Master seminar: Particle tracking and identification at high rates
Report: An introduction to track reconstruction in collider experiments

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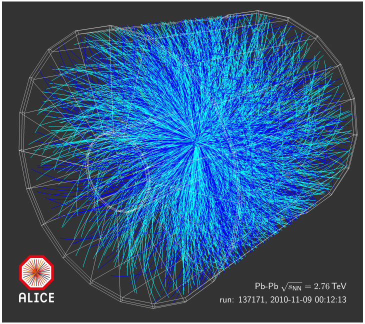
Introduction: particle tracks in collider experiments

Big European Bubble Chamber (CERN)



From this...

to this!

In large collider experiments: particles leave hits on detector layers → huge amount! Goal: **proper linking** of hits.
Big difficulties: high multiplicity of tracks, hit uncertainties, physical effects (multiple Coulomb scattering, measurement errors...), hit ambiguities (geometry of the detector...).

Overview

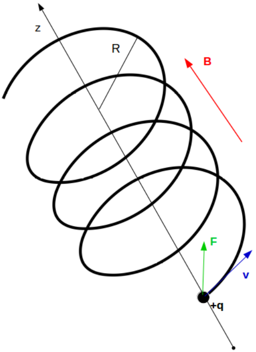
- Particle in field $B \rightarrow$ Lorentz force: $F_L = q(E + v \times B)$. $q :=$ charge, $E :=$ electric field, $v :=$ velocity.
- Case of most collider experiments: $E = 0$, $B = (0, 0, B_z)$:

$$\Rightarrow F = qvB_z$$

$$\leftrightarrow \frac{mv^2}{R} = qvB_z \text{ (in circular motions)}$$

$$\leftrightarrow \frac{p}{q} = \frac{1}{k} B_z .$$

With $R = \frac{1}{k}$, $k :=$curvature.



Introduction: particles in a magnetic field

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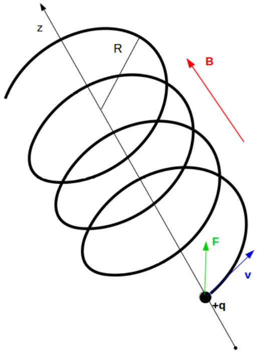
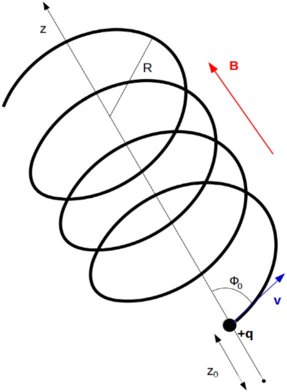


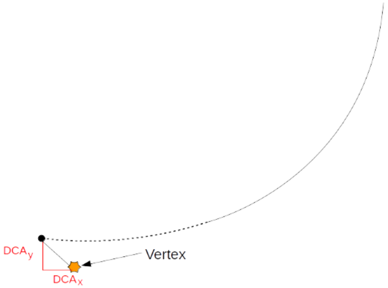
Conclusion

One can deduce *rigidity* (momentum over charge) of particle with known magnetic field, and unknown curvature.

Trajectory of a primary particle: helix parametrized by $(k, \eta, \phi_0, z_0, d_0)$ with:

- Pseudorapidity $\eta = -\ln[\tan(\theta_0/2)]$,





Introduction: Parametrization

Trajectory of a primary particle: helix parametrized by $(k, \eta, \phi_0, z_0, d_0)$ with:

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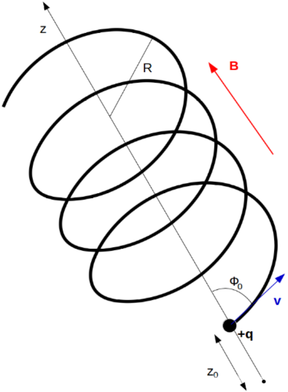

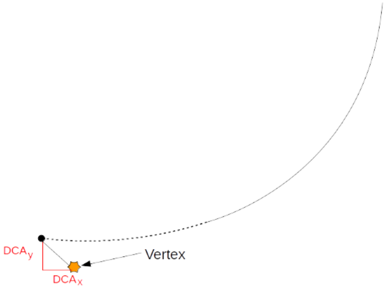

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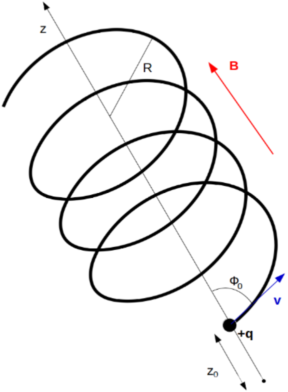

