Lecture:

Standard Model of Particle Physics

Heidelberg SS 2016

Experimental Tests of QED
Part 1

give FEEDBACK!

ETH zürich — PAUL SCHERRER INSTITUT PSI — UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386 — JG|U JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Practical Course in Particle Physics

at the Paul Scherrer Institut (PSI, Switzerland), Summer 2016

Perform a real particle physics experiment at a PSI beam-line.

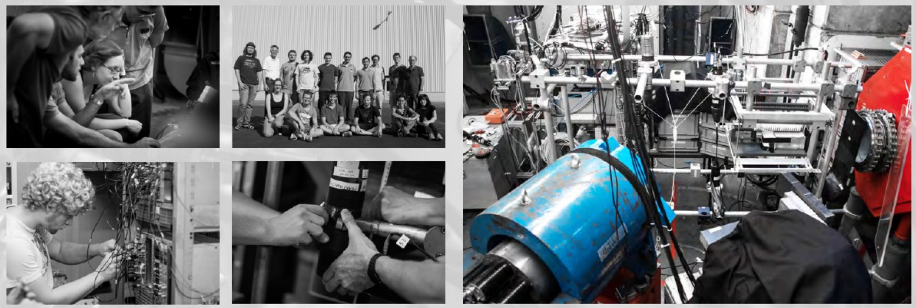
Learn experimental particle physics hands-on.

Join a group of 10-12 students from ETH Zürich and Heidelberg and Mainz Universities for one week of preparartions in Heidelberg and two weeks of beam time at PSI.

Design and build your experiment from available detector components, take data during a 24/7 beam time. Analyse the data and write up the results.

Example measurements:

- Branching ratio $B(\pi \to e\nu)/B(\pi \to \mu\nu)$
- Panofsky ratio $B(\pi^- p \to n\pi^0)/B(\pi^- p \to n\gamma)$
- Lifetimes and decay parameters of muons and pions



Next course: 22.8.-9.9.2016

Please contact:

André Schöning (schoning@physi.uni-heidelberg.de)

Limited number of places, please register early!

The course (MVPSI) is part of the master programme of the faculty for physics and astronomy in Heidelberg

Overview

PART I

- Cross Sections and QED tests
- Accelerator Facilities + Experimental Results

PART II

- Tests of QED in Particle Decays and Resonances
- QED Radiative Effects

Measurement of Cross Sections

$$e^+e^- \rightarrow X$$
$$(e^- e^- \rightarrow X)$$

- **test predictions of QED**
- α_{em}=1/137 is small → perturbative theory!
- Evolution at higher energies?

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Reactions depend on center of mass → many different accelerators

Synchrotron Radiation Law::

$$P \propto \frac{E^4}{R^2}$$

- **large accelerators required for high energies**

List of ee-Accelerators

Accelerator	Location	Years of operation	Shape and circumference	Electron energy	Positron energy	Experiments	Notable Discoveries
AdA	Frascati, Italy; Orsay, France	1961–1964	Circular, 3 meters	250 MeV	250 MeV		Touschek effect (1963); first e^-e^- interactions recorded (1964)
Princeton-Stanford (e^-e^-)	Stanford, California	1962–1967	Two-ring, 12 m	300 MeV	300 MeV		e^-e^- interactions
VEP-1 (e^-e^-)	INP, Novosibirsk, Soviet Union	1964–1968	Two-ring, 2.70 m	130 MeV	130 MeV		e^-e^- scattering; QED radiative effects confirmed
VEPP-2	INP, Novosibirsk, Soviet Union	1965–1974	Circular, 11.5 m	700 MeV	700 MeV	OLYA, CMD	multihadron production (1966), $e^+e^-\to\varphi$ (1966), $e^+e^-\to\gamma\gamma$ (1971)
SPEAR	SLAC	1972-1990(?)				Mark I, Mark II, Mark III	Discovery of Charmonium states
VEPP-2M	BINP, Novosibirsk	1974–2000	Circular, 17.88 m	700 MeV	700 MeV	ND, SND, CMD-2	e^+e^- cross sections, radiative decays of ρ, ω, and φ mesons
DORIS	DESY	1974–1993	Circular, 300m	5 GeV	5 GeV	ARGUS, Crystal Ball, DASP, PLUTO	Oscillation in neutral B mesons
PETRA	DESY	1978–1986	Circular, 2 km	20 GeV	20 GeV	JADE, MARK-J, PLUTO, TASSO	Discovery of the gluon in three jet events
CESR	Cornell University	1979–2002	Circular, 768m	6 GeV	6 GeV	CUSB, CHESS, CLEO, CLEO-2, CLEO-2.5, CLEO-3	First observation of B decay, charmless and "radiative penguin" B decays
PEP	SLAC	1980-1990(?)				Mark II	
SLC	SLAC	1988-1998(?)	Addition to SLAC Linac	45 GeV	45 GeV	SLD, Mark II	First linear collider
LEP	CERN	1989–2000	Circular, 27 km	104 GeV	104 GeV	Aleph, Delphi, Opal, L3	Only 3 light ($m \le m_Z/2$) weakly interacting neutrinos exist, implying only three generations of quarks and leptons
BEPC	China	1989–2004	Circular, 240m	2.2 GeV	2.2 GeV	Beijing Spectrometer (I and II)	
VEPP-4M	BINP, Novosibirsk	1994-	Circular, 366m	6.0 GeV	6.0 GeV	KEDR	Precise measurement of Υ-meson masses
PEP-II	SLAC	1998–2008	Circular, 2.2 km	9 GeV	3.1 GeV	BaBar	Discovery of CP violation in B meson system
KEKB	KEK	1999–2009	Circular, 3 km	8.0 GeV	3.5 GeV	Belle	Discovery of CP violation in B meson system
DAΦNE	Frascati, Italy	1999-	Circular, 98m	0.7 GeV	0.7 GeV	KLOE	Crab-waist collisions (2007)
CESR-c	Cornell University	2002–2008	Circular, 768m	6 GeV	6 GeV	CHESS, CLEO-c	
VEPP-2000	BINP, Novosibirsk	2006-	Circular, 24.4m	1.0 GeV	1.0 GeV	SND, CMD-3	Round beams (2007)
BEPC II	China	2008-	Circular, 240m	3.7 GeV	3.7 GeV	Beijing Spectrometer III	

AdA Accelerator

- First $e^+ e^-$ collider ever
- AdA = Anello di Accumulazione (Frascati/Orsay, 1961-64)
- Energy: 250 MeV Electrons x 250 MeV Positrons

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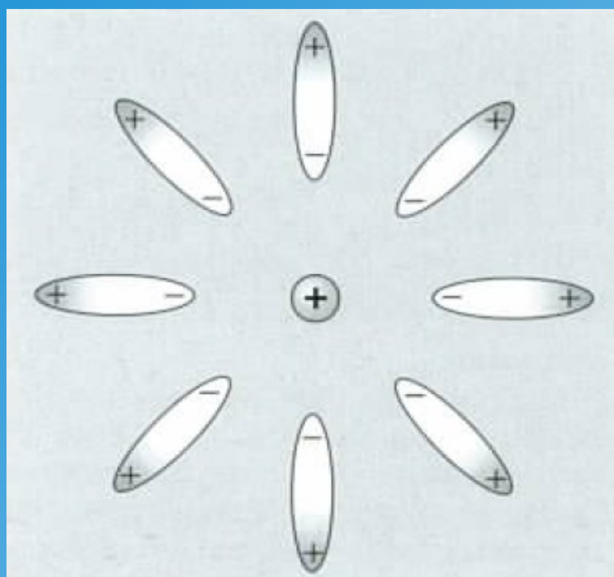
- Bruno Touschek: excite the dielectric vacuum to create vector mesons (e.g. rho meson predicted to be light!)

Note: at that time all new particles had been discovered in hadronic interactions (ie. proton beams)!

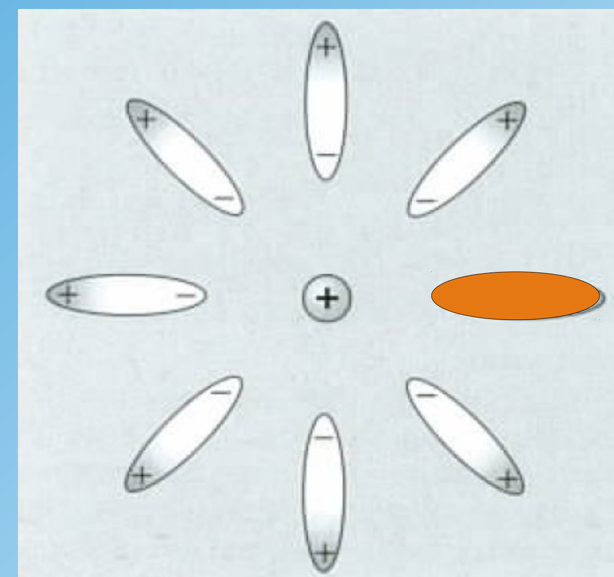
Dielectric Vacuum

classical dielectric



high energy

"excited dielectric"



bare electrical charge shielded
by induced dipoles

bare charge shielded
by vacuum polarisation

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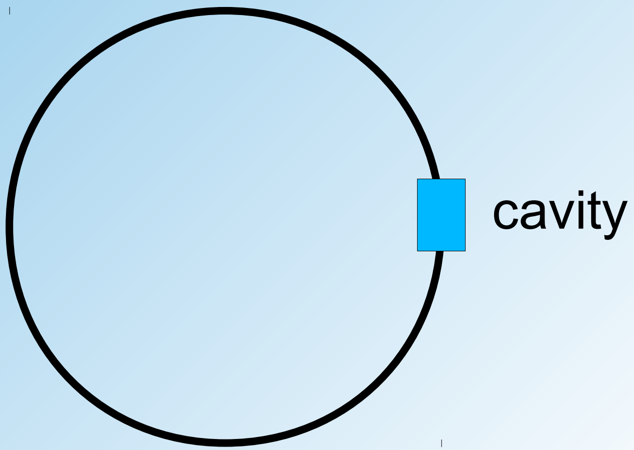
“Revolutionary” concept as the rho-meson is electrically neutral and was predicted to explain (as carrier) strong interactions

Remark: Indeed, Touschek was right. The strong force can be tested in $e^+ e^-$ collisions. But not in AdA (too low luminosity, too low energy)

AdA Challenges I

- How to store electrons and positrons?
  - magneto-optical storage ring
    (→ mastered at this time, synchrotron radiation facilities)



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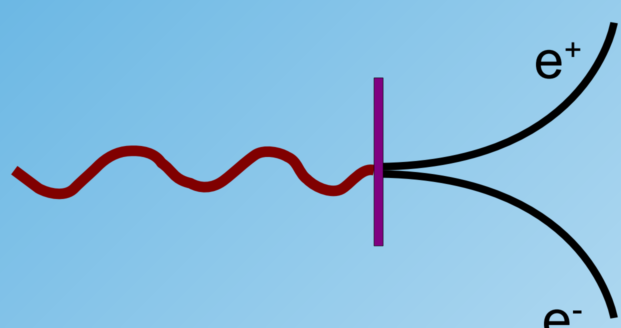

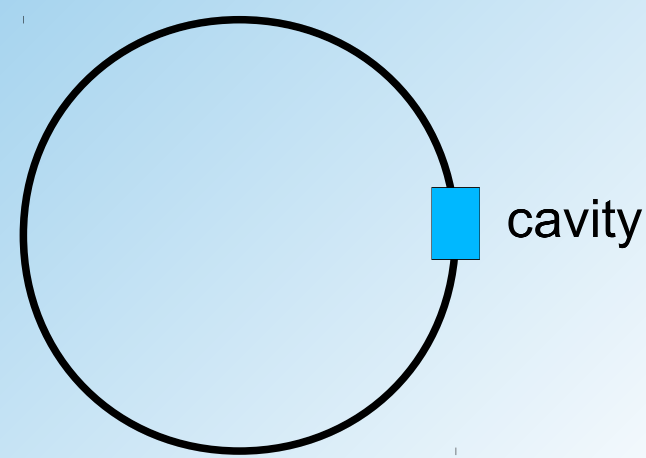

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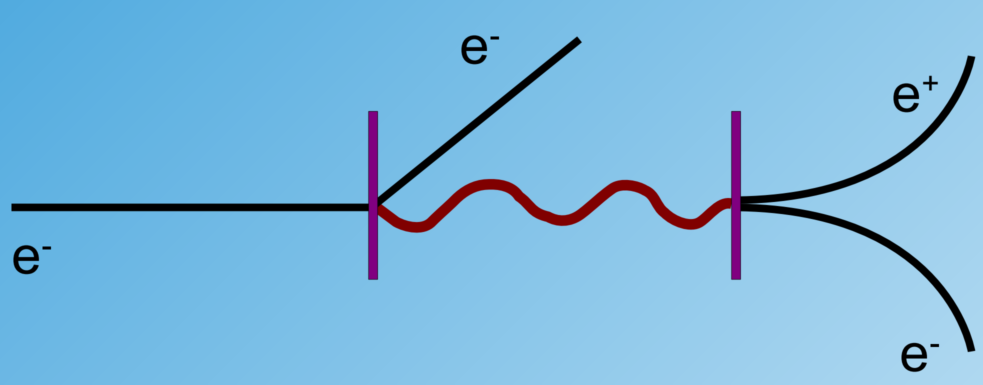
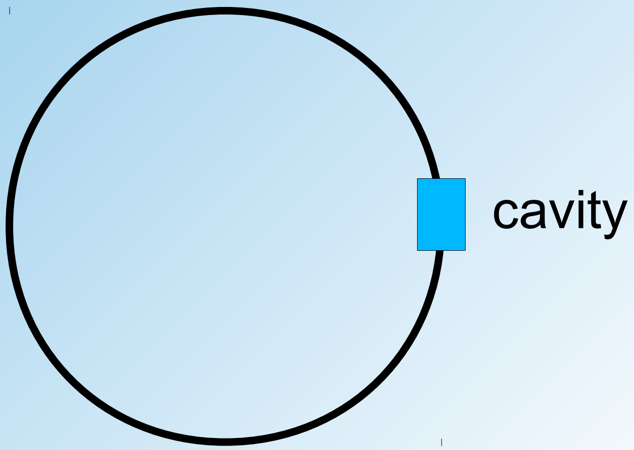
  - Bremsstrahlung from high energetic electrons at target
    $e^- N \to \gamma e^- N$ using a linear electron accelerator (→ also known)





