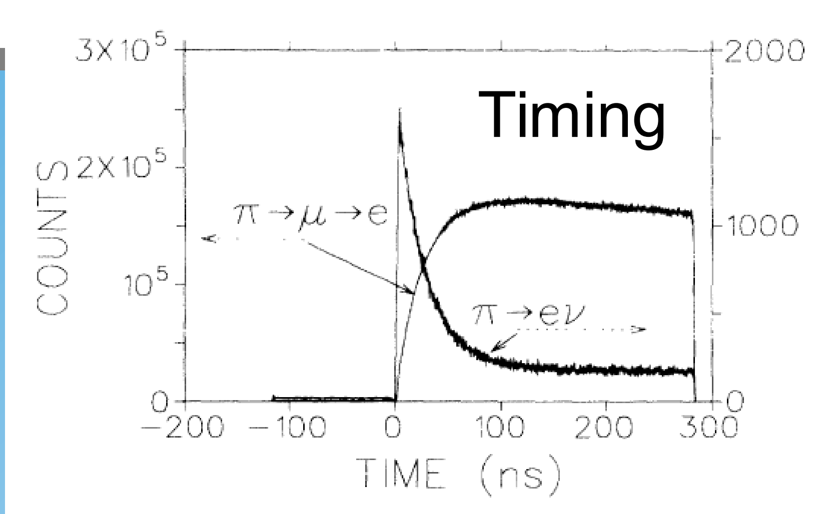
$$\frac{\Gamma(\pi^+ \to e^+)}{\Gamma(\pi^+ \to \mu^+)} = \frac{m_e^2(1-m_e^2/m_\pi^2)}{m_\mu^2(1-m_\mu^2/m_\pi^2)} \sim 10^{-4}$$

Measurement of $\pi^+ \rightarrow e^+ \nu$

(Britton PRL 68, 20, 1992, 3000)







Pion lifetime 25 ns

Muon lifetime 2 μs

- Result:

$B(\pi^+ \rightarrow e^+ \nu) = 1.23 \cdot 10^{-4}$

Conclusion: no scalar coupling!

Gamma Matrices II

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(in other representations γ^5 is diagonal)