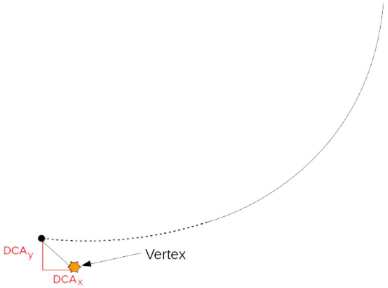

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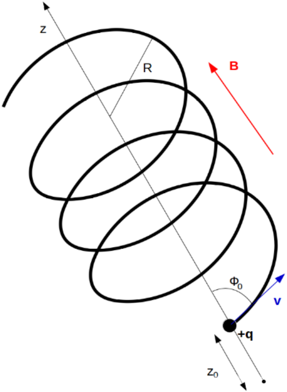
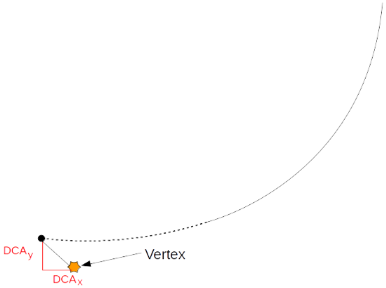
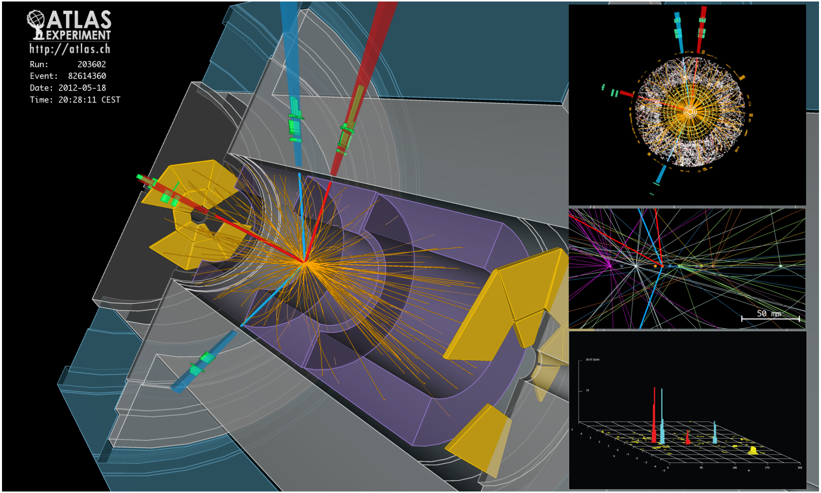
- Pseudorapidity $\eta = -\ln[\tan(\theta_0/2)]$,
- Azimuthal angle of the initial momentum vector ϕ_0 (θ_0 polar angle),
- Initial position on the z (beam) axis z_o,
- Distance of closest approach $d_0 = \sqrt{\mathrm{DCA}_x^2 + \mathrm{DCA}_y^2}$ (often called DCA_{xy}).





FITTING


ATLAS
EXPERIMENT
http://atlas.ch
Run: 203602
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Date: 2012-05-18
Time: 20:28:11 CEST
50 mm

Finds circle curvature, direction and position parameters of a track with least square method. Assumptions are:

- Only spatial hit uncertainties (no multiple scattering, no inhomogeneous magnetic field),
- All hits are part of the track.

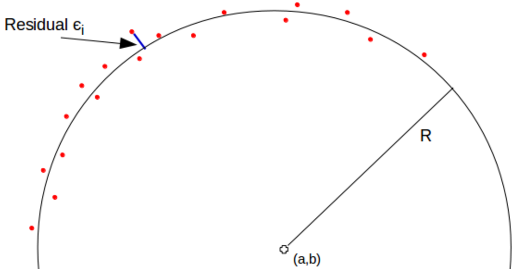
Least square method is the minimization of:

$$\chi^2 = \sum_i w_i \epsilon_i^2$$

where w_i := weights and ϵ_i := measurement residuals:

$$\epsilon_i = \pm\left[\sqrt{(x_i - a)^2 + (y_i - b)^2} - R\right].$$

x_i and y_i := measured coordinates.



The Karimaki fit

Minimizing problem solved with approximation:

$$\epsilon_i \approx \pm\frac{1}{2}R^{-1}\left[(x_i-a)^2+(y_i-b)^2-R^2\right]$$

under the condition that $|\epsilon_i \ll R|$.