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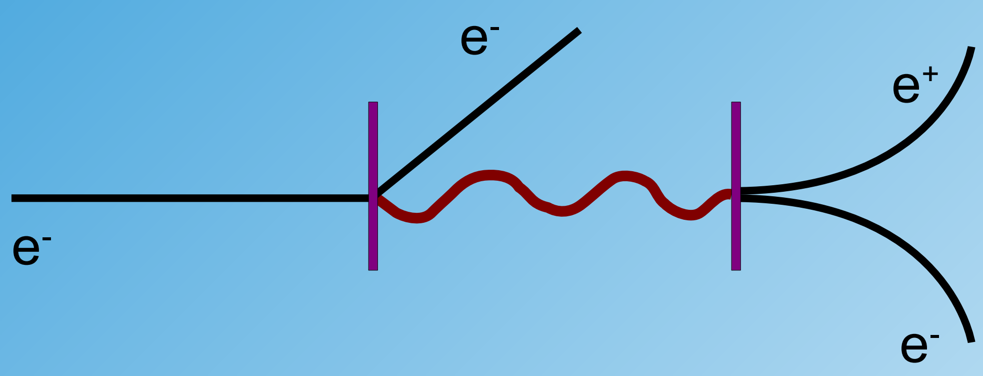
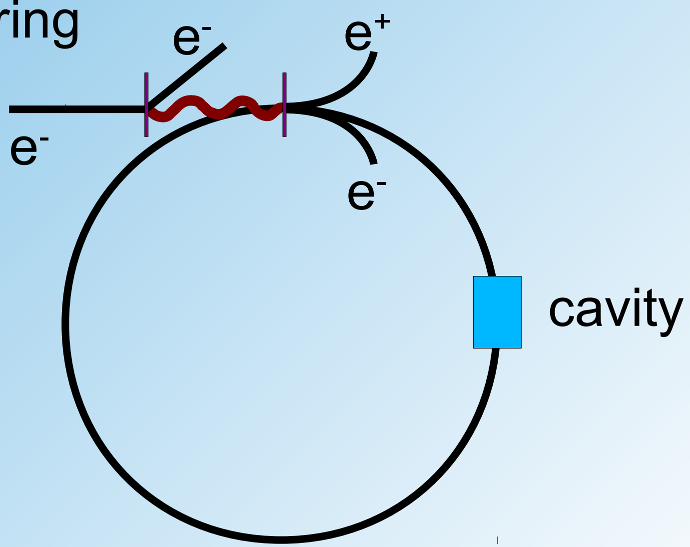
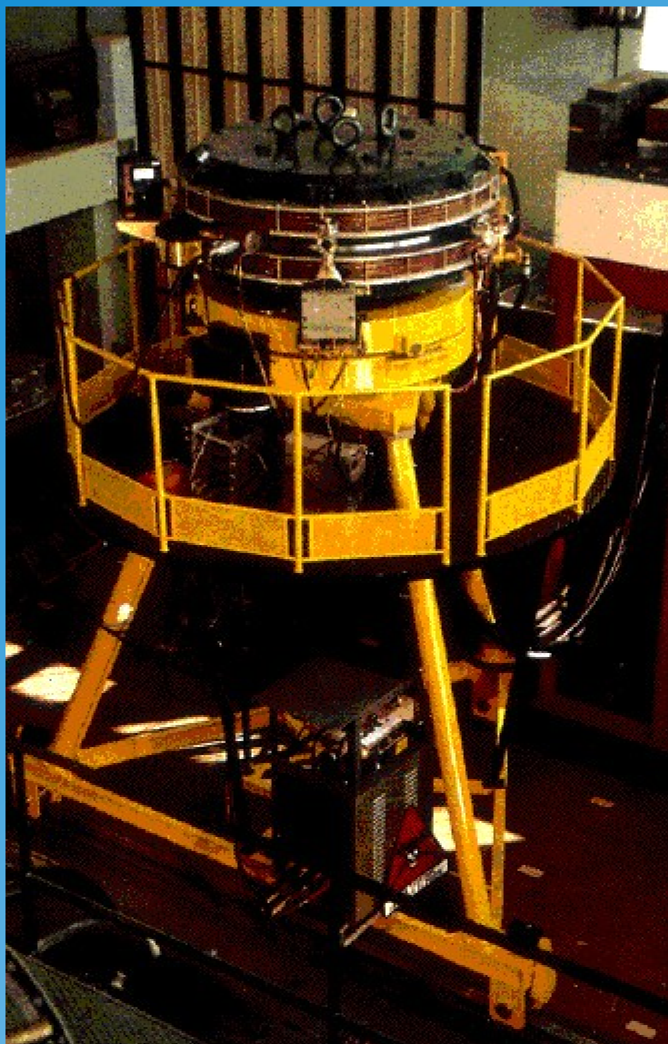
- How to fill the storage ring with electrons and positrons???
  - place the conversion target inside the storage ring

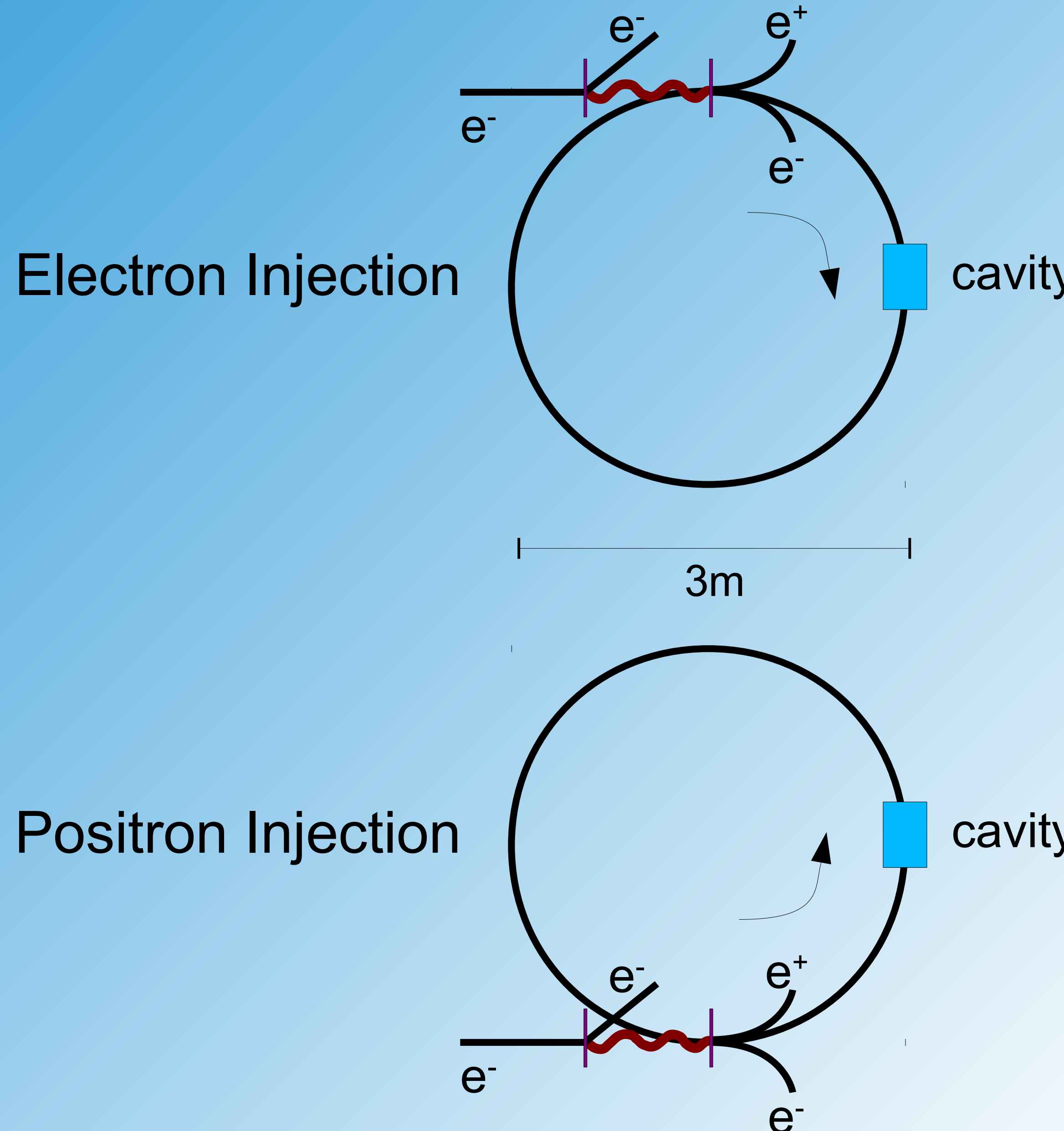




MAGNETIC DISCUSSION

AdA Concept



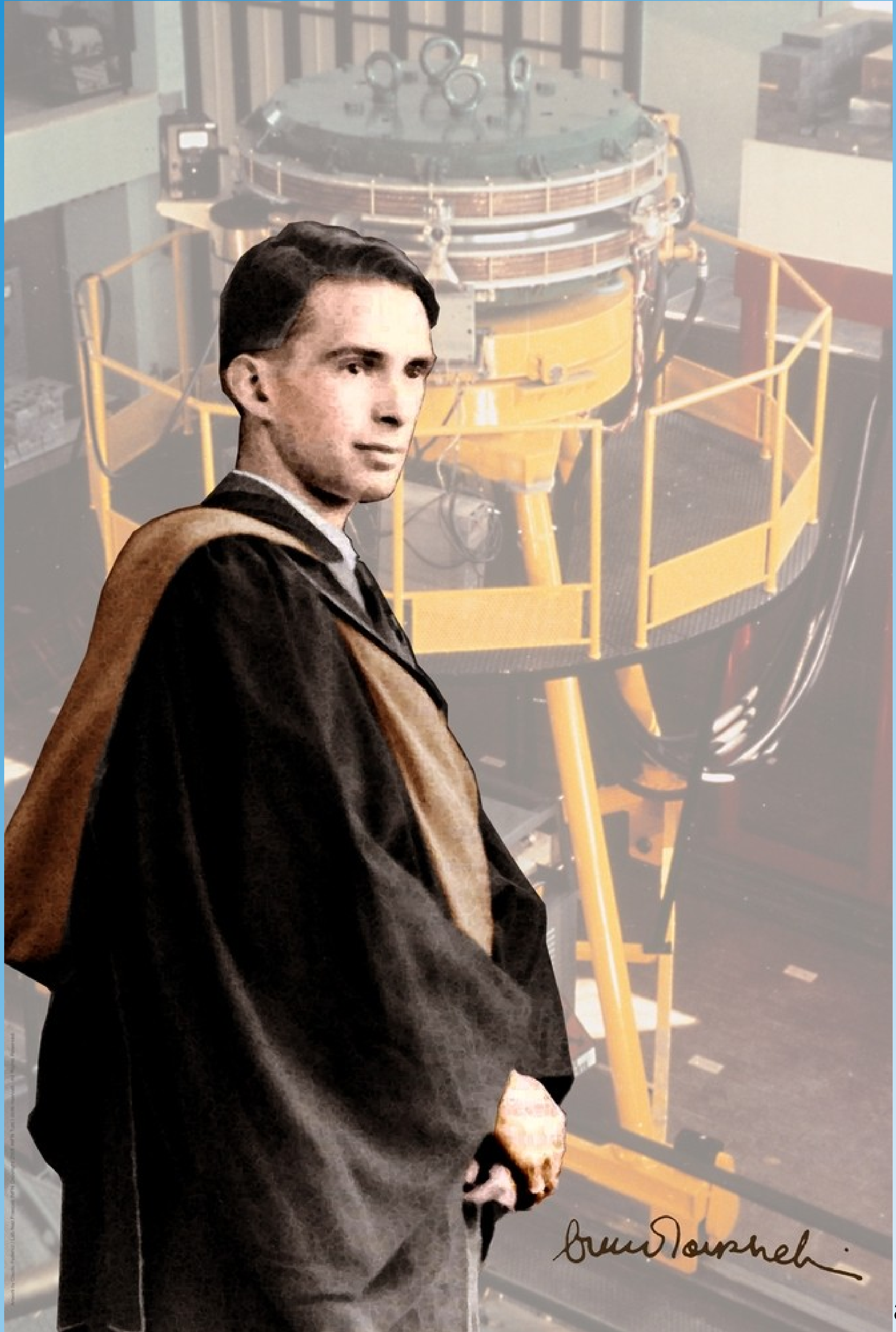

e-
e+
e-
e-
Electron Injection
cavity
3m
Positron Injection
cavity
e-
e+
e-
e-

AdA Challenges II

- How to make electrons and positrons collide?