⚠Many drawbacks: non-Gaussian...

Solution:

- Use curvature k instead of R (⚠most important change!),
- Distance of closest approach d_0 and direction of propagation Φ_0,
  - Used because not correlated and good straight track limit behavior (when $R \to \infty$).

Equation becomes:

$$\epsilon_i = \frac{1}{2}kr_i^2-(1+kd_0)r_i\sin(\Phi_0-\phi_i)+\frac{1}{2}kd_0^2+d_0$$

with r_i and ϕ_i polar coordinates of i.

The Karimaki fit

Parameters are Gaussian and behave well at the straight track limit.

Rewrite now ϵ_i as:

$$\epsilon_i = (1 + kd_0)\eta_i \,,$$

with $\eta_i = kr_i^2 - r_i \sin(\Phi_0 - \phi_i) + \delta$, $\kappa = \frac{1}{2}\frac{k}{1+kd_0}$ and $\delta = \frac{1+\frac{1}{2}kd_0}{1+kd_0} d_0$.

$$\Rightarrow \chi^2 = (1 + kd_0)^2 \widehat{\chi^2} \,,$$
$$\widehat{\chi^2} = \sum_i w_i \eta_i^2 \,.$$

In practice, minimizing $\widehat{\chi^2}$ with k, d_0, Φ_0 $\rightarrow$ minimizes the true χ^2 !

Procedure:

1. minimize the $\widehat{\chi^2}$ with respect to κ, Φ_0 and δ,

Result:
State vector (k, d_0, Φ_0).

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Procedure:

1. minimize the $\widehat{\chi^2}$ with respect to κ, Φ_0 and δ,
2. solve the curvature and distance parameters k and d_0 by inverting their expressions.

Result:

State vector (k, d_0, Φ_0).

Parameters are Gaussian distributed, error estimation done in respect to that.
Error matrix V:

$$(V^{-1})_{jk} = \sum_i w_i \frac{\partial \epsilon_i}{\partial p_j} \frac{\partial \epsilon_i}{\partial p_k},$$

with $p_{j,k} :=$ parameter.

- Corrections on error on the fitting of k, Φ_0 and d_0, due to the minimization of $\widehat{\chi^2}$ and not χ^2, are added.
  - Can be implemented simply and computed quickly.

The Karimaki fit: event reconstruction

In event reconstruction: fit first particle trajectories, then determine vertex from it → **iterative procedure**.

Need:

- algorithm which calculates parameters and errors
  - according to a new vertex (x'_0, y'_0).

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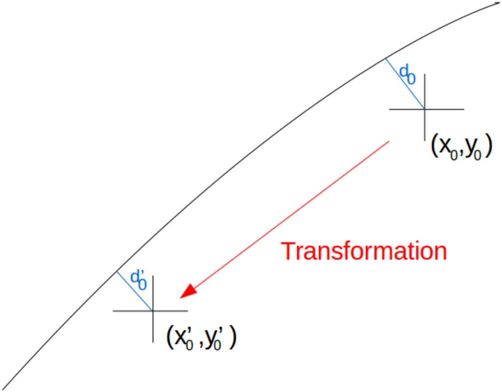
Solution → transformation:

- $d_0 \rightarrow d'_0$,
- $\Phi_0 \rightarrow \Phi'_0$,
- k is invariant.



Φ_0 is also transformed, not shown here since not on the same plane.

The Karimaki fit: event reconstruction

Mathematically (not very important for us):

$$\Phi_0' = \arctan\frac{B}{C},\ \ d_0' = \frac{A}{1+U}$$

k is invariant.

$$\begin{aligned}
A &= 2\Delta_\perp + k(\Delta_\perp^2 + \Delta_\parallel^2),\\
B &= k(x_0 - x_0') + (1 + kd_0)\sin\Phi_0,\\
C &= -k(y_0 - y_0') + (1 + kd_0)\cos\Phi_0,\\
U &= \sqrt{1 + kA},\\
\Delta_\perp &= (x_0 - x_0')\sin\Phi_0 - (y_0 - y_0')\cos\Phi_0 + d_0,\\
\Delta_\parallel &= (x_0 - x_0')\cos\Phi_0 + (y_0 - y_0')\sin\Phi_0.
\end{aligned}$$

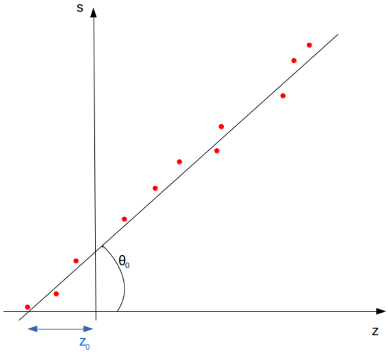
The new error matrix is then:

$$V' = JVJ^T$$

where J is simply the Jacobian derivative matrix.