Note: AdA is a single storage ring: electrons and positrons see same optics but in reverse direction

B.Touschek: *It is guaranteed that an electron and a positron necessarily meet in a single orbit because QED is CP (charge-parity) invariant*



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If a ring collider works, then CP(T) invariance of QED is confirmed!!!

Note: CP(T) invariance says that a positron can be regarded as an electron traveling in reverse time direction.

Touschek was right, in a very short time AdA was commissioned and electron-positron collisions were observed.

This is much more than just a technical (engineering) achievement!

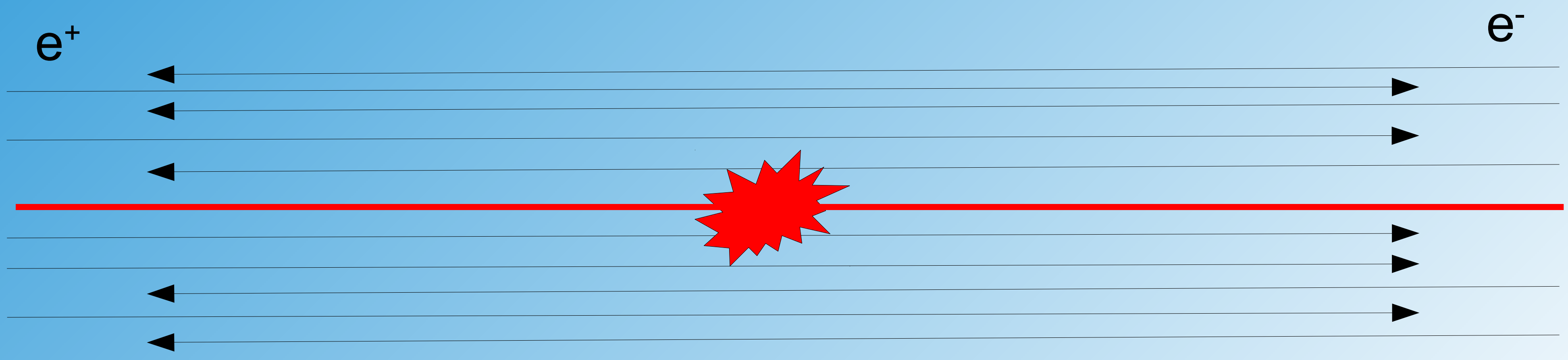
AdA Challenges III

- How to measure that electron-positron collisions take place?
- How many collisions?

Definition of "Luminosity":

$$R = L\,\sigma$$

Relation between rate of events and cross section of process



Luminosity Measurement in Ring

Collider (gaussian beams):

$$L = \frac{N_1 N_2 f}{4 \pi A}$$

N_1 and N_2 and beam cross section A are unknown and have to be precisely measured → difficult

More practical ansatz – use reference process(es):

$e^+ e^- \to e^+ e^-$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}(\cos\theta)} = \frac{\pi\alpha^2}{s}\left(u^2\left(\frac{1}{s}+\frac{1}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + \left(\frac{s}{t}\right)^2\right)$$

ultrarelativistic approx. (Bhabha 1936)

$e^+ e^- \to \gamma\gamma$

$$\frac{d\sigma}{d\Omega}(e^+e^- \to \gamma\gamma) = \frac{\alpha^2}{2s}\frac{u^2+t^2}{tu}$$

annihilation process (Compton-like)

Both processes are forward peaked!

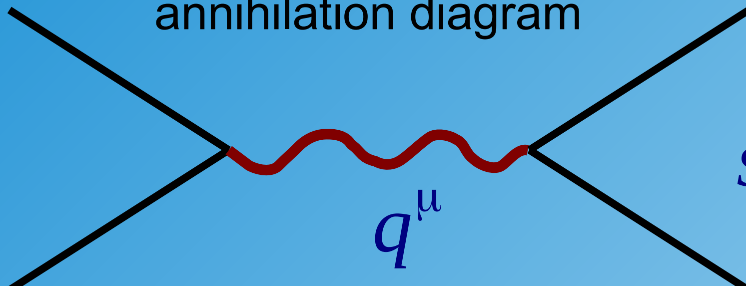
t-pole $t = -s\sin^2(\theta/2)$

Bhabha Scattering

$$\frac{d\sigma}{d\Omega}\left(e^+ e^- \rightarrow e^+ e^-\right) = \frac{\alpha^2}{2s}\left(\frac{u^2+t^2}{s^2}+\frac{s^2+u^2}{t^2}+\frac{2u^2}{st}\right)$$

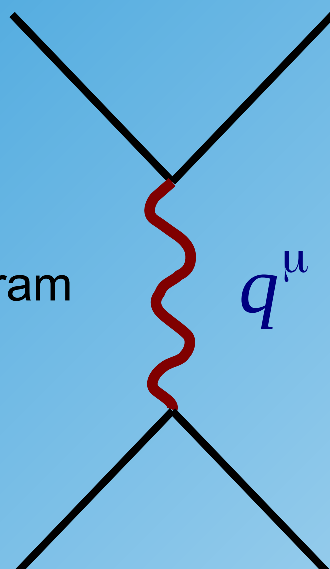
annihilation scattering interference

annihilation diagram



$s = q^\mu q_\mu$ s-pole from photon propagator

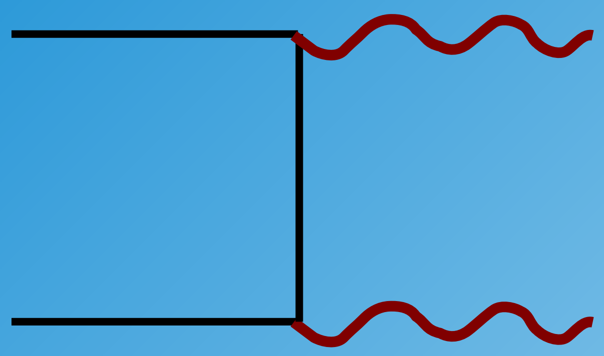
scattering diagram



$t = q^\mu q_\mu$ t-pole from photon propagator

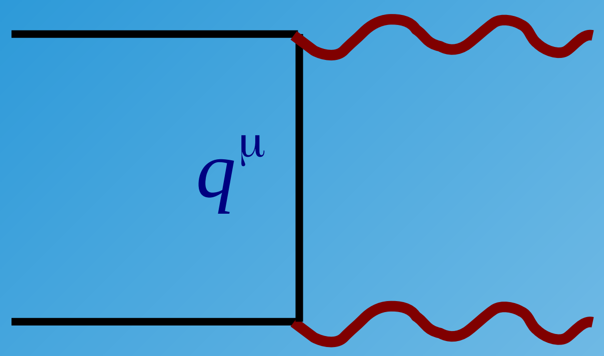
Photon Pair Production

$$\frac{d\sigma}{d\Omega}\left(e^+ e^- \rightarrow \gamma\gamma\right) = \frac{\alpha^2}{2s}\left(\frac{u^2 + t^2}{tu}\right)$$



Photon Pair Production

$$\frac{d\sigma}{d\Omega}(e^+ e^- \rightarrow \gamma\gamma) = \frac{\alpha^2}{2s}\left(\frac{u}{t} + \frac{t}{u}\right)$$



$$t = q^\mu q_\mu$$

t-pole from electron propagator

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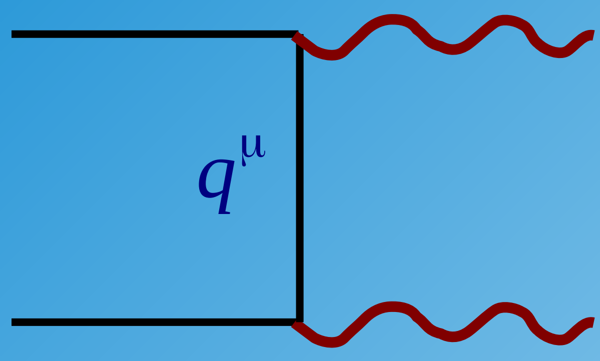

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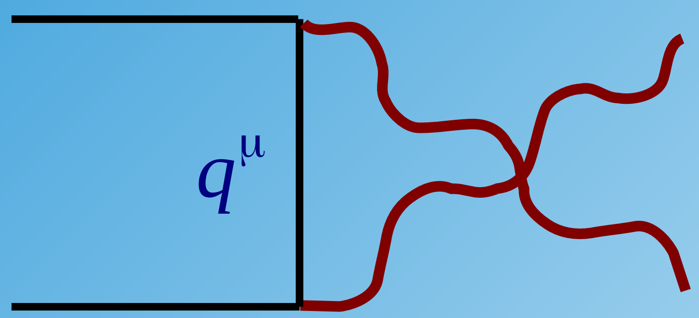
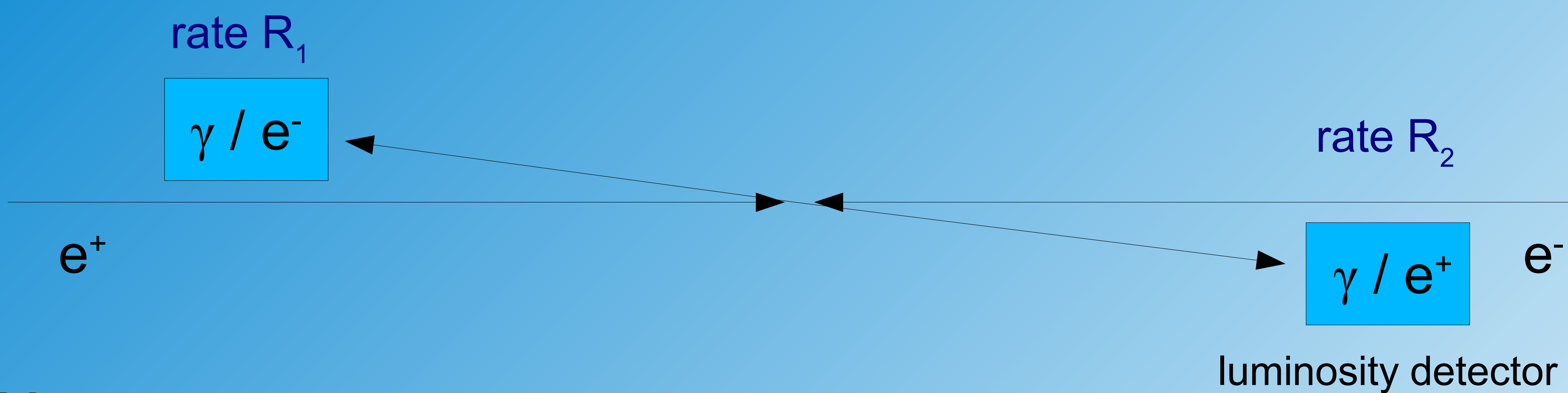


u-pole from crossed diagram

Sketch of Luminosity Measurement



Measurement: rates R_1 and R_2 (in counts/s)

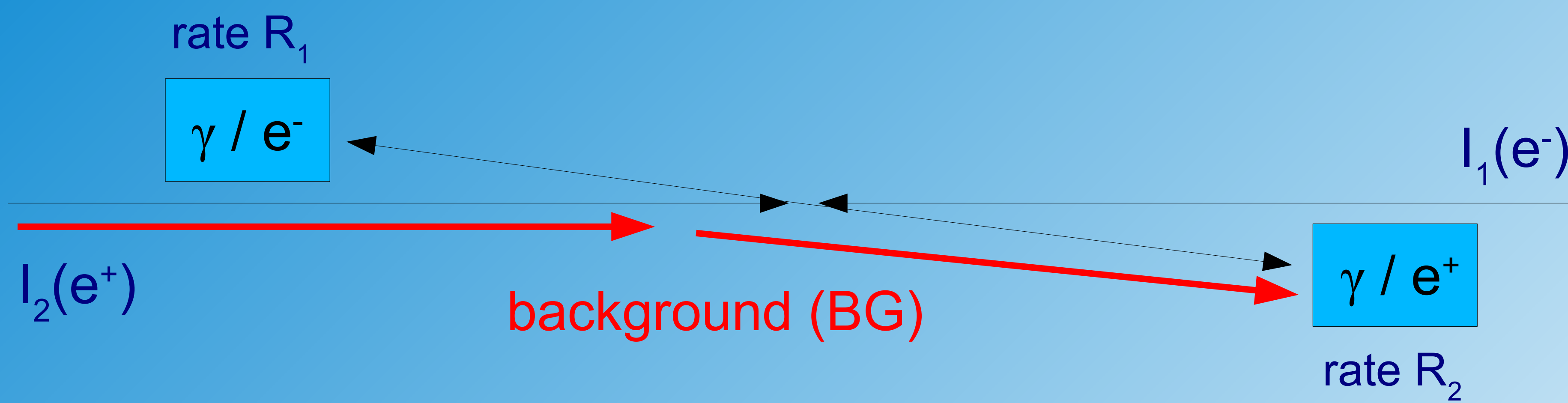
Use: $\mathbf{R = L\,\sigma_{Detector}} \leftrightarrow \mathbf{L = R/\,\sigma_{Detector}}$

$$\sigma_{Detector} = \int_{Detector} \frac{d\sigma}{d\Omega}\, d\Omega$$

acceptance calculation is an experimental task!

$\boldsymbol{\sigma_{Detector}}$ is the **observed** cross section ≠ total cross cross section

Background for Luminosity Measurement



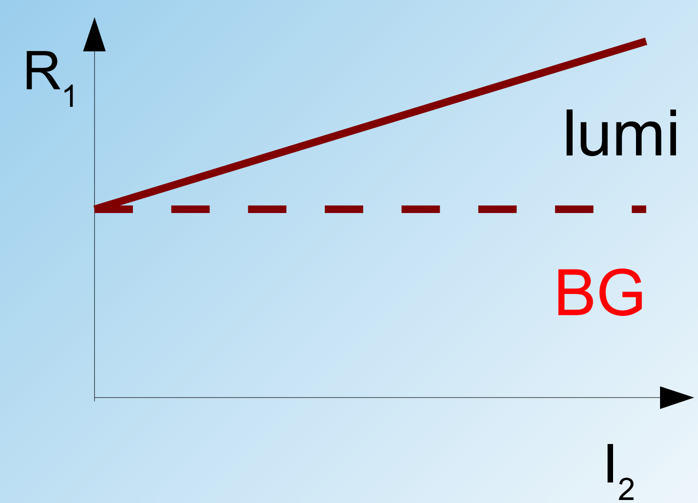
Problem: beam induced background, e.g. electron-rest gas scattering)

Ansatz:

$$R_1 = a_1 I_1 + b I_1 I_2 = I_1 (a_1 + b I_2)$$

$$R_2 = a_2 I_2 + b I_1 I_2 = I_2 (a_2 + b I_1)$$

BG lumi

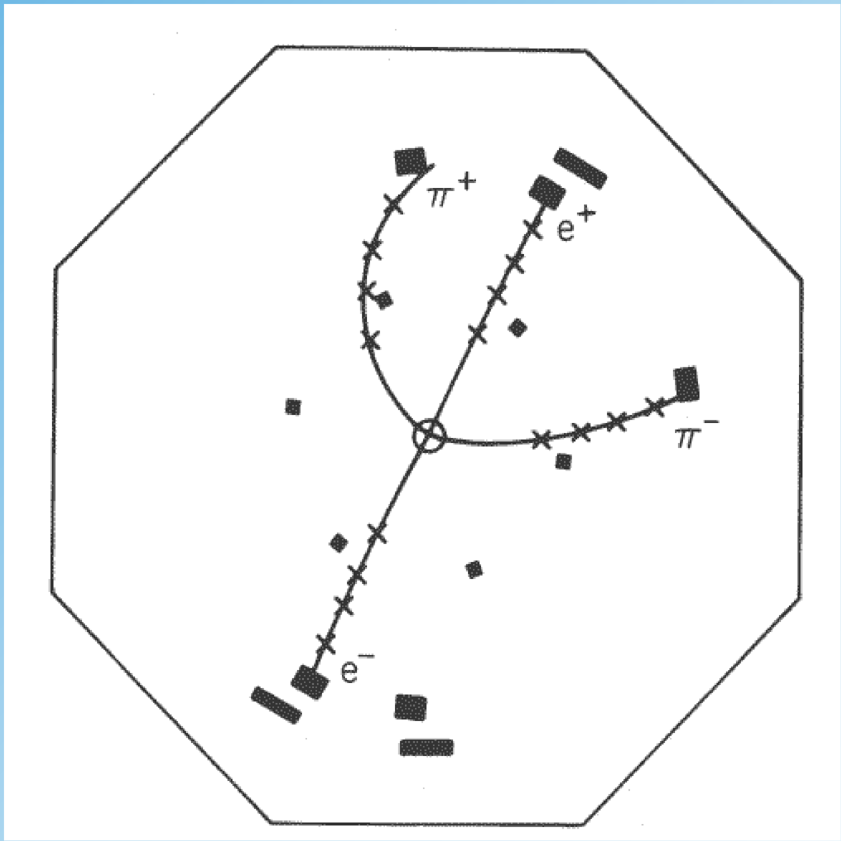


The Big e^+e^- Accelerators

- **SPEAR** (Stanford Positron Electron Accelerator Ring) at SLAC (1974-1990), $s^{1/2}$=3-8 GeV, Discovery of the Charm Quark
- **PETRA** (Positron Electron Tandem Ringanlage) at DESY (1978-1986), $s^{1/2}$=38 GeV, Discovery of Gluon-Jets
- **TRISTAN** at KEK, Japan (1986-1989) $s^{1/2}$=50-64 GeV (discovery of the “desert”)
- Large Electron-Positron Collider, Geneva (1988-2000): $s^{1/2}$=90 GeV (**LEP I**, Z-factory), $s^{1/2}$=200 GeV (**LEP II**, WW factory)
- Stanford Linear Accelerator at SLAC, Stanford (1991-1998) $s^{1/2}$=90 GeV (**SLC**, Z-factory)

SPEAR at SLAC

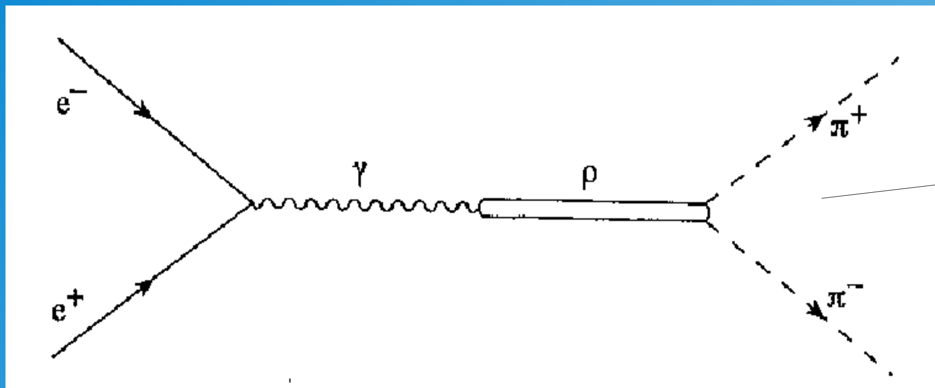
- Stanford Positron Electron Accelerator Ring (1974-1990), $s^{1/2}$=3-7 GeV, Discovery of the J/Psi



Discovery of the Charm Quark

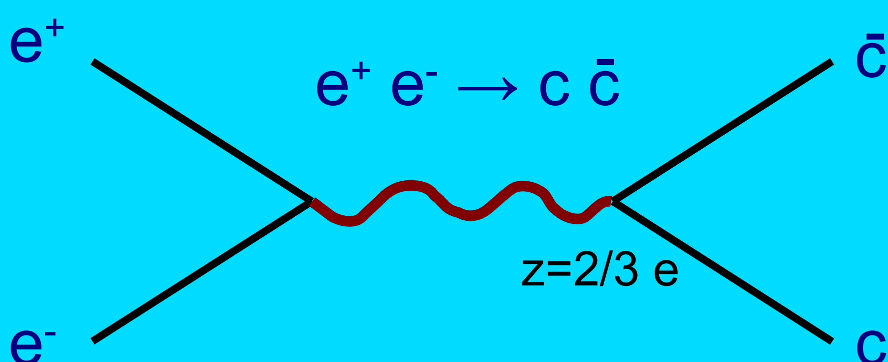
$$\Psi(2S) \to J/\Psi \pi^+ \pi^- \to e^+ e^- \pi^+ \pi^-$$

Quark-Pair Production



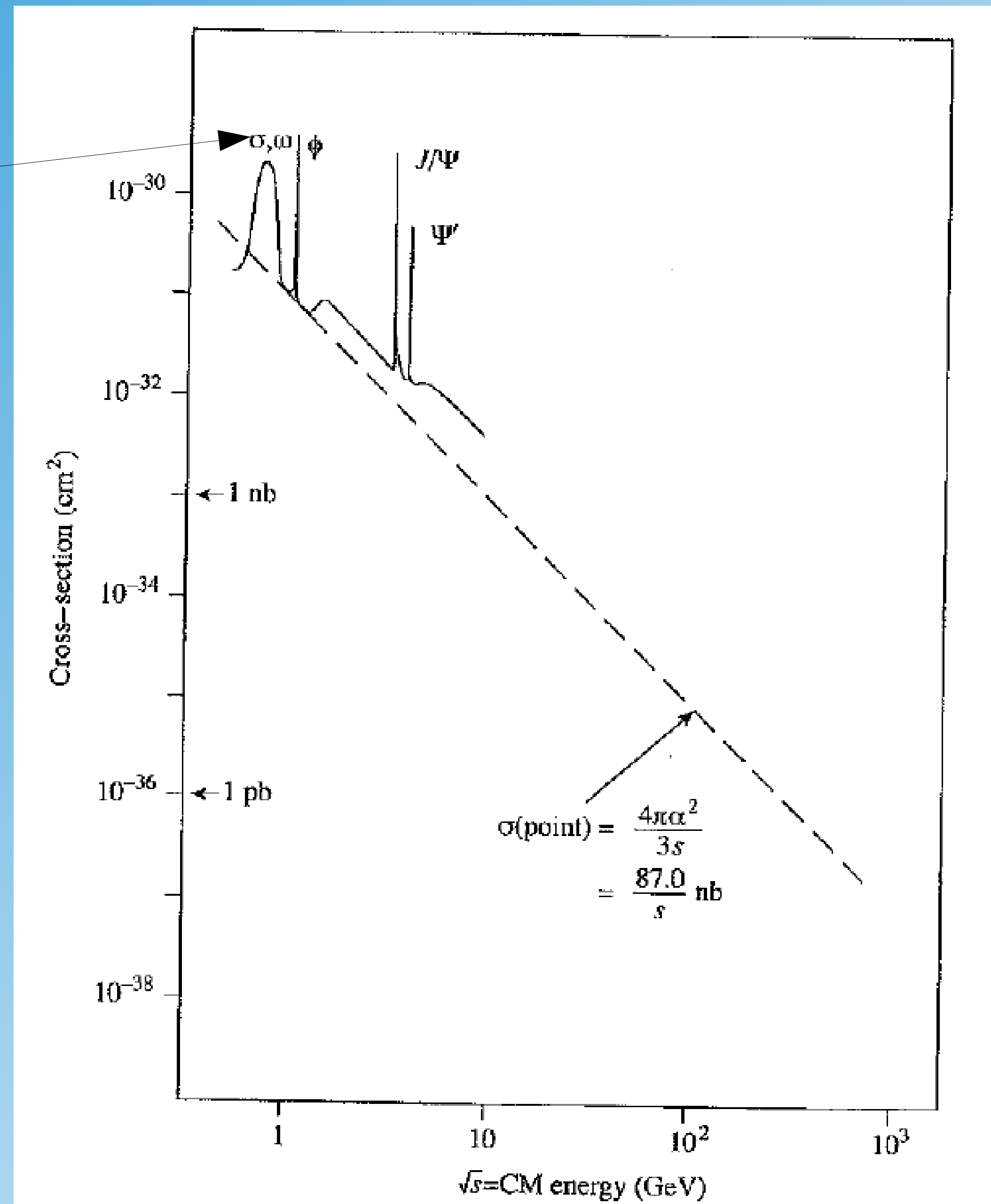
Resonant Rho production

➜ later



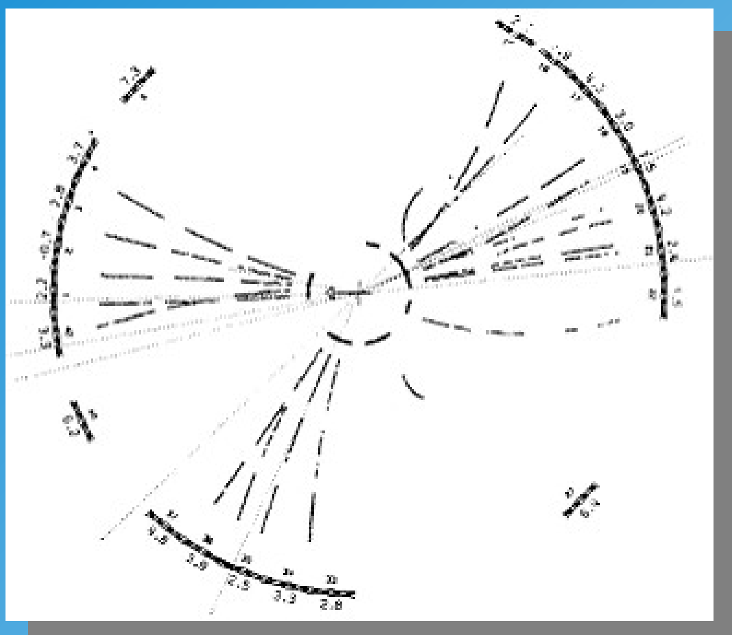
similar to muon-pair production

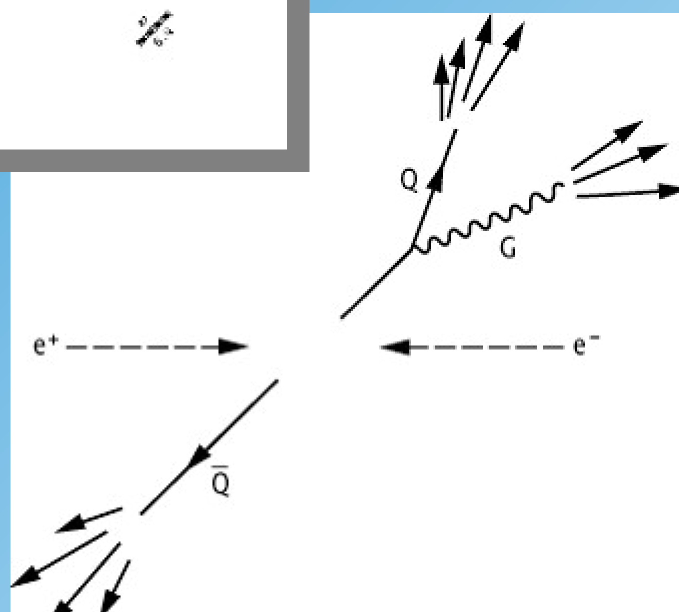
$$\frac{d\sigma}{d\Omega}(e^+ e^- \to c\bar{c}) = \frac{\alpha^2 q^2}{2s}\left(\frac{u^2+t^2}{s^2}\right)$$