The Karimaki fit: from circle fitting to helix fitting

- ⚠Karimaki: fits a circle, not a helix!
- To get a helix, need θ_0:



$$\tan\theta_0 = \frac{\Delta s}{\Delta z}$$

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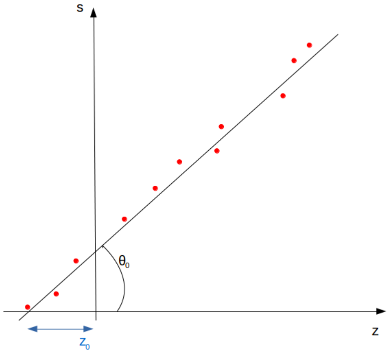


$\tan\theta_0 = \frac{\Delta s}{\Delta z}$

Conclusion

The Karimaki fit solves a nonlinear least squares problem with an elegant solution:

- Gaussian behaved fitted parameters,
- Computes value of the χ^2 quickly (one of the quickest fits!),
- Calculates the covariance matrix of the fitted parameters.

Physical effects

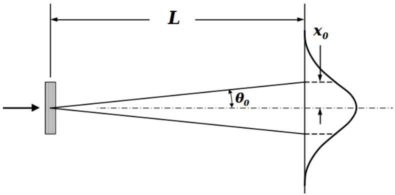
Most dominant effect that will modify particle path:

- Multiple Coulomb scattering,
- Described roughly by a Gaussian distribution.

Standard deviation of the planar scattering angle (Highland equation):

$$\theta_{\text{estimated}} := \theta_0 = \frac{13.6\,\text{MeV}}{\beta c p} Z \sqrt{L/X_0}\,[1 + 0.038 \ln(L/X_0)]$$

Where $p :=$ momentum, $\beta c :=$ velocity, $Z :=$ charge in electron charge units and $L/X_0 :=$ thickness of the scattering material in radiation length.



Hit uncertainties→ also affects proper track reconstruction.

- Drift chamber resolution: about 100 μm
- Silicon detector resolution: about 10 μm

The Kalman filter: case of no present magnetic field

The Kalman filter and the Karimaki fit: same basic principles, but Kalman filter has more advantages.

Random process noise w (e.g Coulomb scattering) is added to the transformation:

$$x_i = F_{i-1} x_{i-1} + w_{i-1}\,.$$

x_i := state vector, F_{i-1} := transformation.

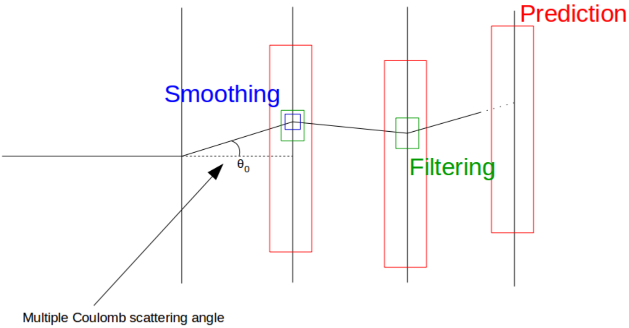
State vector not measured directly. Quantities m_k are measured by the detector k, related to the state vector with linear function h_k:

$$m_k = h_k(x_k) + \epsilon_k\,.$$

- ϵ_k is a measurement noise,
  - Implementation of discrepancy between a measured and true value of the detector.
- ⚠This part extremely dependent on the experiment!

The Kalman filter: the three operation steps

1. **Filtering**: $x_k = x_k^{k-1} + K_k \cdot (\text{residual of prediction})$,
   - $K_k :=$ Kalman gain matrix.



Operations also applied on the covariance matrix! Also update of $\chi 2$ after each iteration.