PETRA at DESY

- Positron Electron Tandem Ring Anlage (1978-1986), $s^{1/2}$=38 GeV, Discovery of Gluon Jets





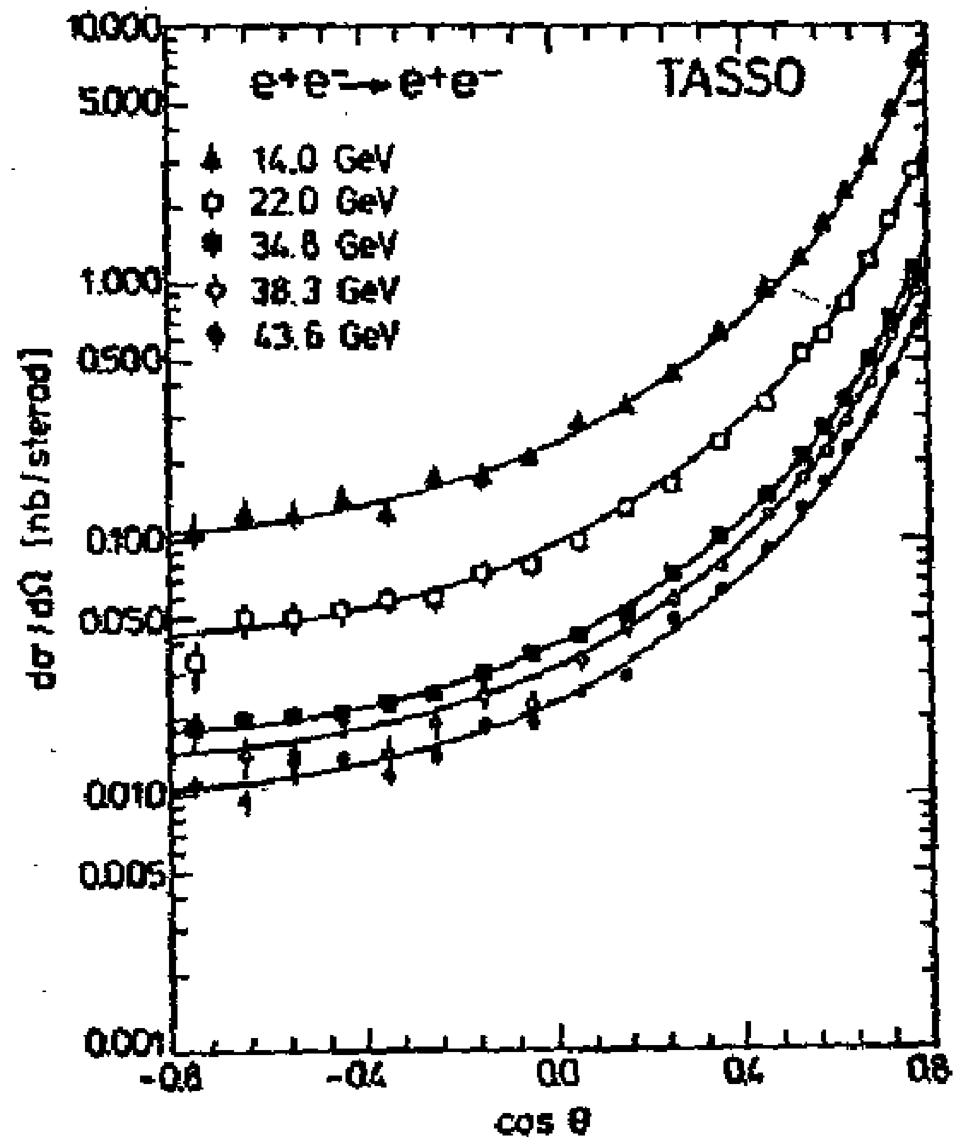
predicted by QCD!!!

Tasso at PETRA

<u>QED Test:</u>

Bhabha scattering

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{s}\left(u^2\left(\frac{1}{s}+\frac{1}{t}\right)^2+\left(\frac{t}{s}\right)^2+\left(\frac{s}{t}\right)^2\right)$$



Total Muon Pair Production C.S.

derivation:

$$\frac{d\sigma}{dt} = -\frac{2\pi\alpha^2}{s^2}\frac{t^2+u^2}{s^2}$$

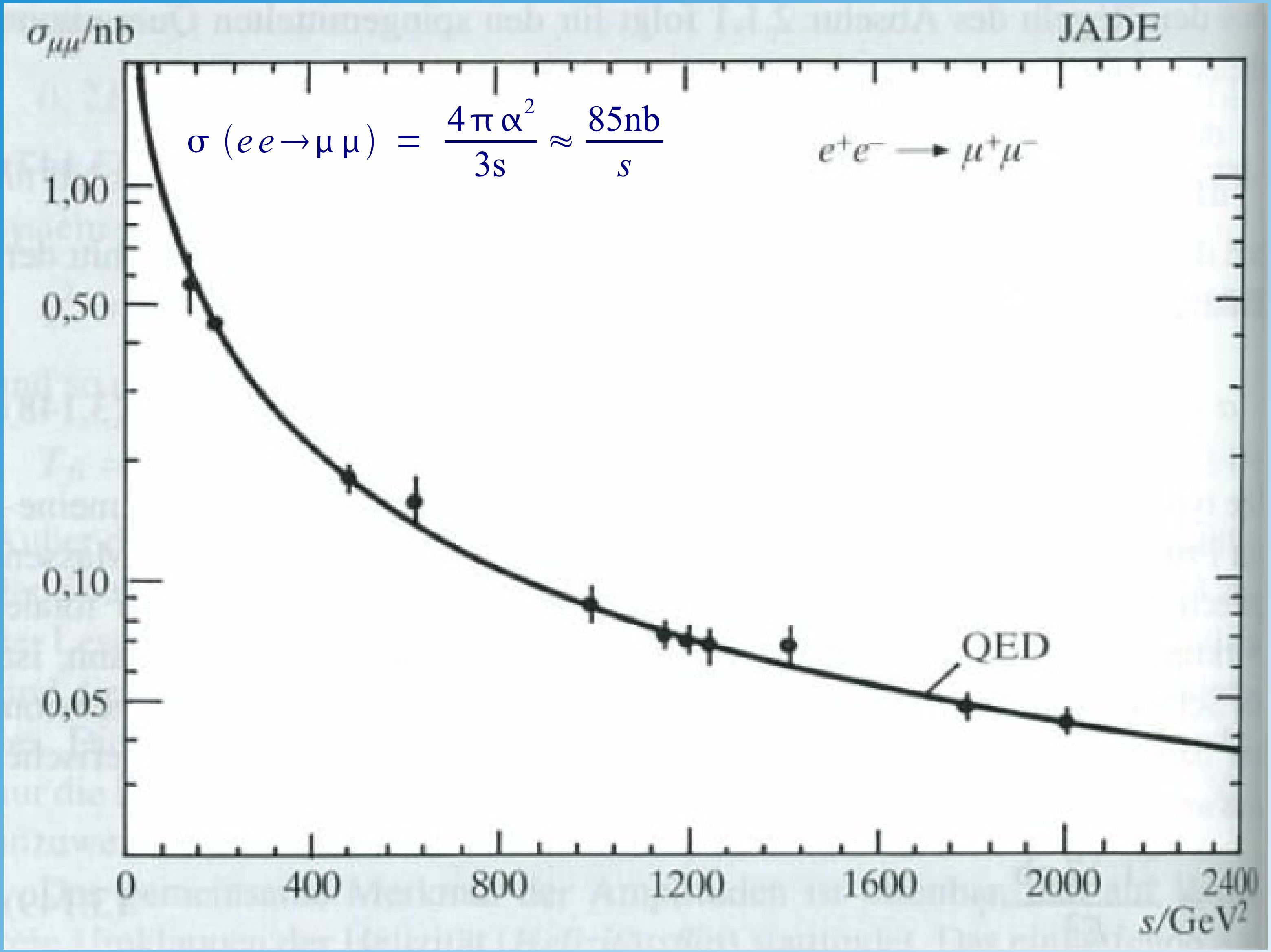
$$s+t+u=\sum m_i^2 \approx 0$$ (only two independent)

$$\rightarrow u^2=t^2+s^2+2ts$$

$$\frac{d\sigma}{dt} = -2\pi\alpha^2 \frac{2t^2+s^2+2ts}{s^4}$$

$$\sigma = -\int_{-s}^{0} 2\pi\alpha^2 \frac{2t^2+s^2+2ts}{s^4} = -2\pi\alpha^2 \frac{-2/3s^3-s^3+s^3}{s^4} = \frac{4\pi\alpha^2}{3s}$$

Myon Pair Production



PETRA accelerator (DESY)

Quark Pair-Production

Difficulty:

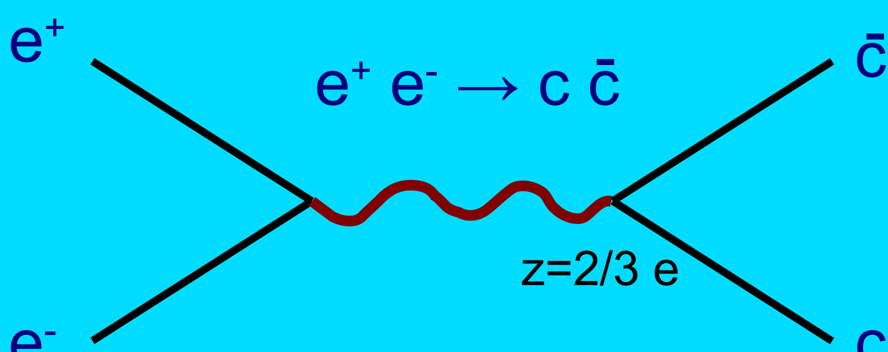
quarks and anti-quarks are experim. difficult to distinguish

$$t = -\frac{s}{2}(1 \mp \cos\theta)$$
$$u = -\frac{s}{2}(1 \pm \cos\theta)$$

different signs for quarks and antiquarks

$$t^2 + u^2 = \frac{s^2}{2}(1 + \cos^2\theta)$$

quarks and antiquarks averaged!



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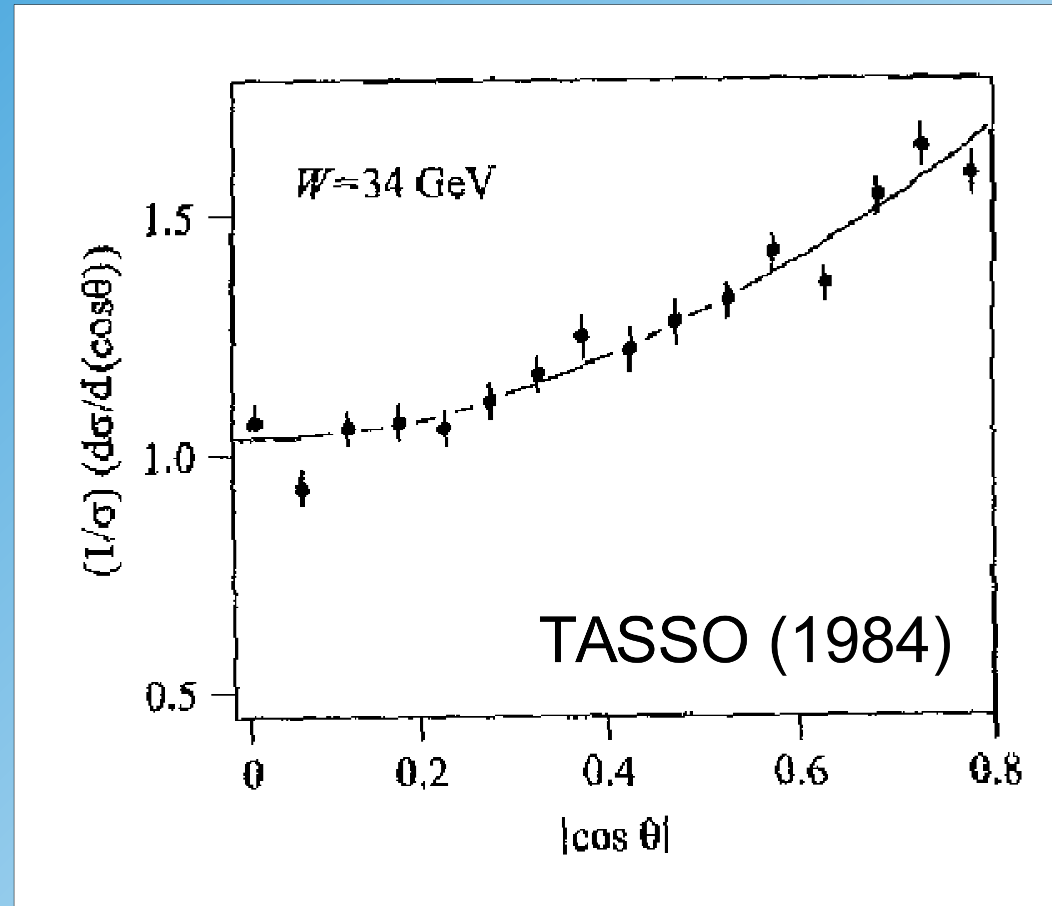
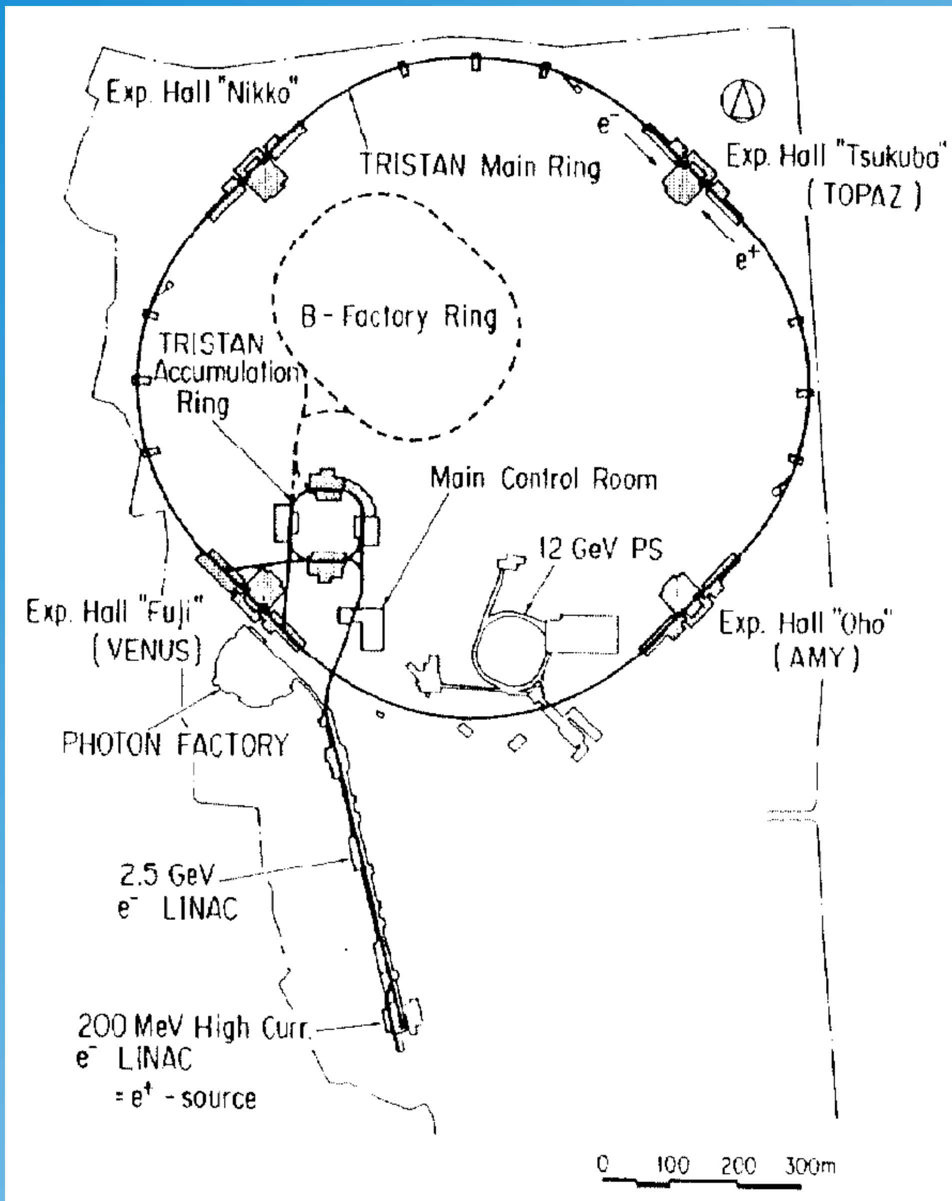
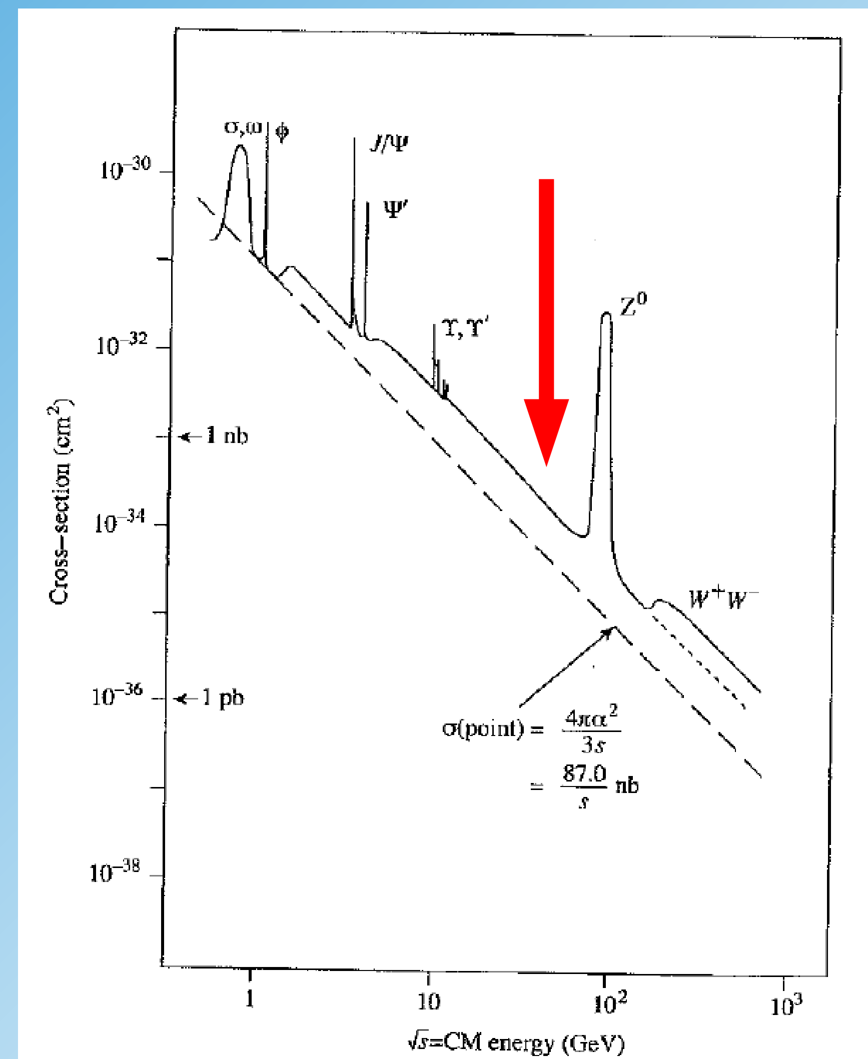


Tristan Collider at KEK

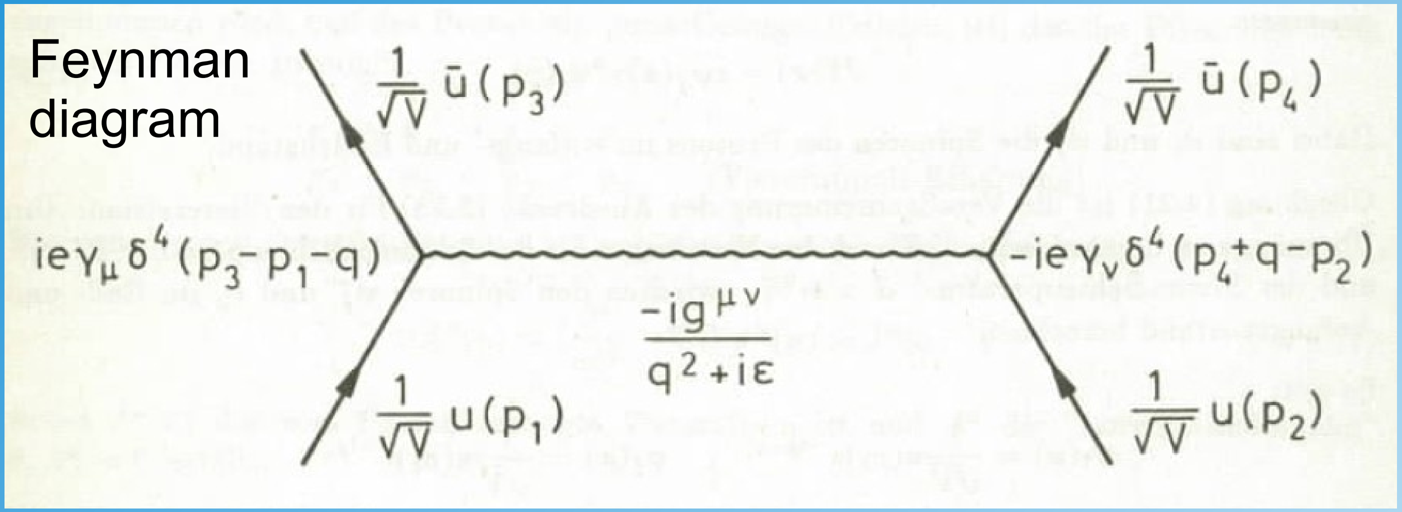
1986-1989: $s^{1/2}$=50-64 GeV

Search for the top-quark in the “desert”





QED Tests in e^+e^- collisions



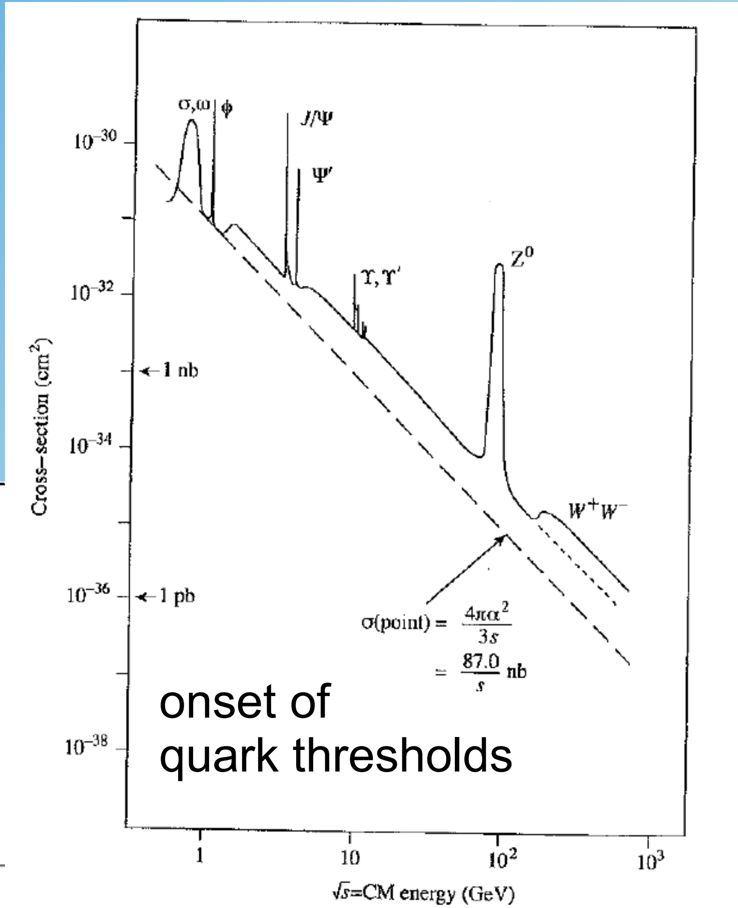
Possible tests:

- universality of charges (leptons, quarks, ...)
- energy dependence of coupling ("running")
- test of perturbation theory
- Lorentz structure of coupling
- propagator effect → new physics
- test crossing symmetries (→ gauge invariance)