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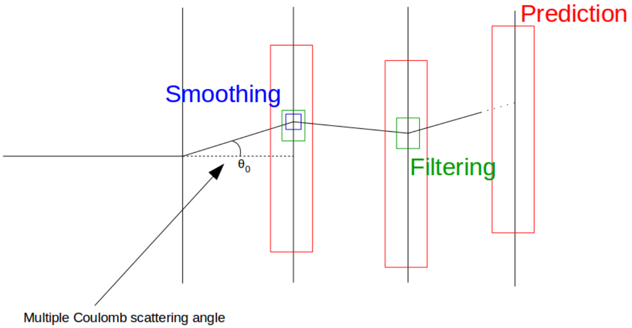

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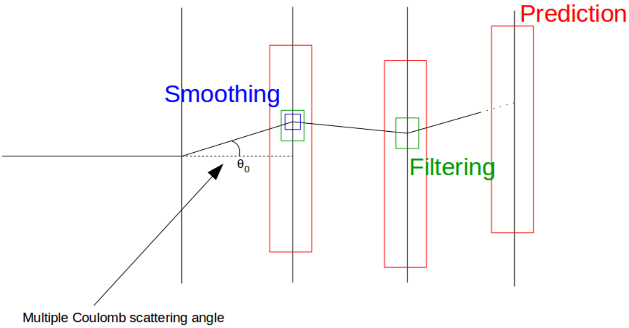
2. **Prediction**: $x_{k+1}^k = F_k x_k$.
3. **Smoothing**: $x_k^n = x_k + A_k \cdot (x_{k+1}^n - x_{k+1}^k)$,
   - A_k := Smoother gain matrix.



Operations also applied on the covariance matrix! Also update of $\chi 2$ after each iteration.

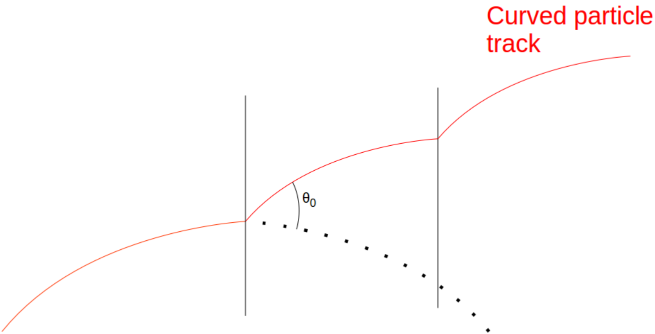
The extended Kalman filter

In magnetic field, track propagator is nonlinear:

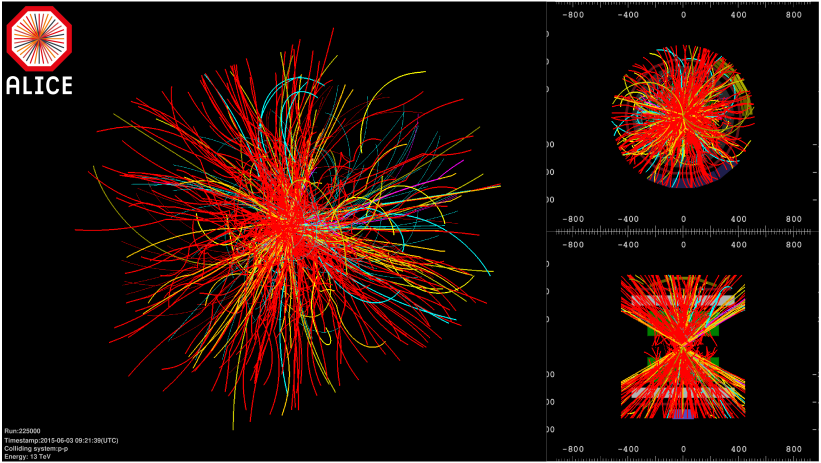
- Has to be approximated by a linear function,
- Done in the usual way by replacing it by first two terms of Taylor expansion.

→ $x_{k+1}^{k} = F_k x_k$ (extrapolation of state vector) becomes:

$$x_{k+1}^{k} = f_k(x_k)$$

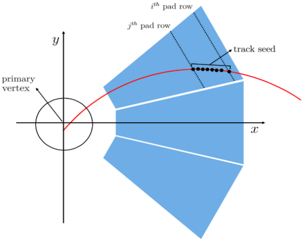


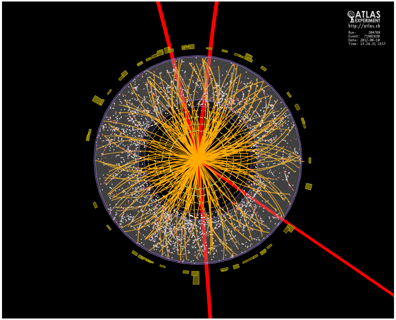
PATTERN RECOGNITION



First step: *Seed finding*, in regions of low track density (outer detector regions).