Measurement of R_{had}

Measurement of Quark Charges:

$$R = \frac{e^+ e^- \rightarrow hadrons}{e^+ e^- \rightarrow \mu^+ \mu^-}$$



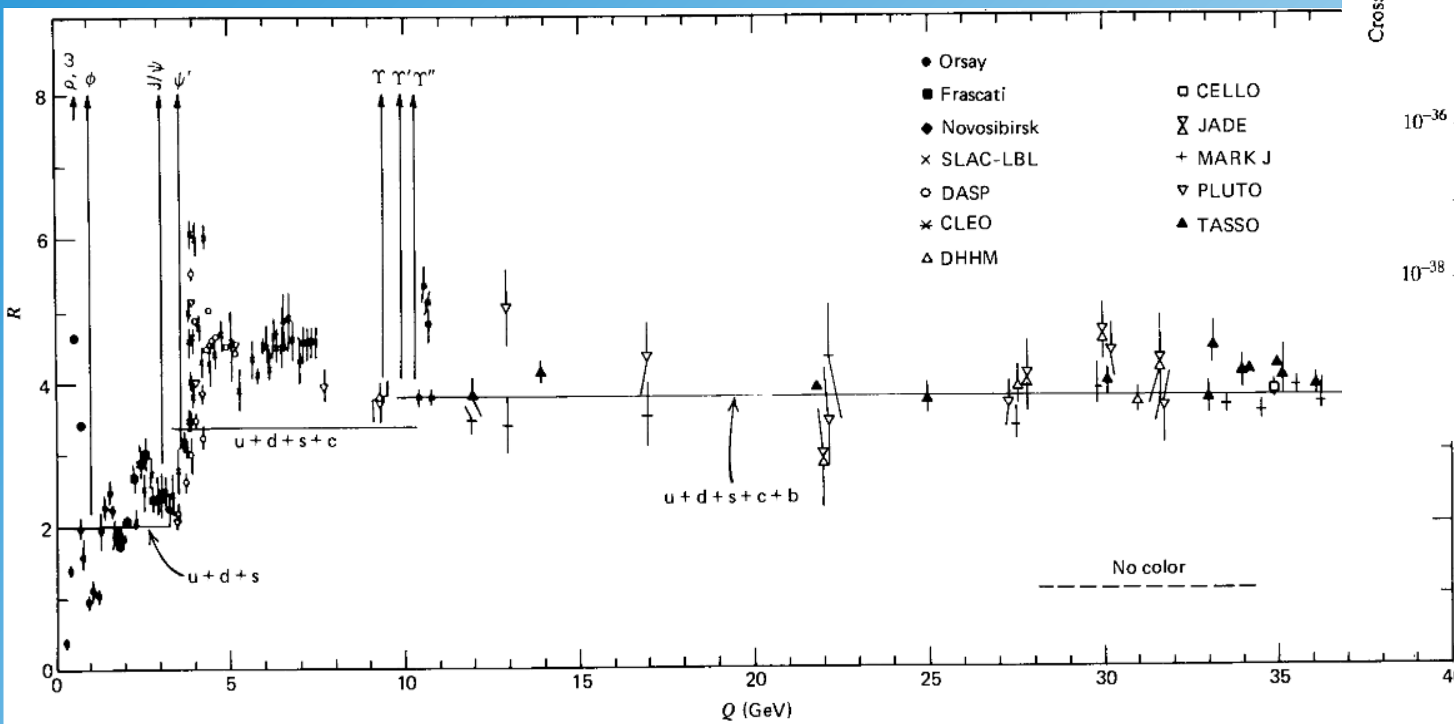
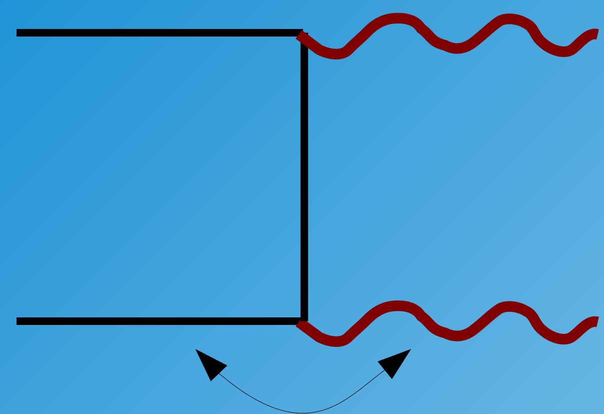


Fig. 11.3 Ratio R of (11.6) as a function of the total e^-e^+ center-of-mass energy. (The sharp peaks correspond to the production of narrow 1^- resonances just below or near the flavor thresholds.)

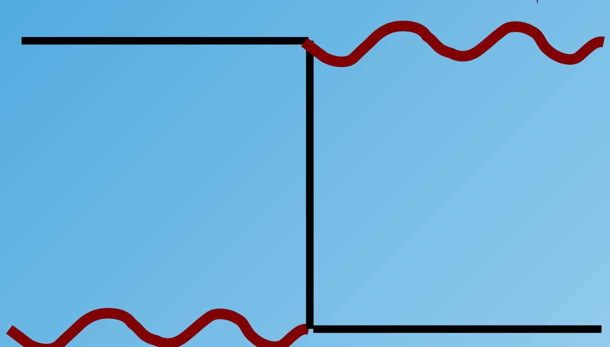
Crossing Symmetries

$$\frac{d\sigma}{d\Omega}(e^+ e^- \to \gamma\gamma) = \frac{\alpha^2}{2s}\left(\frac{u}{t} + \frac{t}{u}\right)$$



$$t \to -t$$
$$s \to u$$

$$\frac{d\sigma}{d\Omega}(\gamma e^- \to \gamma e^-) = \frac{\alpha^2}{2s}\left(\frac{-s}{u} + \frac{-u}{s}\right)$$



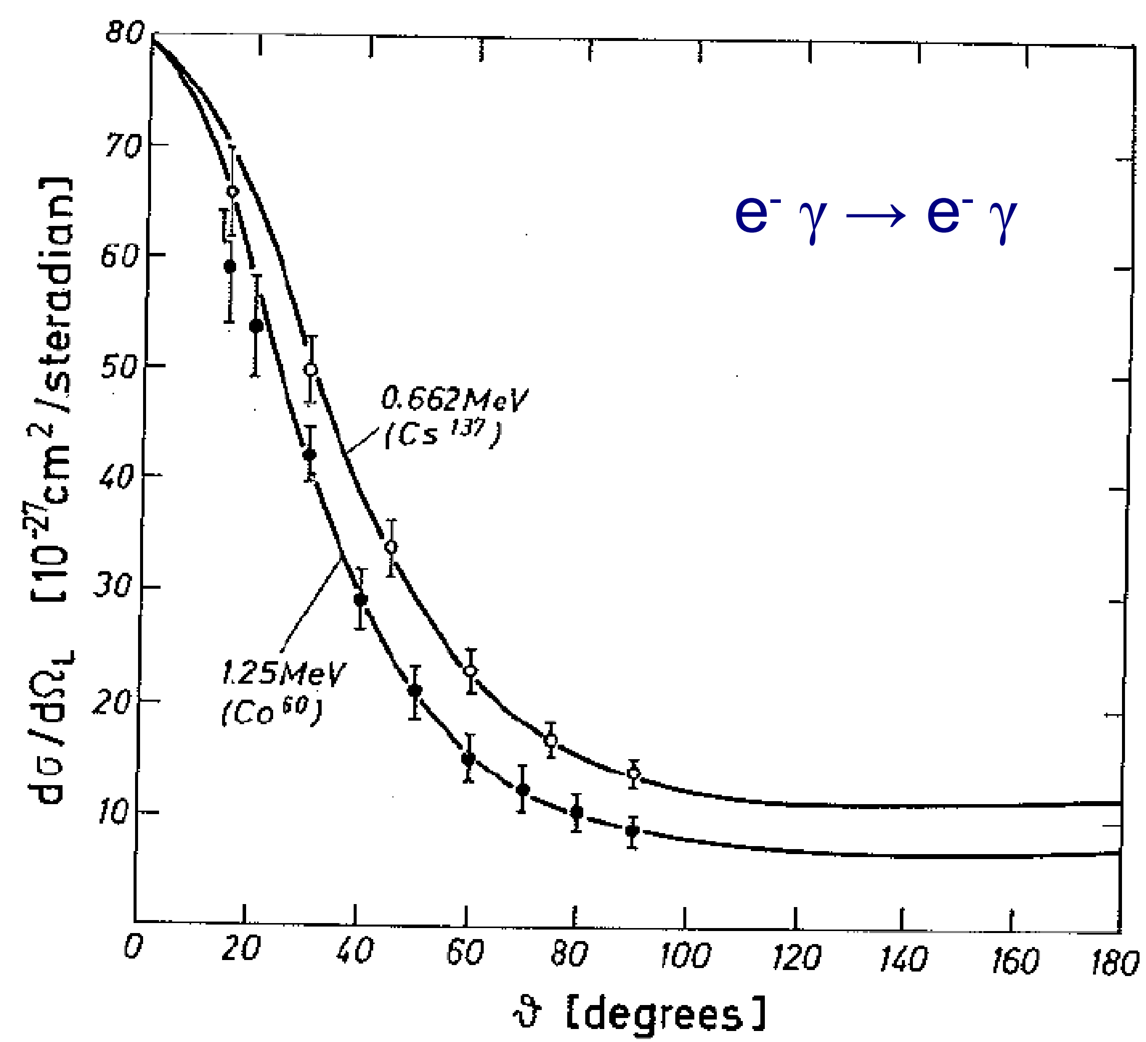
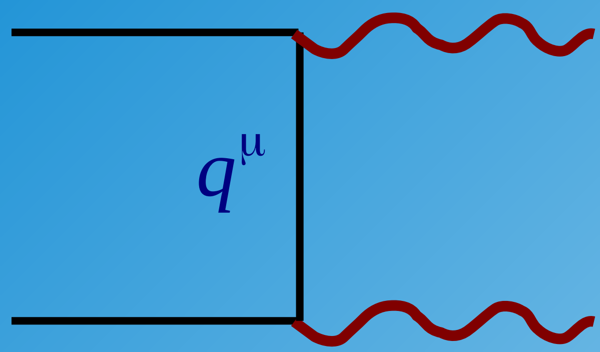


Figure 10.7 Experimental results for Compton scattering. The curves correspond to the Klein–Nishina formula (10.41) for photon energies $\omega = 0.662$ MeV and $\omega = 1.25$ MeV. The experimental data are from Hofstadter (1949) and Bernstein (1956) (after Evans 1958).

The Low Energy Limit

Electromagnetic coupling at low energy:



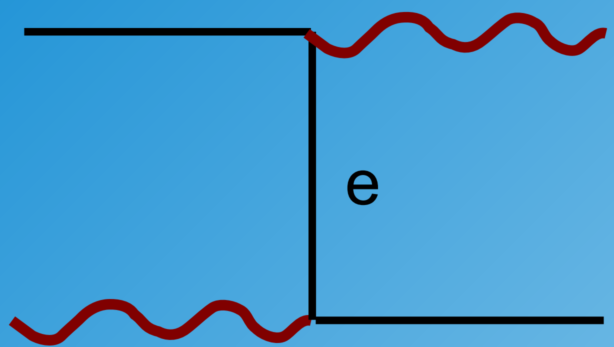
$$Q^2 = -t = -(q^\mu q_\mu) \rightarrow 0$$

Thompson scattering cross section $\sigma_t = \frac{8\pi}{3 r_e^2} = \frac{8\pi}{3}\left(\frac{\alpha \lambda_c}{2\pi}\right)^2$

used to determine α

The Low Energy Limit

Electromagnetic coupling at low energy:

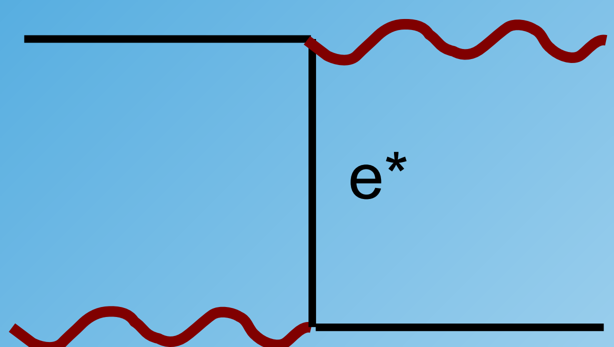


$$Q^2 = -t = -(q^\mu q_\mu) \rightarrow m_e^2$$

Thompson scattering cross section $\sigma_t = \frac{8\pi}{3 r_e^2} = \frac{8\pi}{3}\left(\frac{\alpha \lambda_c}{2\pi}\right)^2$

not dependent on energy! Used to determine α

General cross section:



$$\frac{d\sigma}{d\Omega}(\gamma e^- \rightarrow \gamma e^-) = \frac{\alpha^2}{2s}\left(\frac{-s}{u} + \frac{-u}{s}\right)$$

two terms

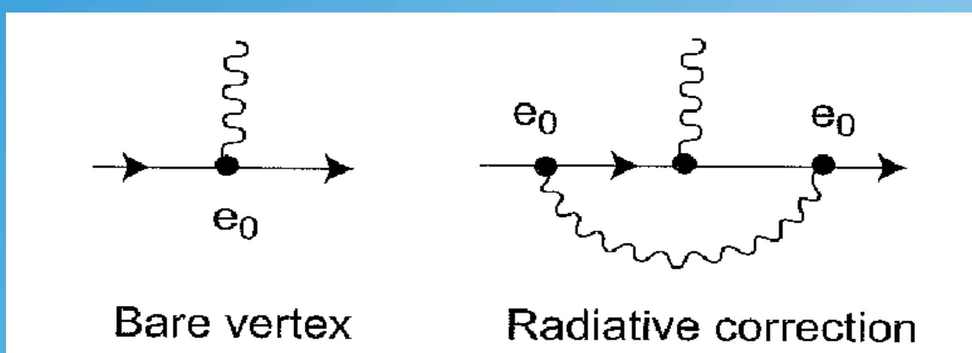
Running of alpha_{em}

$$\alpha(Q=0)=1/137 \longrightarrow \alpha(Q=90\,GeV)=1/128$$

self-energy corrections

alpha is not a constant!

vertex corrections



dressed charge!

- measure em. coupling for different (high) energies
- search for new physics effects at mass scale Λ

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma(QED)}{d\Omega}\left(1+\frac{s}{\Lambda^2}\right)$$

Lorentz-Structure of Electromagnetic Interaction

From Maxwell Equations:

$$\partial_\nu \partial^\nu A^\mu(x) = e\, J^\mu(x)$$

in QED: $\partial_\mu j^\mu_V = 0$ (conservation of currents)

electromagnetic interaction described by vector currents!

Also true at high energies?

Vector Current:

$$j^\mu_V = \bar{\psi}\gamma^\mu\psi$$

Axial-vector Current:

$$j^\mu_A = \bar{\psi}\gamma^\mu\gamma^5\psi$$

scalar coupling:

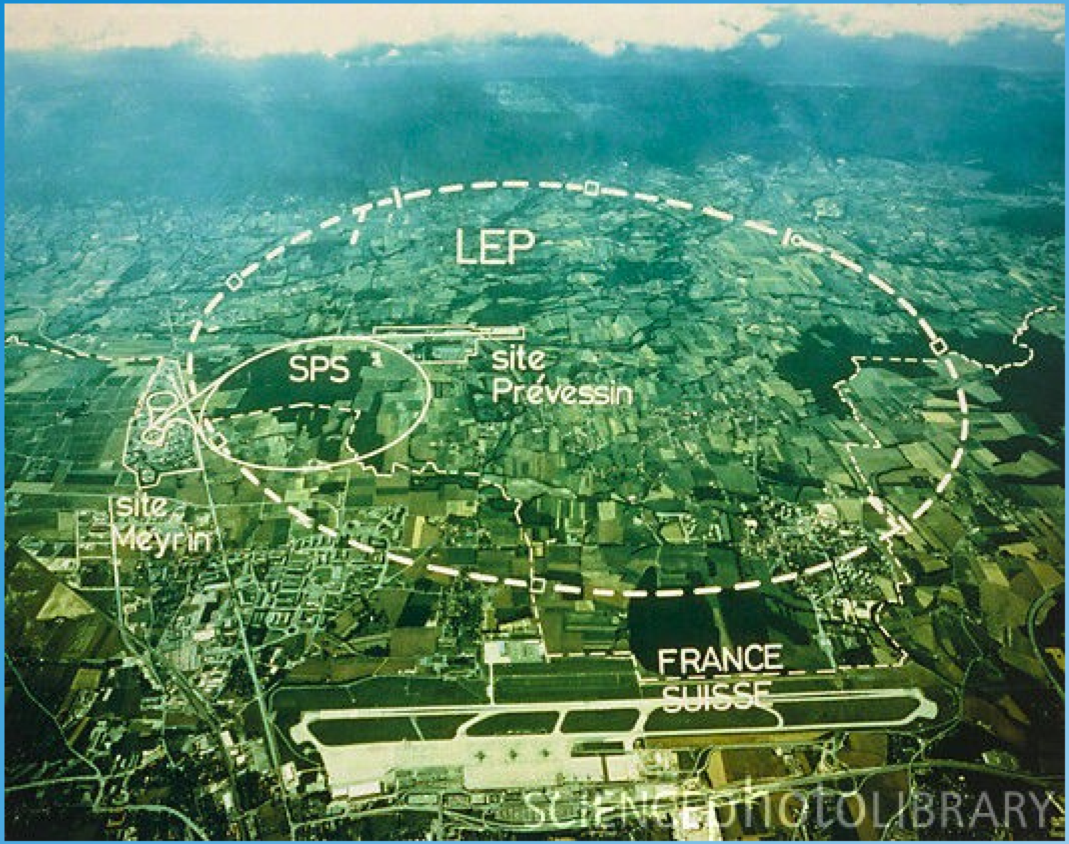
$$\lambda = \bar{\psi}\psi$$

pseudoscalar coupling:

$$\lambda = \bar{\psi}\gamma^5\psi$$

➔ lead in general to different angular distributions!

LEP Collider



biggest electron-positron collider with up to 200 GeV centre of mass

4 experiments:
ALEPH, DELPHI, L3, OPAL

LEP1: “Z-factory”

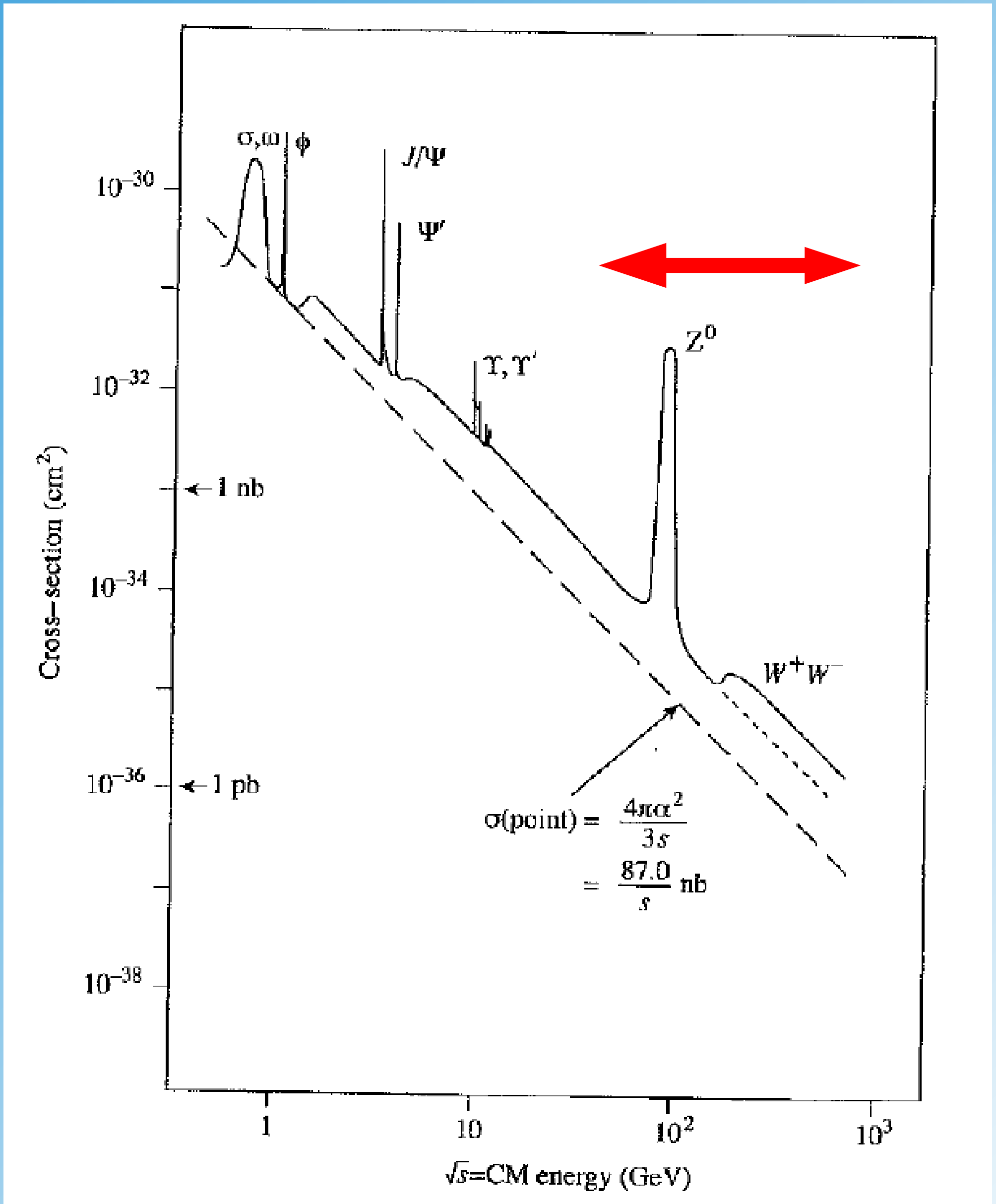
LEP2: “WW factory”

LEP Regime

Processes:

$e^+ e^- \rightarrow Z$

$e^+ e^- \rightarrow W^+ W^-$



at LEP energies radiative and electroweak effects play an important role!

International Linear Collider

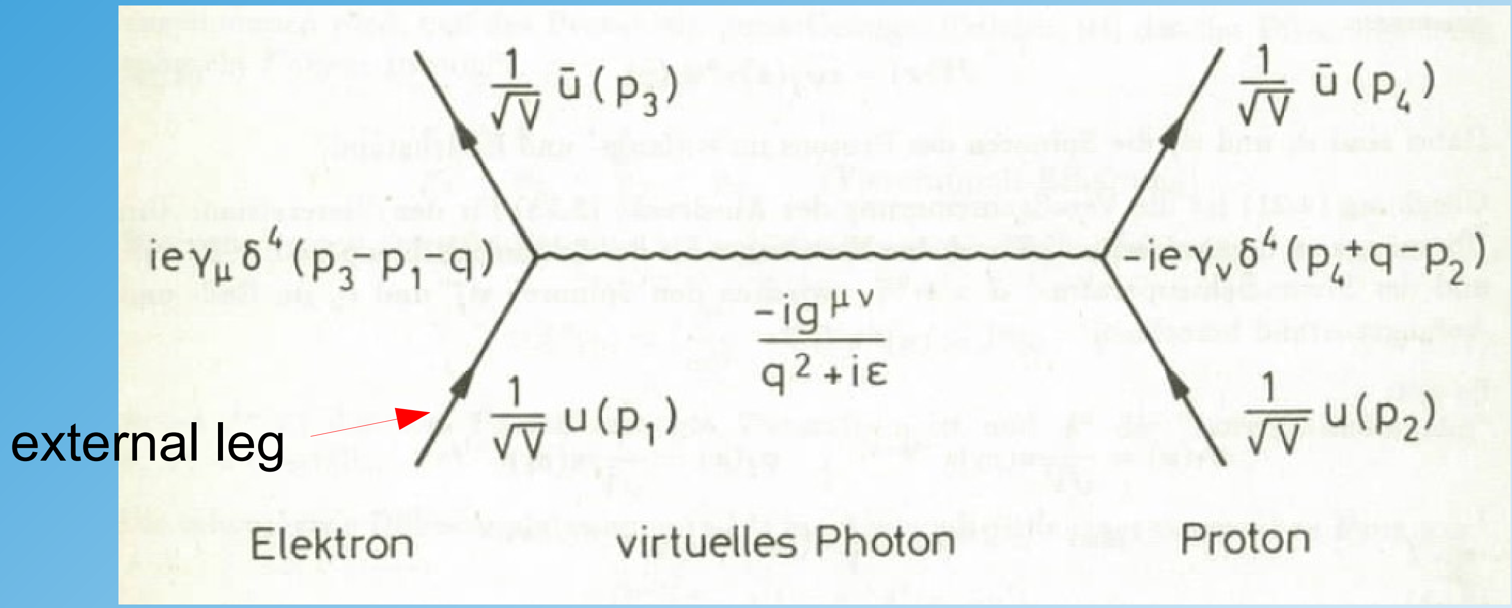
500 GeV electrons x 500 GeV positrons

Backup

Fermion-Fermion Scattering

$$S_{fi}^{(1)} = ie^2 \int \frac{d^4q}{(2\pi)^4}\, \delta^4(p_3 - p_1 - q)\, \delta^4(p_4 + q - p_2)\, (2\pi)^8 \cdot \frac{1}{V^2} \overline{u}(p_3)\gamma_\mu u(p_1) \cdot \frac{-g^{\mu\nu}}{q^2 + i\epsilon} \cdot \overline{u}(p_4)\gamma_\nu u(p_2)$$

particle-particle (here electron-proton) scattering



lowest oder perturbation theory: leading order graph (Born)

Gamma Matrices I

Gamma matrices γ^μ are chosen such that γ^0 is hermitian while γ^k (k=1,2,3) are anti-hermitian

$$(\gamma^0)^\dagger = \gamma^0, \ (\gamma^0)^\mu = 1,$$

$$(\gamma^k)^\dagger = -(\gamma^k), \ (\gamma^k)^\mu = -1 \ (k=1,2,3)$$

We define γ^5 as the hermitian matrix: $\gamma^5 = i\,\gamma^0\gamma^1\gamma^2\gamma^3$, $\quad (\gamma^5)^2 = 1$,

The 4x4 gamma matrices can be represented by (representation where γ^0 is diagonal):

$$\gamma^k = \begin{pmatrix} 0 & \underline{\sigma}^k \\ -\underline{\sigma}^k & 0 \end{pmatrix}, \quad \gamma^0 \stackrel{\text{def}}{=} \beta = \begin{pmatrix} \underline{1} & 0 \\ 0 & -\underline{1} \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & \underline{1} \\ \underline{1} & 0 \end{pmatrix}$$

(note several representations exist)

With the 2x2 Pauli matrices:

$$\underline{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \underline{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \underline{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Gamma matrices anti-commute: $\gamma^i\gamma^k + \gamma^k\gamma^i = 0 \quad$ for $i \neq k$

Gamma Matrices II

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

(in other representations
g^5 is diagonal)