Schematic view of the seeding algorithm.





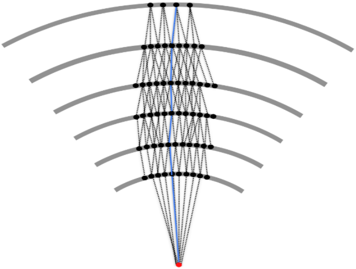
Track seed: neighbor clusters roughly compatible with a track shaped as a helix pointing to the interaction.

Pattern recognition: track extension

Next step: track extension.

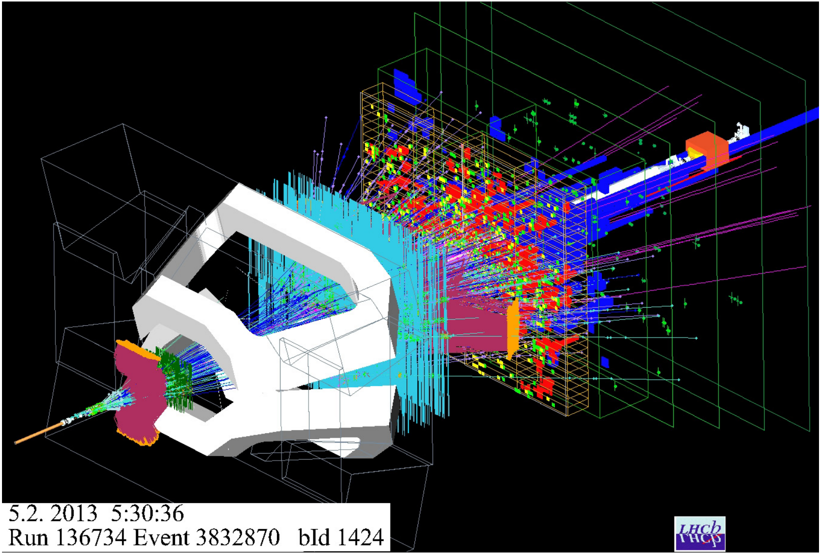
- When another layer is reached, a "tree" of possible extensions calculated, most probable candidate eventually selected with Kalman filter,
- When a cluster is associated with existing track, track parameters and covariance matrix are updated,
- Tracks usually refitted by the Kalman filter in the outward direction,
- Sometimes a final step of second inward propagation done.

Tree of possible extensions to a track.



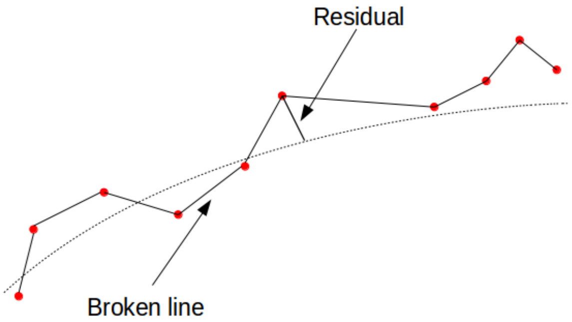
Prolongation hypotheses are made, taking into account multiple scattering. The best track candidate (in blue) is chosen according to the quality of the whole track.

CONCLUSION



Conclusion: new tracking techniques

- Kalman filter: most used→ implementation of multiple scattering effects, measurement errors, (inhomogeneous) magnetic field.
- Has some drawbacks: very complicated implementation, iterative→ starts from the beginning when something wrong.
- New techniques available→ *broken line fit*:

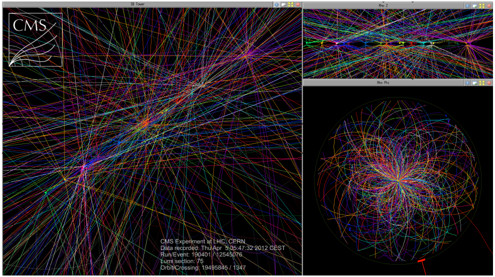


Fast track fit (Karimaki)+detailed fit (reduction of residuals) allowing *kinks*.

Conclusion

What did we learn today?

- Particles in large collider experiments follow a helix.
- Fitting a circle or a helix is difficult (nonlinear problem) $\rightarrow$ use new parametrization!
- Physical effects make track reconstruction even more difficult.
- Kalman filter takes physical effects into account and
  - Predicts,
  - Filters,
  - Smooths the state vectors.
- A lot more of methods and techniques available: broken line fit, Hough transformations, multiple scattering fit, use of templates...



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